EXCITATION OF HIGH NUMBERS HARMONICS BY CHARGE PARTICLES IN A TIME-PERIODIC ELECTRIC FIELD AND A SPACE-PERIODIC POTENTIAL

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Regular and chaotic dynamics of a charged particle, which moves in an external time-periodic electrical field and in a field of space periodic potential, are investigated. We have obtained a system of integro-differential equations, which describes the non-linear self-consistent dynamics of excitation of electromagnetic waves by flows of charged particles. The analytical and numerical analysis of a full self-consistent set of equations is carried out. The results of this analysis qualitatively well agree with the experimental data.

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1. INTRODUCTION

It is known that in a vacuum an oscillator effectively radiates high numbers of harmonics only if it has a large energy. So, for the synchrotron radiation the maximum of radiation is obtained at harmonics with number $\nu \sim \gamma^3$ [1]. Many authors (see, for example [2-6] and referenced therein) studied the radiation of relativistic particles in a periodically inhomogeneous medium. The interest to such radiation is conditioned by the fact that due to the Doppler effect there is a possibility to excite effectively the short-wave radiation $\lambda \sim d/\gamma^2$.

It is possible to name one more mechanism for short-wave radiation generation which does not require the use of high-energy beams. This is the radiation of high numbers harmonics by nonrelativistic oscillators moving in the periodically inhomogeneous medium as well as by charged particles moving in an external time-periodic electrical field, and in the field of an external, space-periodic, potential [7-9].

Meaning features of the radiation mechanism, which we investigate, one can expect the creation of sources of an intensive coherent X-radiation with the wavelength $\lambda \approx d/\beta$.

In this paper we have investigated, analytically and numerically, the capabilities of excitation of high numbers harmonics by ensembles of charged particles. We carried out the analytical and numerical analysis of a full self-consistent set of equations. The analytical results are in good agreement with the numerical results, and qualitatively well agree with the data of experiments [10].

2. MOTION OF A CHARGED PARTICLE IN A FIELD OF STRATIFIED - INHOMOGENEOUS POTENTIAL AND IN A PERIODIC ELECTRICAL FIELD

Let a charged particle to move in the external time-periodic electrical field $E(0,0,E_z)$ $E_z(t) = -E_{ext} \cdot \sin(\omega_{ext} \cdot t)$ and in the field of periodic potential $U(z) = U_0 + g \cdot cos(k_z \cdot z)$.

The nonrelativistic equations of electron motion in such a field can conveniently be presented as:

$$\begin{cases} dV_z/d\tau = -\varepsilon \sin(\Omega\tau) - \omega_0^2 \sin(\varsigma), \\ d\varsigma/d\tau = V_z, \end{cases} \tag{1}$$

here: $V_z = V_z/$, $\tau = k_z c \cdot t$, $\varepsilon = eE/mc^2 k_z$, $\omega_0^2 = eg/mc^2$, $\Omega = \omega_{ext}/k_z c$, $\varsigma = k_z z$.

System (1) is reduced to the equation of mathematical pendulum with external periodic force

$$\ddot{\varsigma} + \omega_0^2 \sin(\varsigma) = -\varepsilon \sin(\Omega\tau). \tag{2}$$

The radiation power of harmonic oscillator $z = a\sin(\omega_{os} t)$ can be expressed by the formula (see, for example, [1]):

$$\partial W/\partial t \approx e^2 \omega_{os}^4 a^2 / 3c^3 \tag{3}$$

and radiation takes place at a frequency of oscillator ω_{os}. The higher harmonics are small.

In our case, under the approximation $\varepsilon >> \omega_0^2$, for $n>>1$ from (1) it is possible to obtain the maximum amplitude of particle displacement at n-th Fourier harmonics $a = \omega_0^2 J_n(\mu)(n\Omega)^{-2} k_z^{-1}$ ($\mu \sim n, \mu = \varepsilon/\Omega^2$).

