HiSOR MULTIMODE UNDULATOR AS A CIRCULARLY POLARIZED LIGHT SOURCE

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A linear/helical multimode undulator, which is installed at storage ring in Hiroshima Synchrotron Radiation Center, is initially designed for the utilizing of circularly polarized light from its fundamental harmonic in the spectral range of 4 – 40 eV. Operating of the undulator in elliptical configuration may considerably extend this spectral region. This paper presents a study for the optimisation of operating parameters of a linear/helical multimode undulator with the aim to maximize the circularly polarized high-energy photons output. The comparison with the correspondent characteristics of synchrotron radiation from bending magnet is performed. The results presented here were obtained for the general case that provides a means of applying derived expressions for any kind of elliptical wiggler as well as bending magnet.

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INTRODUCTION

Many efforts were recently made to satisfy demands for intense beams of circular polarized electromagnetic radiation in a wide range of experiments in materials science and molecular biology. For this purpose, among many other works, a linear/helical multimode undulator [1] was installed at one of two straight sections of a compact racetrack-type 700-MeV storage ring (HiSOR) of Hiroshima Synchrotron Radiation Center (HSRC) [2]. This multimode undulator was designed to enhance the intensity and degree of circular polarization of photon beam in comparison with those radiating from bending magnet (BM). However, while the undulator is operating in the helical mode, the available photon energy is restricted to the range of 4 – 40 eV, since only the first harmonic is generated along the undulator axis. With the aim to extend the photon energy range, it was proposed [3] to use the linear/helical multimode undulator in so-called elliptical multipole wiggler (EMPW) mode [4-6], since the configuration of the multimode undulator allows the continuous transformation from linear mode through elliptical to helical mode. In that event energy of elliptically polarized photons generated at high harmonics along the undulator axis is considerably beyond the fundamental energy and its intensity is reasonably strong for utilization.

The main features of EMPW radiation can be understood through analogy to standard synchrotron radiation from bending magnets. More precisely, almost the same relations describe the on-axis radiation from elliptical wiggler and the off-plane synchrotron radiation. It gives direct insight into physics of the elliptical wiggler's radiation. At the same time, it should be pointed out that the corresponding variables in these relations have different physical meanings for the case of elliptical wiggler and bending magnet.

A detailed study has been conducted to evaluate and optimise the spectral performance of the HiSOR linear/helical multimode undulator at elliptical mode for photon energies above 40 eV. This paper includes a summary of the procedure and results; some of them are derived for the general case and thus may be used for optimisation of other electromagnetic radiation sources, both to elliptical wigglers and to bending magnets.

RADIATION SOURCES

The HiSOR storage ring [2] with the electron beam operational energy 700 MeV has two 180° normal-conducting bending magnets with maximum magnetic field 2.7 T. The radius of the electron beam's trajectory is $R = 86$ cm and critical wavelength of synchrotron radiation is $\lambda_c = 1.42$ nm (critical energy is 873 eV). It generates an electromagnetic radiation with intensity high enough to satisfy experimental needs within the photon energy range of about $1 - 10^3$ eV. It is generally known that the elliptically polarized off-plane component of synchrotron radiation from bending magnet can be used as a source of circularly polarized photons. The degree of circular polarization is zero at the median plane and increases gradually as the vertical observation angle increases. However, in this process the radiation intensity falls rapidly down at high observation angles.

The HiSOR linear/helical multimode undulator has a similar design to those of the elliptical wiggler for Spring-8 [7,8] and the helical undulator for UVSOR [9]. Its magnetic structure consists of two jaws, which are placed above and below the storage ring median plane, similar to the planar undulator. However, each of these jaws consists of three standard Halbach-type permanent magnet arrays (see Fig. 1). The four outer arrays may slide simultaneously along the stationary middle arrays. In doing this, the pair of arrays 1 and 6 move together in one direction through the distance d and the pair of arrays 3 and 4 move together through the same distance, but in the opposite direction. As it is shown in Fig. 1, d is the relative displacement of the side arrays in reference to their non-shifted position. Depending on its value the undulator configuration is linear $(d = 0)$, elliptical $(0 < |d| < \lambda_u/4)$ and helical $(|d| = \lambda_u/4)$. Here λ_u is the length of the undulator period.

