

# QUASI STEADY-STATE DISTRIBUTIONS FOR COLLISIONAL PLASMAS IN THE PRESENCE OF ENERGY AND PARTICLE SOURCES

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The formation of a non-equilibrium distribution function (DF) of particles with the power-law interaction potentials is studied. Consideration is based on the non-linear kinetic equation of a Landau - Fokker - Planck type in the presence of particle (energy) sources. We compared our results with experimental data.

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## 1. INTRODUCTION

Non-equilibrium states of various physical systems and the analysis of the non-equilibrium (NE) distributions attract increasing interest particularly in connection with the development and wide use of high-power particle and energy sources. The energy (particle) source and sink can be provided by ion beams, high-power laser radiation, emission currents, fluxes of charged particles produced in fusion or fission reactions. A comparison of the characteristic times of ionization and relaxation shows that, in the case at hand, the steady-state electron DF should be determined mainly by electron - electron collisions [1,2]. The analytical consideration of the NE DF formation in the case of nonlinear equation and non-stationary, non-localized, mismatched on intensity sources and sinks is very problematic, thus numerical treatment must be used. The original completely conservative difference schemes having a very high accuracy and allowing to provide the numerical calculation during hundreds collisional times without error accumulation, except for machine errors, are used. Hence, it can be obtained from the condition for the Boltzmann collision integral (CI) (for a semiconductor plasma, the Landau or Fokker - Planck (LFP) collision integral) to be zero. It follows from the above analysis [1] that, for a semiconductor plasma in the energy range  $E - E_F > E_F$ , (where  $E_F$  is the Fermi energy) a power-law DF with a nonzero flux of energy or particles in momentum space (MS) can be established. This DF is formed both due to collisions with electrons whose energy is in the range  $E - E_F > E_F$  and background (equilibrium) electrons. A specific feature of systems of particles interacting via the Coulomb potential is that the scattering cross section increases without bound as the momentum transferred tends to zero. For gaseous and semiconductor plasmas with a large Coulomb logarithm  $\ln \Lambda = 10 - 15$ , one can restrict himself to the expansion of the integrand in the collision integral in small momenta transferred (a diffusion approximation) and to represent the CI in the LFP form [2-4], which are model representations of the Boltzmann CI.

## 2. BASIC CONTENT

Below, we will consider potentials ( $U \sim r^{-\beta}$ , with  $1 \leq \beta \leq 4$ , where  $r$  is the distance between the interacting particles), for which a local NE particle DF can form. Note that the dynamics of particles interacting via the Coulomb repulsion potential ( $\beta = 1$ ) can be considered using a kinetic equations in either the LFP form (see [2,4]). The DF  $f(v, t)$  is bounded at  $v = 0$  and quite rapidly decreases as  $V \rightarrow \infty$  at  $t \rightarrow \infty$ . Below, we use dimensionless variables: the velocity in units of the thermal velocity  $V_T$  and time in units of the electron - electron relaxation time  $\tau_{ee}$ , which is

$$\tau_{ee} = \frac{v_T^3 m^2}{4\pi n_p e^4 \ln \Lambda} \quad \text{in the case of Coulomb}$$

interaction. The DF  $f(v, t)$  is normalized so that the particle density and the total energy and the constant  $\Gamma$  are equal to unity. We consider the formation of a steady-state NE solution in the presence of particle or energy flows in velocity space using the versions of the LFP collision integrals. In this case, the right-hand side of the kinetic equation is added with the terms accounting for the presence of a particle (energy) source (sink):

$$\begin{aligned} \frac{\partial f}{\partial t} &= I_{FP,L}[f, f] + S_+ - S_-, \quad \text{where} \\ S_{\pm} &\sim I_{\pm} \exp\{-\alpha_1(v - v_{\pm})^2\}, \quad \text{or} \\ S_{\pm} &\sim I_{\pm} \delta(v - v_{\pm}) / v^2 \quad \text{or} \\ S_{\pm} &\sim I_{\pm} \frac{\delta(v - v_{\pm})}{v^2} f(v_{\pm}, t). \end{aligned} \quad (1)$$