The formula (3) will become as:

$$\partial W/\partial t \approx \left(e^2 \omega_{ext}{}^2/3c\right)\left(eg/mc^2\right)^2 n^2 J_n^2(\mu) \tag{4}$$

So, one can see that conditions of maximum radiation $kc\beta = m\omega_{ext} = \omega$ in this case completely coincide with the condition of oscillator radiation in a periodically inhomogeneous dielectric [7], i.e. in both cases the maximum radiation corresponds to the same frequency. In the cases $\omega_0^2 \geq \varepsilon$ the role of the periodical potential is more significant.

3. LIMITATIONS ON ENERGY AND WAVELENGTH OF RADIATION

Let's define the conditions, under which the process of radiation is possible. For particle radiation it is necessary, at least, that its energy should exceed the energy of radiated quantum

$$E = mV^2/2 \geq \hbar\omega. \tag{5}$$

From (5) we find the condition for the length of radiated wave

$$\lambda \geq 2h/mc\beta^2, \tag{6}$$

here c – velocity of light; m – particle mass; $\beta = V/c$ – its dimensionless velocity (Fig.1).

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As it is clear from the figure, the wavelengths, accessible to the radiation, are below the point of intersection of curves and above the line $\lambda = 2h/mc\beta^2$.

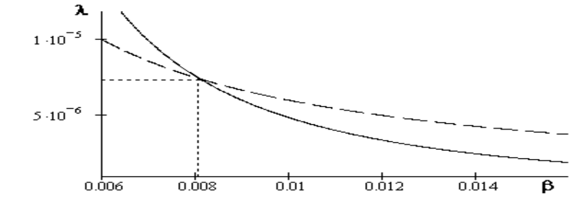

Fig.1. Dependence of radiated wavelength $\lambda = 2h/mc\beta^2$ (firm line) and dependence $\lambda = d/\beta$ (dash line) versus β for $d = 6\cdot10^{-8}$ cm

4. RADIATION OF OSCILLATORS FLOW

It is particularly interesting to investigate the induced radiation of an ensemble of such oscillators, not the radiation of one oscillator. The fullest description of the self-consistent process of interaction of charged particles with the exciting field implies the simultaneous solution of the Maxwell equations for the electromagnetic field and equations of charged particles motion in the exited fields.

$$\partial \vec{B}/\partial t = -c\,\mathrm{rot}\,\vec{E},\ \partial \vec{E}/\partial t = c\,\mathrm{rot}\,\vec{H} - 4\pi \vec{j},$$
$$d\vec{p}/dt = e\vec{E} + (e/c)[\vec{v}\times\vec{B}] + F_{ext}\sin\omega_0 t - e\nabla U,\ d\vec{r}/dt = \vec{v}, \quad (7)$$

where ω_{ext}, F_{ext} are the frequency and the amplitude of the external force, which acts upon the oscillator (produces oscillator). Let's suppose that the oscillations of the oscillator occur along the axis Z.

While investigating the elementary mechanism of the oscillator radiation it was found out, that a directional diagram corresponds to a dipole radiation, i.e. the radiation is directed in a transverse direction with respect to direction of the oscillator oscillation. Therefore, we will search for such solution for an exited wave

$$E = Re\, A(t)\exp(ikx). \quad (8)$$

We will study a time evolution of the electromagnetic field, in which the only E_x, E_z, H_y components are nonzero. Let's substitute the field expressions (8) into the set of equations (7). By averaging the obtained equations on space phase of perturbation, we get the following set of equations for the fields and characteristics of the oscillator:

$$dp_x/d\tau = \mathrm{Re}\,\varepsilon_x\exp(ix) - v_z\,\mathrm{Re}\,h_y\exp(ix),$$
$$dp_z/d\tau = \mathrm{Re}\,\varepsilon_z\exp(ikx) + v_x\,\mathrm{Re}\,h_y\exp(ix) - f_0\sin\Omega\tau - w\sin(\kappa z),$$
$$dx/d\tau = v_x,\quad dz/d\tau = v_z,\quad \vec{v} = \vec{p}/\sqrt{1+p_x^2+p_z^2}, \quad (9)$$
$$d\varepsilon_z/d\tau = -ih_y - \left(2\omega_b^2/2\pi\right)\int_0^{2\pi} v_z\exp(-ix)dx_o,$$
$$dh_y/d\tau = i\varepsilon_z,\quad d\varepsilon_x/d\tau = -\left(2\omega_b^2/2\pi\right)\int_0^{2\pi} v_x\exp(-ix)dx_o.$$

