

THE ION CORE IN ELECTRON STORAGE RINGS WITH CLEARING ELECTRODES

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Theoretical estimations of the ion core parameters inside a bending magnet with clearing electrodes at its ends are presented.

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INTRODUCTION

It is well known that positive ions produced from a residual gas and confined in beams of negatively charged particles limit performance of these beams. Effects of the confined ions are similar to those caused by the beam space charge but much larger in magnitude [1, 2] due to relativistic effects. Hence, a small fraction of ion population surviving when the clearing applied could cause a significant effect on the circulating beam.

One of the most efficient and simple ways to reduce the ion core density down to proper limits is the usage of electrostatic electrodes providing transverse electric field that extracts ions from within the beam. This techniques is mostly applied in storage rings–synchrotron light sources. Nevertheless the density of the ion core in that beams has been estimated rather approximately. In particular, it concerns the relation of the ion core parameters to the density of the residual gas in the vacuum chamber of a machine.

Technical conditions restrict clearing electrodes from covering more than a small fraction of the orbit length. Thus the ion core density is strongly dependent on the longitudinal (along the beam orbit) motion of ions. This motion is caused by so called drift of a charged particle in the crossed fields (e.g. the electrostatic field of the beam and the magnetic field of a bending magnet). Specific for the storage rings is that ions are being produced with initial velocities much less than the maximum velocity gained in the potential well of the beam space charge: $\dot{x}_0 \ll \max \dot{x}$, because depth of the well reaches hundreds volts whereas the initial (thermal) kinetic energy of ions is about 0.03 eV. Hence, as is expected, the ion motion under the storage ring conditions differs significantly from the drift [3], where the advancing velocity is much smaller than that of gyration [4].

Present work is aimed at theoretical estimation of the ion core parameters inside a bending magnet with clearing electrodes, installed at its ends.

ION TRAJECTORY

Technical conditions restrict clearing electrodes from covering more than a small fraction of the orbit length. Thus the ion core density is strongly dependent on the longitudinal (along the beam orbit) velocity of ions. This motion is caused by so called ‘drift’ of a charged particle in the crossed fields (e.g. the electrostatic field of the

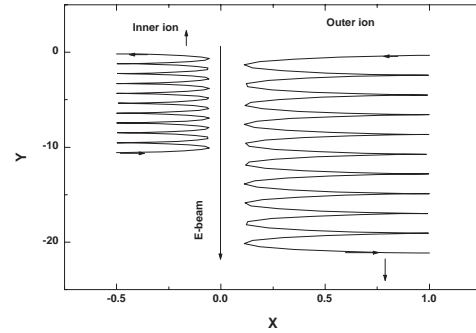


Fig 1: Electron and ions trajectories. The center of magnet curvature is leftward.

beam and magnetic field of a bending magnet). The exact trajectories of a charged particle are known for only a few kinds of field [3, 5]. For more complex fields the trajectory is described within framework of the drift approximation [3].

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Under considered conditions ion motion differs from the drift one (where the advancing velocity is much smaller than the orbital one) [6].

Problem settings. Hamilton function \mathcal{H} , which describes the motion of the nonrelativistic particle in the electric and magnetic fields, is as follows [7]:

$$\mathcal{H} = \frac{1}{2M} \left[(\vec{p} - e\vec{A})^2 \right] + e\phi, \quad (1)$$

where M is the ion mass; \vec{p} — a canonical momentum; e — the ion charge; \vec{A} — a vector potential to the magnetic field; ϕ — a scalar potential to the electric field.

The model geometry considered here is sketched in Fig. 1.

We accept simplifying assumptions keeping for the electron storage rings. These are:

a) the curvature of the beam orbit is negligibly smaller than that of an ion, so we consider the beam trajectory being stright;

b) the uniform magnetic field is supposed:

$$A_y = Bx, \quad A_x = A_z = 0 \quad (2)$$

with B being the magnetic field strength.

c) The electric field is uniform along the beam axis (y) and even with respect to the xy plane: $\phi(z) = \phi(-z)$.

d) Depth of the potential well of the beam induced electric field is significantly exceeds the ion initial kinetic energy, which is about the thermal energy of the neutral gas [4].