The flow direction is determined by the positions of  $v$  and  $v_{\pm}$ . Either a Maxwellian DF or a delta - function was used as an initial DF. In [4], the formation of a NE DF was numerically simulated for the kinetic equation with either the LFP collision integral in the presence of energy and particle flows in MS that were sustained by a source and a sink. First, solutions were obtained for the case where the positions of a source and a sink in MS were matched with the direction of a flow sustained by collisions. Note that analytic consideration of equations for the case of a localized source and sink gives a correct

flow direction, a namely, from high to low velocities [1]. It was shown in [4] that, within the interval between the source and sink, a stationary (S) NE DF (of the Kolmogorov kind) of particles is established with time. This DF corresponds to the presence of an energy flow in MS, whereas beyond this interval, the DF is thermodynamically equilibrium. As was noted above, the positions of the source and sink and the direction of flow in MS should be matched with one another. To make sure once again that this requirement is important, we performed calculations with the inter-changed positions of the source and sink in energy space. It turned out that variations in the flow intensity by several orders of magnitude did not influence the equilibrium particle distribution when the source and sink positions were not matched with the flow direction.

It is shown the DF for different flow intensities. It is found that, for low intensities of the source a universal SNEDF is formed in the velocity range  $v \approx v_+$ . This is due to (i) a decrease in the cross section for Coulomb scattering with increasing velocity ( $\sim v^{-3}$ ) and (ii) the always present flow of energy and particles (due to Coulomb diffusion) toward the region of the main ("back-ground") equilibrium DF. Consequently, as the source (sink) intensity increases, a universal NE particle distribution is formed that occupies a progressively larger space between the source and sink. Such behavior is related to a decrease in the fraction of the flow transferred to the background plasma. It is worth noting that the increase in the intensity is accompanied by an increase in the magnitude of the NE DF in proportion to the flux magnitude [1]. Let us examine the form of the DF for power-law interaction potentials with the exponents  $1 \leq \beta \leq 4$ . Note that  $\beta = 1$  corresponds to the Coulomb interaction potential,  $\beta = 2$  corresponds to dipole interaction, and  $\beta = 4$  describes the interaction of so-called Maxwellian molecules. It is shown that NE DFs for the case of a steady-state energy flow with an intensity of  $I = 0.01$  and  $\beta = 1, 2, \text{ and } 4$ , for all these  $\beta$  values, the power indexes of the formed NE power-law DF are close to one another, which agrees with the analytical results [1]. The magnitude of the NE part of distribution function decreases with increasing  $\beta$ . These results are in qualitative agreement with the above analytic predictions. It is well-known that a fast ion, which velocity is commensurable or exceeds the speed of an electron in the Bohr's orbit, interacts mainly with binding and free electrons. In doing so, the ion produces the atom excitation and ionization by the direct electron impact and from the excitation of wake plasma waves by ion, as well. We evaluate the source and sink intensities, their positioning in the MS, the characteristic times of Coulomb relaxation and those of the sources and sink action using parameters corresponded to the conditions of the experiment [2]. Note that, the free path of ion H with energy 1.25 MeV in GaAs is  $l = 3 \cdot 10^{-6}$  m, the ion speed corresponding to the energy of 1 MeV is equal to  $v_H = 7.5 \cdot 10^6$  m/s. From this, we compute the time period while which the proton loses the main part of its energy –  $t_{tr} = 4 \cdot 10^{-13}$  s. For a given test specimen of gallium arsenide the

electron density is  $n_e = 5 \cdot 10^{24}$  m<sup>-3</sup> and the characteristic velocity in such semiconductor plasma is  $v_{th} = 6 \cdot 10^5$  m/s. Then the corresponding characteristic electron - electron relaxation time is in the order of  $\tau_{ee} = 3 \cdot 10^{-14}$  s. We see that sufficient quantity of collisions takes place in a time of the electron free path and under this condition the non-equilibrium distribution function is developed due to the flux presence in the velocity space. Let us estimate the intensity of the sources arisen due to the electron ionization by the direct electron impact and owing to the plasma wave excitation. Along the full free path ion creates about  $10^4$  electrons, in this case the characteristic volume, where ionization will be produced, equals  $3 \cdot 10^{-21}$  m<sup>3</sup>. The density of new electrons being produced by ion in a time unit is about

