EXCITATION OF THE ACCELERATING FIELD BY HIGH-CURRENT ELECTRON BEAM IN A PERIODIC MAGNETIC FIELD

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The excitation of a slow electromagnetic wave by intense relativistic electron beam (IREB) in the cylindrical drift chamber is theoretically investigated. The beam current is modulated in time. Because of propagation of the electron beam in the periodic magnetic field its surface is modulated in the longitudinal direction. Expressions for the amplitude of the accelerating field are obtained, its dependences on parameters of an electron beam (depth of spatial and of temporary modulation, period of modulation) is investigated. The radial structure of longitudinal components of an electromagnetic field of a slow wave and a possibility of acceleration of ions in the field of the excited wave is analysed.

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I. INTRODUCTION

For creation of a spatial modulation of IREB in a collective method of ion acceleration the external periodic constant magnetic field can be used [1]. The sequence of metal rings with various magnetic properties placed in the solenoid, creates a periodic magnetic field with longitudinal and cross components of a vector of a magnetic induction [2]. At injection of a beam in such field the trajectories of its particles become spatially periodically promodulated. The goffering of a lateral surface results in periodic spatial modulation of the charge density and of the current density of a beam. As a result of such periodic beam profiling, a longitudinal component of an electric field arises. Presence of the time modulation will create the slow wave travelling in the direction of a beam. If the frequency of modulation in time is low [3] such slowed down wave can be used for ion acceleration [1].

II. IREB TRANSPORTATION IN THE FIELD OF CYLINDRICAL WIGGLER

The magnetic field of the solenoid at periodic arrangement in it of rings from magnetic and nonmagnetic materials is determined by the following expression:

$$B_z = B_0 - \frac{4}{\pi} B_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \frac{I_0(\kappa_{2n+1} r)}{I_0(\kappa_{2n+1} R)} \cos\kappa_{2n+1}(z - z_0),$$

$$B_r = -\frac{4}{\pi} B_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \frac{I_1(\kappa_{2n+1} r)}{I_0(\kappa_{2n+1} R)} \sin\kappa_{2n+1}(z - z_0),$$

where B_0 is the induction of the homogeneous magnetic solenoid in the absence of rings, $\kappa_n = 2\pi n / L_w$, L_w is the period of structure, I_1, I_0 are the Bessel functions. In the case of obtaining expressions , it was supposed, that to a point $z_0 = 0$ there corresponds the middle of a ring from the magnetic medium.

Let us study the transportation of the IREB in a periodic magnetic field , on the basis of the envelope equations [4]. It is supposed, that at the input of the cylindrical chamber the beam is annular with inner r_{i0} and outer r_{o0} radii and the cross components of velocities of outer and inner boundaries are equal to zero. Each of the set of equations, used for investigation of the evolution of inner and outer beam envelopes has consisted of the equations for radius envelope, radial, azimuthal components of velocity and the relativistic factor of the boundary electron. The inner boundary electron is subjected to the action of the magnetic force from the constant magnetic field only. And the outer one undergoes the action of another electrical and magnetic force of a field of a spatial beam charge.

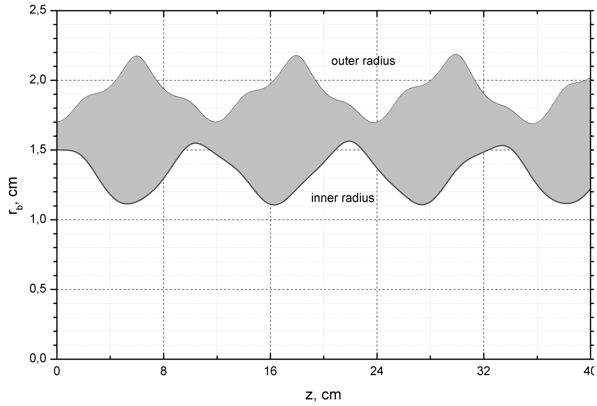

Fig.1. The surface shape of a beam at its transportation in a periodic magnetic field: $B_0 = 1.1kG$, $I_0 = 3kA$, $L_w = 4cm$