This potential can then be decomposed into truncated to second order series around $x = x_0, z = 0$ where an ion produced:

$$\begin{aligned} \phi(x, y, z) &= \phi(x, z) \\ &= \phi_0 + (x - x_0)\phi'_x + \frac{(x - x_0)^2}{2}\phi''_{xx} \\ &\quad + \frac{z^2}{2}\phi''_{zz} + O(\phi'''_{xxx}), \end{aligned} \quad (3)$$

$$\phi_0 \equiv \phi(x_0, 0); \quad \phi'_x \equiv \left. \frac{\partial \phi}{\partial x} \right|_{x=x_0}; \quad \phi''_{zz} \equiv \left. \frac{\partial^2 \phi}{\partial z^2} \right|_{z=0}.$$

Substituting (2) and (3) in (1), we come to the following expression to the Hamiltonian and the corresponding canonical equations:

$$\begin{aligned} \mathcal{H} &= \frac{p_x^2}{2M} + \frac{p_z^2}{2M} + \frac{(p_y - \omega Mx)^2}{2M} \\ &\quad + e \left[\phi_0 + (x - x_0)\phi'_x + \frac{(x - x_0)^2}{2}\phi''_{xx} \right. \\ &\quad \left. + \frac{z^2}{2}\phi''_{zz} \right]; \end{aligned} \quad (4)$$

$$\begin{cases} \dot{x} &= p_x/M \\ \dot{p}_x &= \omega(p_y - M\omega x) - e[\phi'_x + (x - x_0)\phi''_{xx}] \\ \dot{y} &= p_y/M - \omega x \\ \dot{p}_y &= 0 \\ \dot{z} &= p_z/M \\ \dot{p}_z &= -e\phi''_{zz}z \end{cases} \quad (5)$$

Here $\omega = eB/M$ is the cyclotron (Larmour) frequency [5] of the ions in the magnetic field. The Hamiltonian (4) can be read as sum of two parts:

$$\mathcal{H} = \mathcal{H}_x(x, y, p_x, p_y) + \mathcal{H}_z(z, p_z)$$

describing the independent motion in the vertical direction and in (xy) plane.

Ion trajectory. The projection of the ion trajectory into the z -axis represents a harmonic motion with the 'electrostatic' frequency ω_{ez} :

$$z = z_0 \cos \omega_{ez}t + \frac{\dot{z}_0}{\omega_{ez}} \sin \omega_{ez}t; \quad (6)$$

$$\omega_{eu}^2 \equiv e\phi''_{uu}/M,$$

where z_0, \dot{z}_0 are the initial coordinate and velocity, respectively; u stands for x or z .

As it follows from (5), the ion trajectory in the plane orthogonal to the magnetic field, has the form:

$$\begin{cases} x &= x_0 + \frac{Q}{M\Omega}(1 - \cos \Omega t) + \frac{\dot{x}_0}{\Omega} \sin \Omega t \\ y &= y_0 + \frac{\omega Q}{M\Omega^2}(1 - \sin \Omega t) + \frac{\omega \dot{x}_0}{\Omega^2} \cos \Omega t + Vt \end{cases} \quad (7)$$

$$\Omega^2 \equiv \omega^2 + \omega_{ex}^2 = \omega^2(1 + D);$$

$$Q \equiv \frac{M\omega}{\Omega}y_0 - \frac{e\phi'_x}{\Omega};$$

$$D \equiv \frac{\omega_{ex}^2}{\omega^2};$$

$$V \equiv \dot{y}_0 - \frac{\omega Q}{M\Omega}.$$

Projection of the ion trajectory into (x, y) plane represents oscillations with the frequency Ω and amplitudes

$$\begin{aligned} a_x^2 &= \frac{1}{\Omega^2} \left(\frac{Q^2}{M^2} + \dot{x}_0^2 \right); \\ a_y^2 &= \frac{\omega^2}{\Omega^4} \left(\frac{Q^2}{M^2} + \dot{y}_0^2 \right), \end{aligned}$$

which reduces to an 'elliptic' cycloid for the off-axis born ions (see [6]):

$$\left| \frac{M\omega y_0}{e\phi'_x} \right| \ll 1;$$

$$V = -\frac{\omega Q}{M\Omega};$$

$$a_x = \frac{|V|}{\omega}; \quad a_y = \frac{|V|}{\Omega}; \quad x_c = x_0 - a_x.$$

The applicability limits for the obtained expressions are determined by the validity of the potential representations (3) within the ion oscillations $x_c \pm a_x$, indicated by the parameter D . This parameter equals to the ratio of the amplitude to the initial position of an ion with zero kinetic energy:

$$D = \frac{a_x}{x_0}.$$

The case $D \rightarrow \infty$ relates to zero strength of the magnetic field (ions oscillate around the beam axis, $x_0 = 0$); the case $D = 0$ represents zero electrostatic field, the ions are at rest. For $D \geq 1$ the representation (3) is only valid for the cylindrical beam with homogeneous density, $\phi'' = \text{const}$. The case $D < 0$ indicates non periodic motion beyond our consideration.

