EXCITATION OF CORRELATED CHAIN OF NONLINEAR ION-ACOUSTIC SOLITARY PERTURBATIONS OF FINITE AMPLITUDE, ACCELERATING IONS IN CURRENT-CARRYING MAGNETOSPHERE PLASMA

N.A. Azarenkov, A.M. Yegorov, D.Yu. Frolova*, V.I. Lapshin, V.I. Maslov, I.N. Onishchenko*
NSC "Kharkov Institute of Physics & Technology", 61108, Kharkov, Ukraine;
E-mail: vmaslov@kipt.kharkov.ua
**Karazin Kharkov National University, Kharkov, 61108, Ukraine;*

In this paper the chain of correlated solitary perturbations of finite amplitude, accelerating ions in the magnetosphere, is investigated.

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1. INTRODUCTION

Plasma with the electron current velocity lower than the electron thermal velocity is considered. Ion-acoustic perturbations are excited. The homogeneous ion-acoustic turbulence in one-dimensional current-carrying plasma is saturated on the low level. Further this ion-acoustic turbulence is modulated into widely spaced short solitary-type perturbations. The properties of single solitary perturbation have been investigated earlier by the author. In this paper the chain of correlated solitary perturbations of finite amplitude (see Fig.1), accelerating ions in magnetosphere, are investigated. This perturbation is the nonmonotonous double layer, which is represented by the electric potential dip with a shock in its vicinity. The dip reflects electrons with the energy lower than the dip depth. This leads to the dip depth (amplitude) growth.

The equation describing the shape and evolution of the chain of correlated nonmonotonous double layers is derived in this paper. It is obtained that the dip of the electrostatic potential is excited due to current-carrying instability. The case of the large amplitude of excited perturbation is considered. The growth rate of the nonlinear instability development and potential shock in vicinity of the dip are proportional to the distance between solitary perturbations.

A growing interest has been given to plasmas with negative ions (see, for example, [1,2]) due to that negatively charged particles exist frequently in the space plasmas. It is important to investigate effects of these negative ions on formation and properties of chain of the nonmonotonous electric double layers, observed in magnetosphere and accelerating ions. This nonmonotonous electric double layer is the dip of the electric potential with the potential jump near it. There are many papers on stationary solitary perturbations of small amplitudes or nonstationary solitary perturbations in current-carrying plasma or plasma with a hot electron beam (see, for example, [3–6]). In the present paper the formation and properties of this chain of monotonous electrical double layers (electrical potential dip φ with a potential shock in its vicinity) are investigated theoretically. The plasma consists of electrons, positive and negative ions. The electrons propagate relative to negative and positive ions with some velocity. This flow (current) excites nonmonotonous electrical double layer (see Fig.2). The effect of the electron current on excitation and properties of this monotonous electrical double layer is investigated.

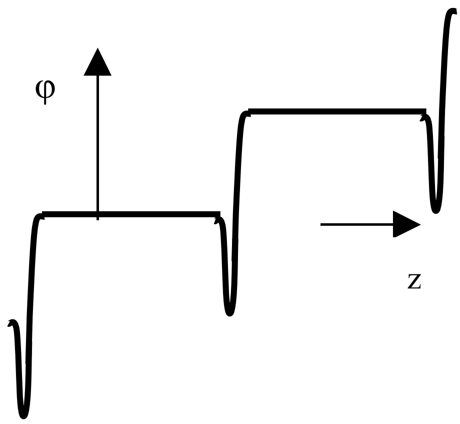

Fig.1. The chain of solitary dips in the electrical potential with potential shocks in their vicinity

The evolution equations describing the shape and time evolution of the electrical field structure, for the case of any amplitude, are derived, when there are not traditional small parameters, permitting to describe properties and excitation of perturbation. It has been shown that the nonmonotonous electrical double layer can be formed on ion-acoustic, on slow ion mode and on ultra-slow dusty ion mode. The conditions have been obtained when the double layer is approximately stationary and fixed in space.