The coordinates $r = r(r_0,\tau)$ and pulses $p = p(r_0,\tau)$ are the Lagrangian coordinates and pulses of particles. The integration in the right part of these equations for fields is the integration over the initial values of oscillator coordinates. The set of equations (9) is written in the dimensionless variables:

$$kct \to \tau,\ k\vec{r} \to \vec{r},\ \frac{\vec{p}}{mc} \to \vec{p},\ k_z/k \to \kappa,\ \vec{\varepsilon} = \frac{eE}{mckc},\ \omega = kc,$$
$$\vec{h} = \frac{eH}{mckc},\quad f_0 = \frac{eE_{ext}}{mckc},\ w = \frac{egk_z}{mc\omega},\quad \omega_b^2 = \frac{4\pi e^2 n_b}{m(kc)^2},$$

$v = V/\omega/k$, where m, e – mass and charge of electrons, n_b – density of oscillators.

5. RESULTS OF THE NUMERICAL ANALYSIS

The numerical analysis of the self-consistent set of equations (9) confirmed the presence of an instability in the considered system. The values of dimensionless parameters were the following: ω_b=0.3, w=0.02, f_0=0.02, n=5. Under these conditions the excitation of 10-th harmonics by oscillators in the periodic potential with $\kappa = 5k$ was observed. The results of simulation are presented in Figs.2,3.

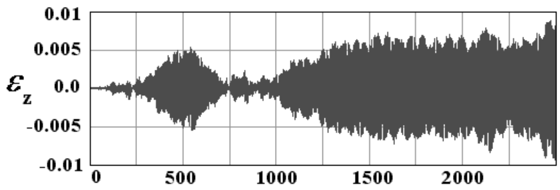

Fig.2. Dependence of amplitude ε_z on time

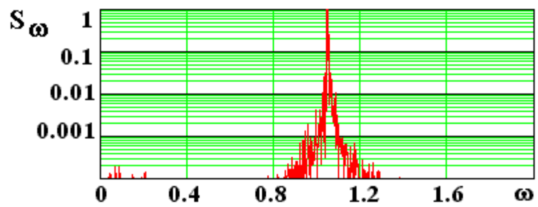

Fig.3. Power spectrum ε_z

The dark filling inside the envelope amplitude corresponds to the high frequency oscillations. Spectrum of the field ε_z has a maximum at the frequency, which equal 1, and that corresponds to the 10-th harmonic of the oscillation frequency of the oscillators. The results of numerical analysis qualitatively well agree with the data of experiments [10].

6. CONDITIONS OF COLLECTIVE RADIATION OF ENSEMBLE OF OSCILLATORS

At the particles motion in the fields of complicated configuration the development of stochastic instability is possible. The presence of such instability in beam systems is similar to the appearance of thermal spread of particles over velocities. The thermal spread can essentially limit the minimal wavelength, which can be excited by a beam in induced way.

We will see that, formally, the conditions of development of stochastic instability are fulfilled. However, the particle dynamics remains practically regular.

The presence of a perturbation results in a formation of a stochastic layer on a phase plane nearby a separatrix – an area in which particle motion is irregular and the separatrix is splitted. The width of such splitted separatrix is proportional to perturbation [11]. The numerical analysis of the Eq. (2) confirms this fact (Fig.4).

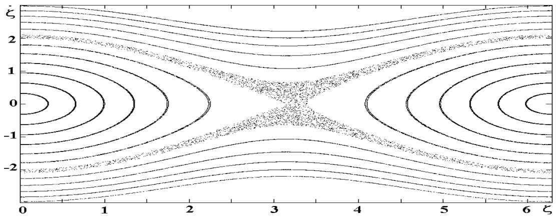

Fig.4. Poincare map of mathematical pendulum: ε *=0.005,* δ *=0.025*

By passing to a moving frame by the variable transformation $\xi = \zeta + (\varepsilon/\delta^2)\sin(\delta\tau)$, the next equation is obtained for ξ from Eq.(2)

$$\ddot{\xi} + \sum_{n=-\infty}^{\infty} J_n(\varepsilon/\delta^2)\sin(\xi + n\delta\tau) = 0, \qquad (10)$$

where $J_n(\varepsilon/\delta^2)$ is the Bessel function of order *n*, $\delta = \omega_{ext}/\omega_0$.