Analysis of ion motion. As seen in the equation of the trajectory (7), motion of ions in the plane (x, y) is the composition of the gyration with the frequency Ω and advancing along the y -axis. Position of the centre of x -oscillation (x_c) is:

$$x_c = x_0 + \frac{Q}{M\Omega}$$

For the potential series expanded around x_c we come to the advancing velocity V having the same form as for

well-known drift in crossed uniform electric and magnetic fields [5]:

$$\begin{aligned} V &= \frac{\varphi'_x}{B}; \\ Q &= \frac{M\Omega}{\omega} \dot{y}_0 - \frac{e\varphi'_x \Omega}{\omega^2}; \\ \varphi'_x &\equiv \left. \frac{\partial \phi}{\partial x} \right|_{x=x_c}. \end{aligned} \quad (8)$$

Thus, the advancing (drift) velocity depends on the electric field strength at the centre of gyration only. So, averaging the ion trajectory over the period of gyration $2\pi/\Omega$, we can involve into consideration virtual objects — ion ellipses (IE) which velocity determined by the electric field strength at their centres.

Canonical form of the equations. The expression for the IE velocity (8) appears to be very important for describing the ion core kinetics in dipole magnets. In fact, in the coordinate system turned for an arbitrary angle around the vertical (z) axis, equations of the IE motion possesses the canonical form

$$\begin{cases} \dot{x}_c = \frac{\partial(\phi/B)}{\partial y_c} \\ \dot{y}_c = -\frac{\partial(\phi/B)}{\partial x_c} \end{cases} \quad (9)$$

where the radial position of IE, x_c , can be regarded as a canonical coordinate and the longitudinal position y_c — as the conjugated canonical momentum.

The Hamiltonian related to (9)

$$\mathcal{H}_{\text{IE}} \equiv \frac{\phi}{B} \quad (10)$$

being explicitly time independent is an invariant of the motion.

Therefore, equicurves of \mathcal{H}_{IE} represent the IE trajectories in the ‘phase space’ (x_c, y_c). The canonical form of the equations of motion enables us to analyze the particle motion without involving solutions to the equations. Small non-uniformity in the magnetic field can be treated by means of the perturbation theory.

DENSITY OF THE ION CORE

Kinetics of the ion core formation. Taking for the grunt the canonical form of the IE motion and considering the stationary state of the ion core, we can present the discontinuity equation describe the dynamic assemble of IEs (9) as the following:

$$\{n_{\text{IE}}(x_c, y_c), \mathcal{H}_{\text{IE}}\} = S_{\text{IE}}(x_c, y_c), \quad (11)$$

where $\{\}$ represent the Poisson brackets; n_{IE} is the density of IEs; the RHS term $S_{\text{IE}}(x_c, y_c)$ stands for power of the IE source, e.g. number of IEs in unit space cell produced per unit time interval. Here, according to (6), we take IEs density averaged over vertical (z) oscillations due to independence of motion in this direction.

Power of IE source can be evaluate as follows. Number of ions n_{ion} producing at (x, y) is

$$\frac{dn_{\text{ion}}}{dt} = n_{\text{gas}} c \sigma n_{\text{beam}} \quad (12)$$

where n_{gas} is the neutral gas density; n_{beam} is the beam density; σ — the ionization cross section of the gas molecules (atoms).

Power of IE source, as follows from (12), is

$$\begin{aligned} S_{\text{IE}}(x_c, y_c) &= S_{\text{IE}}(x_c) = c \sigma n_{\text{gas}} (1 + D) \\ &\times \int_z n_{\text{beam}}(x(x_c), z) dz. \end{aligned} \quad (13)$$

Here $x(x_c) = x_c(1 + D)$.