2. EXCITATION OF THE SOLITARY ELECTRICAL FIELD BY THE ELECTRON CURRENT

We use hydrodynamic equations for negative and positive ions

$$\partial n_\pm/\partial t+\partial(n_\pm V_\pm)/\partial z=0 ,$$

$$\partial V_\pm/\partial t+V_\pm\partial V_\pm/\partial z\pm(q_\pm/M_\pm)\partial\varphi/\partial z=0 . \qquad (1)$$

Here $q_\pm$, $n_\pm$, $V_\pm$, $M_\pm$ are charges, densities, velocities and masses of positive and negative ions.

We use the Vlasov equation. for the electron distribution function f_e

$$\partial f_e/\partial t+V\partial f_e/\partial z+(e/m_e)(\partial\varphi/\partial z)\partial f_e/\partial V=0 \qquad (2)$$

and the Poisson equation.

$$\partial^2\varphi/\partial z^2=4\pi(en_e+q_-n_--q_+n_+) . \qquad (3)$$

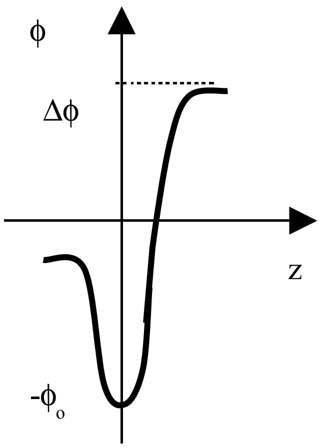

Fig.2. The solitary dip of the electrical potential with a potential shock in its vicinity

Electrons propagate relative to negative and positive ions with some current velocity V_o. The initial electrical potential perturbation is a dip with the width δz. The dip reflects the resonant electrons and obtains the energy from them. The amplitude (depth) $-\varphi_o$ of the dip grows.

Due to reflection of resonant electrons, with nonsymmetrical - relative to the dip velocity V_c -distribution function, the potential jump $\Delta\varphi$ is formed near the dip.

When the dip amplitude increases up to some value the slowdown of its evolution starts. We use slow evolution of the dip for its description using a small parameter $\alpha=\gamma\delta z/V_{tr}$. In zero approximation on α the electron distribution function depends only on the energy ε. Taking into account that the resonant electrons are reflected from the dip one can derive from (2) the expression for the electron distribution function

$$f_e=f_{oe}[-(V^2-2e(\varphi\pm\Delta\varphi)/m_e)^{1/2}\pm V_o] ,$$
$$V^{>}_{<}C(\varphi)\mathrm{sign}(z) , C(\varphi)=[2e(\varphi_o+\varphi)/m_e]^{1/2}. \qquad (4)$$

We use the normalized values: $\phi\equiv e\varphi/T_e$, $N_-\equiv n_{o-}/n_{o+}$, $N_e\equiv n_{oe}/n_{o+}$, $Q_\pm=q_\pm/e$, $V_{s\pm}=(T_e/M_\pm)^{1/2}$. We normalize z on the Debye radius of electrons r_{de}, V_o on the electron thermal velocity V_{th}, time t on the plasma frequency of positive ions ω_{p+}^{-1}, ion velocities and V_c on the ion-acoustic velocity of positive ions $(T_e/M_+)^{1/2}$.

Integrating (4) over the velocity, one can derive the expression for the electron density

$$n_e\approx n_{oe}\exp(\phi)[1-(2\Delta\phi/\sqrt{\pi})\int_o^\beta dx \exp(-x^2)-$$
$$-2V_o(2/\pi)^{1/2}\int_o^\beta dx (x^2-\phi)^{1/2}\exp(-x^2)] . \qquad (5)$$

Far from the dip the plasma is quasineutral $n_e(z)\rvert_{z\to\infty}$ $=n_e(z)\rvert_{z\to-\infty}=1$. From here one can derive, using (5), the expression for the potential jump near the dip

$$\Delta\phi=V_o(2/\pi)^{1/2}(1-\exp(-\phi_o))/[1-(2/\sqrt{\pi})\int_o^{\sqrt{\phi o}} dx\exp(-x^2)] . \qquad (6)$$