$I_+ = 3.3 \cdot 10^{37}$  m<sup>-3</sup> s<sup>-1</sup>. In the CI the normalization units are  $n_e$  and  $\tau_{ee}$ , then the normalized intensity is about 0.01 – 0.1 and normalized time of the source action is of the order of 10. Two different source positions in the velocity space have correlation with the plasmon ionization at the characteristic speed  $v_{+1} = 3.5$  and with the ionization by the electron impact at  $v_{+2} = 7$ . In the considering experimental situation, the principal losses are the ion - electron emission from the film surface. Note that for the case under discussion, the sink is distributed over the MS in such a way that it is equal to zero down to the energy 5.65 eV, which corresponds to the work function. The sink intensity is assumed to be proportional to the developing DF. As have been mentioned above, the DF in the presence of rather intensive sources and sinks cannot be a thermodynamically equilibrium one and has to be found out from the solution of the nonlinear kinetic equation. Thus, we examine the evolution of the DF formation solving the equation for the above parameters. We consider the source intensities' range 0.01- 0.1, which variation can be connected both with the different power of the bombarding ion beams and with the difference in electron density between samples. The sink intensity 0.5 is appropriate in the case of emerging of half of electrons having sufficient energy  $v^2 > 4$  from the sample. It is shown that a DF formation for the sources 1 and 2 with the intensities 0.05. Both sources and losses are acting till the time  $t = 100$ . The quasi stationary non equilibrium distribution is formed in a time about 10 - 15 and has a character which is substantially distinguished from the power-like functional dependence obtained in [1]. The explanation can be attributed to the fact that sources and sinks being distributed over the MS do not provide the constant energy flux. In accordance with the results of previous section, the NE DF establishing is independent of the intensity value. The sources are turned off at  $t=12$ . The total energy in the system is changed approximately to 20%, but the particle density varies from 2 to 5 depending on the source and sink intensities. The source (sink) intensity increasing leads to the corresponding function increasing within the inertial interval and to the function decreasing in the cold region. If the sink acts perpetually and the sources act during a while about 15 - 30 then further developed function takes the Maxwellian-type distribution with the temperature so far lower as the source intensity was higher. Let us compute the emission

current dependence on the retarding potential  $U$ , which is need to analyze in the experiment the energetic spectrum of the electron emission. For the experimental conditions have been studying, the electron energetic spectrum of ion-electron emission would be quasi stationary for the ion beam currents exceeded 1 - 10mA. In this case, the emission current is certainly defined by the different ion tracks dispersed in a space, but the distribution function of emission electrons is practically quasi stationary that leads to the quasi stationary emission current. From the comparison of variants, we see how the difference in the emission electron current dependence is strong for the different endurance of the source action. In the experimental conditions [2], the current of the ion beam does not exceed  $10 \mu\text{A}$ . In this case, the emission current will fully reflect the non-stationary character of the sources. On each ion track, the distribution function has time enough to go through all stages of its formation. That is why, the emission current dependence on the retarding potential observed in the experiment is a superposition of the currents existing in different time stages. Obviously, the electron energetic spectrum differs from one that formed under the stationary source action. From the emission current dependence on the retarding potential it can be seen, that the emission current is non-stationary because of the substantial non-stationary sources. The comparison of the simulation and

experimental results shows that the taking into account of the non-stationary source character may be the determining factor.

### 3. SUMMARY

NE quasi steady-state local DF exist inside the momentum interval between the energy (particle) source and the bulk (or sink) of the particle distribution and has the form of gradually decreasing functions. Numerical simulation is in good agreement with the analytical results and with the results obtained in experiments on irradiation of a thin *GaAs* film by a fast ion beam.

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## КВАЗИСТАЦИОНАРНЫЕ РАСПРЕДЕЛЕНИЯ ДЛЯ СТОЛКНОВИТЕЛЬНОЙ ПЛАЗМЫ ПРИ НАЛИЧИИ ИСТОЧНИКОВ ЭНЕРГИИ И ЧАСТИЦ

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Исследовано формирование неравновесных функций распределения частиц со степенным законом взаимодействия между ними. Рассмотрение основано на нелинейном кинетическом уравнении типа Ландау-Фоккера-Планка при наличии источников частиц (энергии). Проведено сравнение с экспериментом.

## КВАЗИСТАЦИОНАРНІ РОЗПОДІЛИ ДЛЯ ЗШТОВХУВАЛЬНОЇ ПЛАЗМИ ЗА НАЯВНІСТЮ ДЖЕРЕЛ ЕНЕРГІЇ ТА ЧАСТИНОК

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Досліджено формування нерівноважних функцій розподілу частинок зі степеневим законом взаємодії між ними. Розгляд основано на одновимірному нелінійному кінетичному рівнянні типу Ландау-Фоккера-Планка за наявністю джерел частинок (енергії). Наведено порівняння з експериментом.