Because of independence of RHS in (11) and IE velocity on y , the density of IE population is linear respecting to distance along the orbit from the clearing station [8]:

$$\begin{aligned} n_{\text{IE}} &= y n_{\text{gas}} \sigma \frac{B}{\varphi'_x} (1 + D) \\ &\times \int_z n_{\text{beam}} \left[\left(x + \frac{e\varphi'}{M\omega^2} \right), z \right] dz. \end{aligned} \quad (14)$$

Assuming the beam density symmetrical ($n_{\text{beam}}(-x) = n_{\text{beam}}(x)$) and composing together both upstream and downstream halves of the core, the density of ion core per unit of orbit length N_{IE} reads:

$$\begin{aligned} N_{\text{IE}} &= Y n_{\text{gas}} \sigma B (1 + D) \\ &\times \int_s n_{\text{beam}} \left[\left(x + \frac{e\phi'}{M\omega^2} \right), z \right] \frac{dz}{\phi'_x}, \end{aligned} \quad (15)$$

where integration done over the beam cross section, Y is the distance between adjacent clearing sets (the length of the magnet along the beam orbit).

For the round beam model, expression (15) reduces to:

$$\begin{aligned} N_{\text{IE}} &= \frac{Y b n_{\text{gas}} \sigma B (1 + D)}{\pi e} \\ &\times \left\{ \log \left[\frac{1 + \sqrt{1 - y_*^2}}{y_*} \right] - \sqrt{1 - y_*^2} \right\}; \quad (16) \\ y_* &\equiv \frac{x_*}{b} (1 + D). \end{aligned}$$

As it common for this sort of problem, the density of IE raises to infinity when $x_* \rightarrow 0$. It is caused by the ellipses with the centres laying in the beam axis posses zero drift velocity and hence add infinite density to the core. Determination of the minimal offset from the beam centre x_* requires special consideration beyond the considering model.

Determination of x_* and core density distribution.

For the model the only mechanism of removing ions out of the beam is their drift onto the clearing electrodes. This leads to infinite core density in the beam orbit. The model suggestion of small neutralization is violated in this region. Hence, we suppose that in this region ions escape from within the beam in z -direction (along magnetic field lines), the conditions of the free space model [9] are kept. For a typical pressure of the residual gas the free space model yields full neutralization. So, the empty beam model restriction violates in the near-to-axis region.

We suggest to establish the value of x_* phenomenologically based upon the following considerations. Let at

$x_c = x_*$ the density of the ion core reaches that of the beam (full neutralization) at some coordinate x . Thus, the philosophy of determination x_* is as follows. The IE density $n_{IE}(x_c, y)$ (14) corresponds to a certain core density distribution $n_{ion}(x, y, z; x_*)$. We require the maximal core density at the magnet end (entrance to a clearing set: $y = Y$) equal to that of the beam:

$$x_* \Rightarrow \max_x n_{ion}(x, Y, z; x_*) = n_{beam}(x, z). \quad (17)$$

Besides (17), we will also use in calculations an integral criterium:

$$x_* \Rightarrow \max_x \int_0^\infty n_{ion}(x, Y, z; x_*) dz = \int_0^\infty n_{beam}(x, z) dz. \quad (18)$$

As is evident, both criteria (17) and (18) base on knowledge of the core density distribution corresponding to the IE density (14). There exists one more reason to obtain the core density: Some of problems in beam dynamics (e.g., nonlinear betatron motion) require explicit charge distribution for their evaluation or simulation.

Whereas ion motion is independent along the magnet lines and in a plane perpendicular to them, we can separate these two projections. First we will evaluate the vertical density distribution of ions oscillating in a parabolic potential well. The density contributed to the ensemble by a harmonic oscillator can read as:

$$f(u; u_c, a_u) = \frac{H[u - (u_c - a_u)] - H[u - (u_c + a_u)]}{\pi \sqrt{a_u^2 - (u - u_c)^2}}, \quad (19)$$

where $H(u)$ is the Heaviside step function; u_c and a_c is the centre coordinate and the amplitude of oscillations, respectively.

Vertical core shape. For a uniform beam and vertical oscillation of initially rested ions ($u = z$, $u_c = 0$, and amplitude distribution $\alpha(a_z) \cdot b_z = H(a_z) - H(a_z - b_z)$) the (normalized to unity) density distribution function is

$$g(z) = \int_{z_{max}}^{b_z} \frac{da}{\sqrt{a^2 - z^2}} = \frac{1}{\pi} \log \frac{1 + \sqrt{1 - \zeta}}{\zeta}; \quad (20)$$

$$z_{max} \equiv \max(z, z_*)$$

with $\zeta = z/b_z$.