From (1) one can obtain for density perturbations

$$n_{i\pm}=n_{\pm NL}+n_{\pm\tau} , n_{\pm NL}=n_{o\pm}/[1-(\pm q_\pm)2\varphi/M_\pm V_c^2]^{1/2} , \qquad (7)$$
$$\partial n_{\pm\tau}/\partial z=\pm2(\partial\varphi/\partial t)(n_{o\pm}q_\pm/M_\pm V_c^3)[1-$$
$$-(\pm q_\pm)\varphi/M_\pm V_c^2]/[1-(\pm q_\pm)2\varphi/M_\pm V_c^2]^{3/2} .$$

Substituting (5), (7) in the Poisson equation, one can derive the nonlinear evolution equation

$$\partial^3_z\phi+\{Q_+^2V^2_{s+}(1-2(\phi-\Delta\phi)Q_+V^2_{s+}/V_c^2)^{-3/2}(1-(\phi-\Delta\phi)$$
$$Q_+V^2_{s+}/V_c^2)+$$
$$+Q_-^2N_-V^2_{s-}(1+2(\phi-\Delta\phi)Q_-V^2_{s-}/V_c^2)^{-3/2} (1+(\phi-\Delta\phi)Q_-V^2_{s-}$$
$$/V_c^2)\}2\partial_t\phi/V_c^3+$$
$$+(\partial_z\phi/V_c^2)\{Q_+^2V^2_{s+}(1-2(\phi-\Delta\phi)Q_+V^2_{s+}/V_c^2)^{-3/2}+$$
$$+Q_-^2N_-V^2_{s-}(1+2(\phi-\Delta\phi)Q_-V^2_{s-}/V_c^2)^{-3/2}\}-N_e\partial_z\phi\{\exp(\phi)-$$
$$\mathrm{sign}(z)2V_o(2/\pi)^{1/2}\times$$
$$\times[(\sqrt{\phi_o}+A)/2(\phi_o+\phi)^{1/2}\exp(-\phi_o)+\exp(\phi)\int_o^{\sqrt{\phi o+\phi}} dx\exp(-x^2)$$
$$((x^2-\phi)^{1/2}+A)]\}=0, \qquad (8)$$

describing the excitation, evolution and properties of the dip. Here $A\equiv(1-\exp(-\phi_o))/(\sqrt{\pi}-2\int_o^{\sqrt{\phi o}}dx\exp(-x^2))$.

Integrating (8) in quasi-stationary approximation, one can get

$$(\partial_z\phi)^2/2=(V_c/V_{s-})^2N_-((1+Q_-2(\phi-\Delta\phi)V^2_{s-}/V_c^2)^{1/2}-1)+$$
$$+(V_c/V_{s+})^2((1-Q_+2(\phi-\Delta\phi)V^2_{s+}/V_c^2)^{1/2}-1)+N_e\{\exp(\phi)-$$
$$-\exp(\Delta\phi)+\mathrm{sign}(z)2V_o(2/\pi)^{1/2}\times$$
$$\times\{A[\exp(\phi)\int_o^{\sqrt{\phi o+\phi}} dx\exp(-x^2)- \int_o^{\sqrt{\phi o}} dx\exp(-x^2)-$$
$$-\exp(-\phi_o)((\phi_o+\phi)^{1/2}-\phi_o^{1/2})]+$$
$$+\exp(-\phi_o)(\phi_o+1)-1+2\int_o^{\sqrt{\phi o+\phi}} dz\exp(-z^2+\phi)z^2(z^2-\phi)^{1/2}\}\}(9)$$