Eq.(10) describes the phase change of the particle ξ which the high number of waves excites. An interaction of the particle with one of these waves will be most effective under fulfillment of the resonance condition $\dot{\xi} = n\delta$. The typical feature of such interaction is a large number of nonlinear resonances. A parameter, determining the degree of influence of the nearby resonances on each other, is a parameter $K = \Delta_{rez}/\Delta\omega$, a ratio of the resonance width to the distance between resonances, having a simple physical sense of degree of resonances overlapping [11]. In our case, the distance between the resonances is $\Delta\omega = \delta$, and the width of resonance with the number of n is equal

$$\Delta_{rez} = 2\sqrt{J_n(\varepsilon/\delta^2)}. \qquad (11)$$

When condition $K > 1$ is fulfilled all the nearby resonances are overlapped and, from the formal point of view, a chaotic motion must be realized in the whole region of phase plane of Eq.(10). However, when K >> 1, the chaotic motion begins regularizing itself. It is due to the fact that at the parameter values $\varepsilon/\delta^2 >> 1$ ($\delta << 1$) the resonances with the numbers of n $\leq \varepsilon/\delta^2$ remain indistinguishable. Motion of particles here can be described as a motion in a new effective resonance and such motion will be regular practically in the whole region of phase plane except for the region near the split separatrix. The numerical analysis of the Eq.(2) also confirms this fact (see Fig.4).

Thus, as an estimated value of measure that shows the degree of an electron motion chaotization, one can take the ratio of the region of phase plane occupied by the split separatrix to the region of phase plane at which motion of particles is regular.

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ВОЗБУЖДЕНИЕ ВЫСОКИХ НОМЕРОВ ГАРМОНИК ЗАРЯЖЕННЫМИ ЧАСТИЦАМИ В ПЕРИОДИЧЕСКОМ ВО ВРЕМЕНИ ЭЛЕКТРИЧЕСКОМ ПОЛЕ И В ПОЛЕ ПРОСТРАНСТВЕННО-ПЕРИОДИЧЕСКОГО ПОТЕНЦИАЛА

В.А. Буц, В.И. Мареха, А.П. Толстолужский

Исследована регулярная и хаотическая динамика заряженной частицы, которая движется во внешнем периодическом во времени электрическом поле и в поле периодического в пространстве потенциала. Получена система интегро-дифференциальных уравнений, которая описывает самосогласованную нелинейную динамику возбуждения электромагнитных волн потоками заряженных частиц. Проведен аналитический и численный анализ полной самосогласованной системы уравнений. Результаты этого анализа качественно хорошо согласуются с экспериментальными данными

ЗБУДЖЕННЯ ВИСОКИХ НОМЕРІВ ГАРМОНІК ЗАРЯДЖЕНИМИ ЧАСТИНКАМИ У ПЕРІОДИЧНОМУ У ЧАСІ ЕЛЕКТРИЧНОМУ ПОЛІ ТА У ПОЛІ ПРОСТОРОВО-ПЕРІОДИЧНОГО ПОТЕНЦІАЛУ

В.О. Буц, В.І. Мареха, О.П. Толстолужський

Досліджена регулярна та хаотична динаміка зарядженої частинки, що рухається у зовнішньому періодичному у часі електричному полі та у полі просторово-періодичного потенціалу. Одержана система інтегро-диференційних рівнянь, що описує нелінійну самоузгоджену динаміку збудження електромагнітних хвиль потоками заряджених частинок. Проведено аналітичний і числовий аналіз повної самоузгодженої системи рівнянь. Результати цього аналізу якісно добре узгоджуються з експериментальними даними.