In the assumption of a constancy of the longitudinal beam velocity from the equations of movement it is possible to obtain the value of an induction of a magnetic external magnetic field, at which the pulsation of external border is minimum:

$$B_0^c = \frac{\pi}{2}\frac{mc^2}{e}\kappa_0 \left(\frac{J_b}{J_A\gamma\beta}\right)^{1/2} \left(\frac{\pi^2}{32}\kappa_0^2 r_{o0}^2 (1-\alpha_c^2) + \frac{I_1(\kappa_0 r_{o0})}{I_0(\kappa_0 R)}\left(\kappa_0 r_{o0}\frac{I_0(\kappa_0 r_{o0})}{I_0(\kappa_0 R)} - \frac{I_1(\kappa_0 r_{o0})}{I_0(\kappa_0 R)}\right)\right)^{-1/2},$$

where $\alpha_c = 1 - (8/\pi\kappa_0 r_{o0})[I_1(\kappa_0 r_{o0})/I_0(\kappa_0 R)]\cos(\kappa_0 z_0)$, γ is the relativistic factor, $\beta = \sqrt{1-\gamma^{-2}}$, J_b is the beam current, $J_A \approx 17kA$.

For excitation of the high longitudinal electric field it is necessary to achieve steady beam transportation under the high value of longitudinal modulation of a beam. In Fig.1 the example of steady transportation is resulted. The average period of spatial modulation of the beam electrons is not equal to the space period of magnetic field modulation.

III. THE EXCITATION OF THE SLOW WAVE BY MODULATED IREB

Let us consider a metal cylindrical wave guide of a radius R along the axis z of which the thin-walled annular IREB is propagated. Let the time dependence of the beam current I_b is described by the dependence

$$J_b(t) = J_0(1 + \varepsilon \cos\omega_0 t),$$

where I_0 is the average beam current; ω_0 is the frequency, ε is the depth of time modulation. The lateral surface of a beam as a result of its propagation in a periodic magnetic field promodulated under the law

$$r_b(z) = r_0(1 + \alpha \cos k_0 z),$$

r_0 is the average beam radius; $k_0 = 2\pi / L$, L is the period, α – depth of spatial modulation.

At deriving of expression for the field, excited modulated in this way beam, we shall start with Maxwell's equations. From them for the longitudinal components of an electric field the equation follows

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right) + \frac{\partial^2 E_z}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} = 4\pi\left(\frac{\partial \rho}{\partial z} + \frac{\partial j_z}{\partial t}\right),$$

where $\rho = \rho(r,z,t)$ and $j_z = j_z(r,z,t)$ are the space charge density and the longitudinal current density, respectively, determined by expressions:

$$j_z(r,z,t) = \frac{J_b(t)}{2\pi r}\delta(r - r_b(z)),$$

$$\rho(r,z,t) = j_z(r,z,t)/v_z.$$

Here $\delta(x)$ is the Dirac delta function, v_z is the longitudinal electron velocity, assumed as a constant.

Expanding E_z, ρ and j_z, included in equation (6), in Fourier integral on time:

$$(E_z, \rho, j_z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}(E_z^{\omega}, \rho^{\omega}, j_z^{\omega}),$$

and E_z^{ω} in a series on Bessel functions

$$E_z^{\omega} = \sum a_n^{\omega}(z) J_0(\mu_n r/R)$$

after simple transformations we shall come to the equation for definition of amplitude of a longitudinal electric field $a_n^{\omega}(z)$:

$$\frac{d^2 a_n^{\omega}(z)}{dz^2} + \left(\frac{\omega^2}{c^2} - \frac{\mu_n^2}{R^2}\right) a_n^{\omega}(z) = -\frac{4 I_{\omega}}{R^2 J_1^2(\mu_n)} \times$$

$$\times\left[\frac{\mu_n}{v_z R}\frac{dr_b(z)}{dz} J_1\left(\mu_n \frac{r_b(z)}{R}\right) + \frac{\omega^2}{c^2} J_0\left(\mu_n \frac{r_b(z)}{R}\right)\right],$$

where

$$J_{\omega} = 2\pi J_0\left[\delta(\omega) + \varepsilon\delta(\omega - \omega_0)/2 + \varepsilon\delta(\omega + \omega_0)/2\right].$$