From (9) and $\partial_z\phi|_{\phi=-\phi o}=0$ one can show that the dip velocity, V_c, is close to the ion-acoustic velocity of the positive ions, V_{s+}, and essentially depends on ϕ_o. In the limiting case $N_-=0$ one can derive

$$V^2_c=V^2_{s+}N^2_eB^2/2(Q_+(\phi_o+\Delta\phi)+N_eB), \qquad (10)$$

where $B\equiv\exp(\phi_o)-\exp(\Delta\phi)+$ $+\mathrm{sign}(z)2V_o(2/\pi)^{1/2}\{A[-\int_o^{\sqrt{\phi o}}dx\exp(-x^2)+\exp(-\phi_o)\sqrt{\phi_o}]+$ $+\exp(-\phi_o)(\phi_o+1)-1\}$.

From (8) at $\phi\to-\phi_o$ one can get the growth rate of the dip amplitude

$$\gamma=V_c^3N_e\exp(-\phi_o)2V_o(2/\pi)^{1/2}(\sqrt{\phi_o}+A)(N_e\exp(-\phi_o)- \qquad (11)$$
$$Q_+(1+2\beta(\phi_o+\Delta\phi))^{1/2}/2\phi_oQ_+^2V_{s+}^2(1+2\beta(\phi_o+\Delta\phi))^{3/2}\times$$
$$\times(1+\beta(\phi_o+\Delta\phi)),$$
$$\beta\equiv Q_+V_{s+}^2/V_c^2 .$$

3. EXCITATION OF THE SOLITARY ELECTRICAL FIELD ON THE SLOW ION MODE

Let us consider nonmonotonous double layer, propagating with velocity V_i, approximately equal to thermal velocity of positive ions, $V_{thi}=(T_i/m_i)^{1/2}$. The electron density approximately equals to (5). The expression for the perturbation of the positive ion density can be obtained from the Vlasov equation

$$\delta n_i\approx-\theta\phi R(g)+\theta^2(\phi/2)^2[R(g)(3-2g^2)-1] ,$$
$$g\equiv V_i/V_{thi}\sqrt{2} , \theta\equiv T_e/T_i , \qquad (12)$$
$$R(g)=1+(g/\sqrt{\pi})\int_{-\infty}^{\infty}du \exp(-u^2)/(u-g).$$

Substituting (5), (12) into (3), one can derive the nonlinear equation, describing the potential distribution of a nonmonotonous double layer in space:

$$(\phi')^2=\phi^2(1+\theta R(g))+(\phi^3/6)\{2+\theta^2[1+(2g^2-3)R(g)]\},$$

$$\partial/\partial(z/r_d)\equiv"\,'\,". \qquad (13)$$

From (13) and $\phi'|_{\phi=-\phi_o}=0$ one can derive the expression for V_i

$$g=g_o\{1+1/\theta-(\phi_o/6)[\theta+3-2g^2+2/\theta]\}\ ,\ g_o=0.924. \qquad (14)$$

V_i is close to V_{thi} and decreases with the amplitude growth.

The width of the nonmonotonous double layer is approximately determined from (13):

$$(\Delta z)^{-2}=(\phi_o/48)\{2+\theta^2[1+(2g^2-3)R(g)]\}. \qquad (15)$$

Δz decreases with the amplitude growth.

In case of large amplitudes, $\phi_o>1$, we have

$$n_e=n_o\exp(-\phi_o)\ ,$$

$$n_i=(n_o/\sqrt{\pi})\int_{-\infty}^{\infty}du\ \exp(-u^2)/(1-\theta\phi/(g-u)^2)^{1/2}\ . \qquad (16)$$

From (3) and (16) we obtain

$$[\Phi'(\Phi=\Phi^o/2)]^2=\sqrt{2}-2+2\exp(-\Phi^o/2)-\sqrt{2}\exp(-\Phi^o), \quad (17)$$

ϕ_o cannot be more than critical one, ϕ_a , determined by the equation:

$$1-\sqrt{2}+\sqrt{2}\exp(-\Phi^a/2)-\exp(-\Phi^a)=0, \qquad (18)$$

Δz increases with ϕ_o growth. Therefore one needs take into account trapped ions. We assume their distribution function f_{tri}=const=$f_{oi}(V_o)$. Then their density equals

$$n^{tri}=2(-\theta\Phi)^{1/2}(n^o/\sqrt{\pi})\exp(-g^2_o). \qquad (19)$$

From (3), (16), (19) we obtain, that V_i increases with ϕ_o growth.