We have arrived at divergence at $z = 0$ due to Dirac's δ contribution from a zero-amplitude oscillator. This divergence can be eliminated accounting for initial velocity (temperature) of the ions. We take into account the finite temperature of the producing ions in the following way. As is obvious, the initial velocity changes the core distribution significantly just around the median plane ($z = 0$), where the core density reaches its maximal value. Limits of influence are at depth of the beam potential well about the room temperature (0.03 eV). So, the problem of evaluating the maximal core density at this step reduces into obtaining the density distribution functions for ensemble

of oscillators with the uniform initial distribution of coordinates and the thermal distribution of momenta.

Consider the ensemble of particles oscillating with the same frequency around the coordinate origin ($z_0 = 0$), possessed the uniform initial coordinate distribution and the thermal momentum distribution. Let us suggest for the sake of simplicity the momentum distribution being maxvellian up to amplitude a_t and $\delta(p)$ above that:

$$f_v(z) = \frac{1}{\pi b} \begin{cases} \log \left(\frac{b_z + \sqrt{b_z^2 - z^2}}{a_t + \sqrt{a_t^2 - z^2}} \right) + \frac{\pi}{2}; & a_t \geq z \\ \log \left(\frac{b_z + \sqrt{b_z^2 - z^2}}{z} \right); & z_m < z \end{cases} \quad (21)$$

$$\max f_v(z) = f_v(0) = \frac{1}{\pi b_z} \left(\log \left(\frac{b_z}{a_T} \right) + \frac{\pi}{2} \right);$$

$$a_T = \frac{1}{2} \sqrt{\frac{kT}{e^2 n_{beam}}},$$

where k is the Boltzmann constant; T is gas temperature.

Thus, accounting for initial temperature eliminates divergence in density at $z = 0$.

We evaluate the transverse density distribution in the following way. The ensembles with vertical density distribution (21) and density of centres (16) oscillate in the horizontal plane. So, the general density distribution can be evaluated by joining of the elemental density contributing by single oscillator with their density distribution (16) and (21). The ensemble of oscillators with the same initial position, possessing thermal initial velocities, has the density:

$$f_{osc}(a_0, x) = \frac{\Omega}{\sqrt{2\pi s}} \exp \left[\frac{(a_0^2 - x^2) \Omega^2}{2s^2} \right] \times [1 - \nu(a_0, x)], \quad (22)$$

$$\nu(a_0, x) \equiv \begin{cases} \operatorname{erf} \left[\sqrt{\frac{(a_0^2 - x^2) \Omega^2}{2s^2}} \right]; & x^2 < a_0^2 \\ 0 & x^2 \geq a_0^2 \end{cases}$$

with

$$\operatorname{erf}(u) \equiv \frac{2}{\pi} \int_0^u \exp(-t^2) dt,$$

$$s^2 \equiv \frac{kT}{M}.$$

Joining (22) and (21) with $b_z = \sqrt{b^2 - x^2}$, the restored transverse density distribution of the ion core capturing by the uniform beam reads:

$$n_{ion}(x, z) = K_{den} \int_t \frac{V(z_{max}(t), z)}{t} F(\psi(x, t)) dt, \quad (23)$$

$$V(z_*(t), z) = \begin{cases} \log \frac{z_* + \sqrt{z_*^2 - z^2}}{z_i + \sqrt{z_i^2 - z^2}} + \frac{\pi}{2}; & z < z_i \\ \log \frac{z_* + \sqrt{z_*^2 - z^2}}{z}; & z \geq z_i \end{cases}$$