The solution of the equation can be find by expansion of $a_n^{\omega}(z)$ in the infinite Fourier series on the longitudinal coordinate. If the depth of spatial modulation α<<1, then the solution of equation is easy to find, having expand the right part up to item of the first order smallness on the value of spatial modulation α. Having executed the inverse Fourier transformation, we shall obtain the final expression for the value of the longitudinal electric field excited by the relativistic electronic beam modulated in space and time. The full expression for the excited field will consist of the sum of three types of infinite series: 1) a constant electric field non-uniform on the density of the charged layer, 2) a homogeneous field of the electric vibrator and 3) waves travelling in direct and back -to a beam - directions. The wave necessary for ion accelerating is described by the third item. Taking into account, that the velocity of accelerated ions satisfies to a condition of phase synchronism $v_i = v_{ph}$, $v_{ph} = \omega_0 / k_0 << c$, for an accelerating field the following expression is obtained:

$$E_z^{ac} = -\frac{I_0 k_0}{v_z}\alpha\varepsilon\, G(k_0 r, k_0 r_0, k_0 R)\sin(k_0 z - \omega_0 t),$$

where

$$G(x, x_0, X) = x_0 \begin{cases} \dfrac{I_0(x)}{I_0(X)}\left[I_1(x_0)K_0(X) + K_1(x_0)I_0(X)\right], x < x_0, \\ \dfrac{I_1(x_0)}{I_0(X)}\left[I_0(x)K_0(X) - K_0(x)I_0(X)\right], x > x_0, \end{cases}$$

I_0, K_0, и I_1, K_1 are the modified Bessel functions of the zero and first order, respectively. Obtaining we considered, that the infinite sums on Bessel functions in the expression for the excited field can be summed exactly [5]. In the case of long-wave modulation $k_0 R << 1$ the function $G = 1\,(r < r_0)$ and $G = 0\,(r > r_0)$, i.e. the accelerating field depends only on the period of spatial modulation.

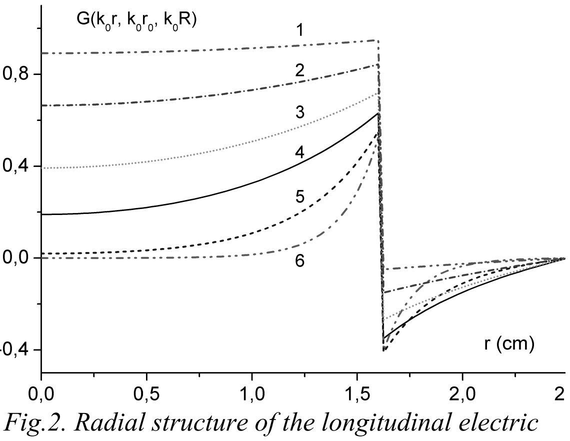

Fig.2. Radial structure of the longitudinal electric field

The radial structure of the accelerating field described by the function G, for various values of the period of spatial modulation L is resulted in fig.2 ($R = 2{,}5$, $r_0 = 1{,}6$ cm; curves 1 - $L = 20$, 2 - $L = 10$, 3 - $L = 6$, 4 - $L = 4$, 5- $L = 2$, 6 - $L = 1$ cm). As follows from the dependences resulted in Fig.2, as the period of spatial modulation decreases, the falling off of the accelerating field is increasing from the beam boundary to the axis of a waveguide. For example, for the period of spatial modulation $L = 4cm$ (line 4 in fig.2) the ratio between the amplitude of the longitudinal electric field on the beam surface and its value on the axis of the system is $E_z(r = r_0)/E_z(r = 0) \approx 3,0$. Homogeneous distribution on the cross section of an electric field is achieved only under condition of k_0r_0<<1 which is practically not realized in our experimental conditions. The magnitude of the function G grows with the period of spatial modulation increasing.