One can see that for a large ϕ_o the dependencies of V_i and Δz on ϕ_o are inverse in comparison with the case of a small ϕ_o.

4. EXCITATION OF THE SOLITARY ELECTRICAL FIELD ON THE ULTRA-SLOW DUSTY ION MODE

Let us consider possibility of nonmonotonous double layer formation on the ultra-slow dusty ion mode, the velocity V_u of which is low in comparison with V_{thi}. The electron density approximately equals to (5). The potential jump near the dip is formed similar to (6).

We determine the density perturbations of negative ions δn_- from (1) and positive ions δn_+ from the Vlasov equation

$$\delta n_+/n_{o+}\approx-\phi/\theta+\phi^2/2\theta^2,\ \delta n_-'/n_{o-}\approx -\phi'/V_u^2+3\phi\,\phi'/V_c^4-N_-\dot{\phi}/V_u^3 \qquad (20)$$

Here «’»≡∂/∂z, «.»≡∂/∂t.

Substituting (5), (20) in (3) one can derive the nonlinear equation. in partial derivatives for the nonhomogeneous and nonstationary potential

$$\begin{aligned} &N_-^2\dot{\phi}/V_u^3-\phi'(1+N_+/\theta-N_-/V_u^2)-\\ &-\phi\phi'(1-N_+/\theta^2+3N_-/V_u^4)+\phi'''+\\ &+V_o(2/\pi)^{1/2}\phi'\{[\phi_o^{1/2}+\\ &+(1+2(\phi_o/\pi)^{1/2})\phi_o/\pi^{1/2}]/(\phi_o+\phi)^{1/2}+\\ &+\ln[(-\phi)^{1/2}/(\phi_o^{1/2}+(\phi_o+\phi)^{1/2})]\}=0 \end{aligned} \qquad (21)$$

From (21) and the condition $\phi'|_{\phi=-\phi_o}=0$ one can find V_u

$$V_u\approx(T_{i+}/M_{i-})^{1/2}(n_{i-}/n_{i+})^{1/2}/(1+n_{oe}T_{i+}/n_{i+}T_e)^{1/2} \qquad (22)$$

So, it has been shown that the dip propagates with very low velocity.

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ВОЗБУЖДЕНИЕ КОРРЕЛИРОВАНОЙ ЦЕПОЧКИ НЕЛИНЕЙНЫХ ИОННО-ЗВУКОВЫХ СОЛИТОННЫХ ВОЗМУЩЕНИЙ КОНЕЧНОЙ АМПЛИТУДЫ, УСКОРЯЮЩИХ ИОНЫ В ТОКОВОЙ ПЛАЗМЕ МАГНИТОСФЕРЫ

Н.А. Азаренков, А.М. Егоров, Д.Ю. Фролова, В.И. Лапшин, В.И. Маслов, И.Н. Онищенко

Исследована цепочка коррелированных солитонных возмущений конечной амплитуды, ускоряющих ионы в магнитосфере.

ЗБУДЖЕННЯ КОРЕЛЬОВАНОГО ЛАНЦЮЖКА НЕЛІНІЙНИХ ІОННО-ЗВУКОВИХ СОЛІТОННИХ ЗБУРЮВАНЬ СКІНЧЕНОЇ АМПЛІТУДИ, ЩО ПРИСКОРЮЮТЬ ІОНИ В ТОКОВІЙ ПЛАЗМІ МАГНІТОСФЕРИ

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Досліджено ланцюжок корельованих солітонних збурювань скінченої амплітуди, що прискорюють іони в магнітосфері.