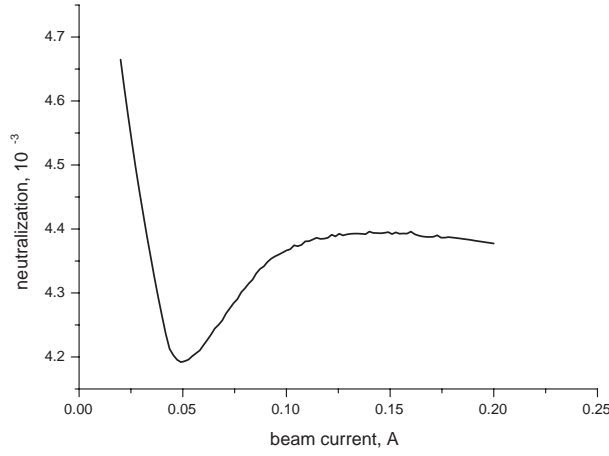


Fig 2: Dependence of the neutralization factor upon the beam current

$$\begin{aligned}
 z_*(t) &\equiv \sqrt{1 - t^2(1 + D)^2}, \\
 F(u) &\equiv \frac{1}{\sqrt{\pi}} \exp(u^2) (1 - \operatorname{erf}(u)), \\
 \psi(x, t) &\equiv \frac{\sqrt{t^2 D^2 - (x-t)^2}}{x_i}, \\
 z_i^2 &= \frac{kT}{N_b r_e m c^2}, \\
 x_i^2 &= \frac{A}{(D+1) r_p B^2}, \\
 K_{\text{den}} &= Y \frac{\sigma_i B (D+1) n_{\text{gas}}}{\pi e},
 \end{aligned}$$

where $r_{e,p}$ is the electron (proton) classical radius; A is the mass number; all the coordinates are dimensionless, normalized on the beam radius b .

COMPUTATION OF THE ION CORE DENSITY

To study the ion core density response upon variation of some parameters, we wrote a computer code that solves numerically the equations (23) with the both criteria (17) and (18) [8]. Calculating procedure is as follows. By variation of x_m the maximal relative ion density (17) at $z = 0$ or optionally the averaged over z density is equalized to unity. Obtained by this procedure the value of x_m is provided to calculate the neutralization factor η (16) and density distribution (23).

The results of computing are as follows:

a) For the typical beam parameters ($a = 0.05$ cm, $Y = 1$ m, $B = 1$ Tesla, residual gas pressure 1 nTorr of Nitrogen) ion clearing keeps the neutralization factor below 0.004 (Fig. 2).

b) The transverse density distribution of ions is essentially nonlinear, the main part of ions is confined in the near-to-axis region (Fig. 3).

c) The density of ions increases slightly when the beam current is built up, the neutralization factor decreases as depicted in Fig. 2.

d) The core density decreases when the magnetic field strength is increased, Fig. 4.

CONCLUSION

Clearing electrodes are capable to reduce significantly the ion core density. Remaining ions occupy the near-to-axis region of the beam. Therefore additional focusing of the beam particles provided by the ion core is sufficiently nonlinear. Density of ion population under the conditions considered is almost independent of the beam current.

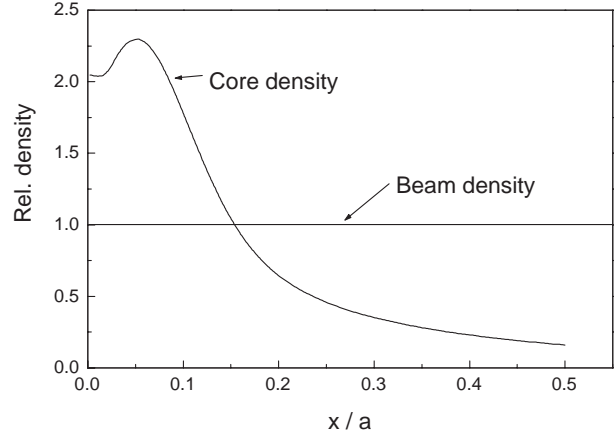


Fig 3: The ion core transverse density shape

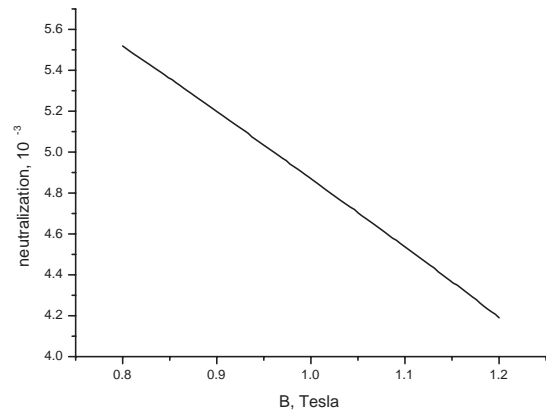


Fig 4: Dependence of the neutralization factor upon the dipole field

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