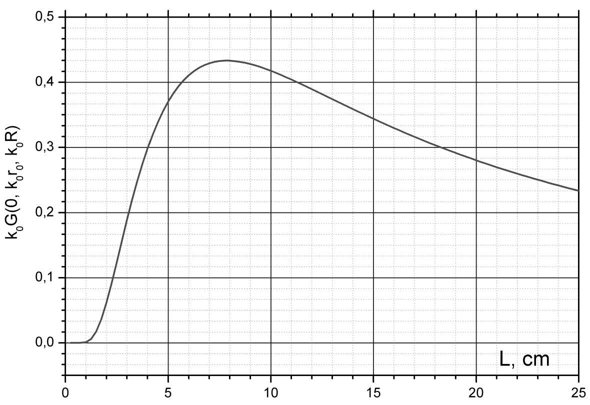

Fig.3. The normalized amplitude of a longitudinal electric field on the axis of a waveguide versus the period of spatial modulation

The acceleration gradient (see Fig.3) is determined by the product of function G and of function k_0. When the period of spatial modulation L increases the function G grows and function k_0 falls. Hence, at the certain value L the maximum of an accelerating field is achieved. In Fig.3 the dependence of the normalized value of an accelerating field on an axis of the system from the period of spatial modulation (other parameters are the same, as in Fig.2) is resulted. At the value of the period of spatial modulation $L = 7.5$ cm the maximum of an accelerating field is achieved.

IV. CONCLUSION

Let us estimate the value of accelerating field for typical parameters of the experimental installation "Duet" $v_z/c = 0,76$, $L_w = 4$ cm. Taking into account Fig.1 and Fig.3 for the field amplitude on the axis of the system we obtain $E_z[\text{kV/cm}] \approx 50\alpha\varepsilon \cdot I_0[\text{kA}]$. At values of spatial modulation $\alpha \approx 0.3$ and temporal modulation $\varepsilon \approx 0.3$, and an average current $I_0 \approx 3.0$ kA the magnitude of the accelerating gradient is 13 kV/cm.

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ВОЗБУЖДЕНИЕ УСКОРЯЮЩЕГО ПОЛЯ СИЛЬНОТОЧНЫМ ЭЛЕКТРОННЫМ ПУЧКОМ В ПЕРИОДИЧЕСКОМ МАГНИТНОМ ПОЛЕ

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Теоретически исследуется возбуждение медленной электромагнитной волны в цилиндрическом волноводе сильноточным релятивистским электронным пучком при его транспортировке во внешнем периодическом магнитном поле. Получены выражения для амплитуды продольного электрического поля, исследована ее зависимость от параметров электронного пучка (глубины пространственной и временной модуляции, периода пространственной модуляции). Анализируется радиальная структура электромагнитного поля медленной волны и возможность ускорения ионов в поле возбужденной медленной волны.

ЗБУДЖЕННЯ ПРИСКОРЮЮЧОГО ПОЛЯ ПОТУЖНОСТРУМОВИМ ЕЛЕКТРОННИМ ПУЧКОМ У ПЕРІОДИЧНОМУ МАГНІТНОМУ ПОЛІ

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Теоретично досліджується збудження повільної електромагнітної хвилі в циліндричному хвилеводі потужнострумовим релятивістським електронним пучком при його транспортуванні в зовнішньому періодичному магнітному полі. Отримано вираз для амплітуди подовжнього електричного поля. Досліджена її залежність від параметрів електронного пучка (глибини просторової і часової модуляції, періоду просторової модуляції). Аналізується радіальна структура електромагнітного поля повільної хвилі і можливість прискорення іонів у полі збудженої повільної хвилі.