At the median plane, two central lanes (Nos. 2 and 5 in Fig. 1) generate only vertical magnetic field, same as for a conventional undulator. In linear configuration the horizontal components (along X-axis) of the magnetic fields from outer arrays cancel each other and the resulting field has only vertical component. In this case the undulator will generate the linearly polarized radiation. At the helical configuration the four outer arrays generate the horizontal magnetic field only since vertical fields of side magnets cancel each other. With the combination of the vertical field of the central arrays, it produces a spiral-type magnetic field. For the chosen geometry of magnetic blocks [1], the amplitudes of the horizontal (x) and vertical (y) magnetic field components are close to each other at any gap. Radiation in this situation is almost fully circularly polarized. Between these two configurations the undulator field is of elliptical type with different amplitudes of horizontal and vertical components depending on d value. The resultant magnetic field is described as a sum of the fields generated by six arrays. The field of each array in the regular part is sinusoidal-type with respect to the longitudinal coordinate z. Considering for simplicity the first harmonic only, one can get:

$$B_p = \sum^{6} b_{ip} \cos\left(2\pi\left(z - d_i\right)/\lambda_u\right),$$

where B_p are horizontal ($p = x$) and vertical ($p = y$) components of the undulator's fields, b_{ip} are the amplitudes of the fields, generated by the i-th array, d_i are the corresponding array shifts: $d_1 = d_6 = -d_3 = -d_4 = d$ and $d_2 = d_5 = 0$. From symmetry considerations the following relations are fulfilled on the undulator axis:

$$b_{2x} = b_{5x} = 0, \quad b_{2y} = b_{5y}, \quad b_{1x} = b_{6x} = -b_{3x} = -b_{4x},$$
$$b_{1y} = b_{3y} = b_{4y} = b_{6y}.$$

Substituting these relationships into Eq. , the following expression for the undulator magnetic field on its axis can be derived:

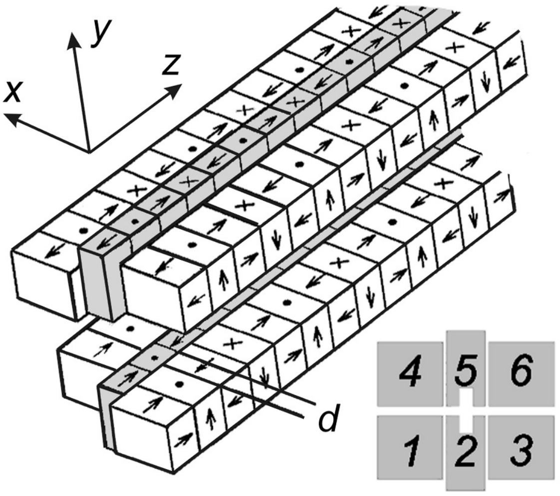

***Fig. 1.** The scheme of the multimode undulator, operated in EMPW mode together with its vertical cut*

$$\begin{cases} B_x = B_x^{\max}\cos\left(2\pi z/\lambda_u\right), \\ B_y = B_y^{\max}\sin\left(2\pi z/\lambda_u\right). \end{cases}$$

Here, $B_x^{\max} = u\sin\varphi$ and $B_y^{\max} = v + w\cos\varphi$ represent the amplitudes of the horizontal and vertical fields, $\varphi = 2\pi d/\lambda_u$ is the phase shift of the outer arrays, $u = 4b_{1x}$, $v = 2b_{2y}$ are the amplitudes of the horizontal and vertical magnetic fields at helical configuration, and $w = 4b_{1y}$. It is important to note that the horizontal component of the resulting field (2) is always out of phase by $\pi/2$ relative to the vertical regardless of the array shift d.

The following main parameters of the linear/helical multimode undulator were chosen to satisfy the users needs: the length of period $\lambda_u = 10$ cm, number of periods $N = 18$, gap varies in the range of $30 : 200$ mm. It provides maximum magnetic field 0.347 T in the helical mode and 0.597 T in the linear mode [1]. The corresponding deflection parameters are 4.58 for helical mode and 5.57 for linear mode. They are defined here by the following standard way:

$$K_{x,y} = eB_{x,y}^{\max}\lambda_u\Big/\left(2\pi mc^2\right) \text{ and } K^2 = K_x^2 + K_y^2.$$

The energy of fundamental harmonic in helical mode is varied from 4 eV up to 40 eV by increasing the undulator gap from 30 mm up to 90 mm, with keeping the fundamental harmonic circularly polarization above 99%. It is also found [3] that an operation of the undulator with minimum gap in the elliptical configuration, when shift d is about a few mm, provides the regime of an elliptical multipole wiggler wherein the photons with energy higher than 40 eV are effectively generated at higher harmonics.

CHARACTERISTICS

OF ELECTRO – MAGNETIC RADIATION

Let us consider synchrotron radiation generated by zero-emittance electron beam moving through a bending magnet along a circular trajectory. The number of photons (flux density I) radiated per second, per unit solid angle $d\Omega$ per unit relative wavelength interval $\left(d\lambda/\lambda\right)$, is given by the following well-known expression [10]:

$$I = \frac{d^2\dot{n}}{d\Omega\left(d\lambda/\lambda\right)} = \frac{i}{e}\frac{3\alpha\gamma^2}{\pi^2}\xi^2\left[F_x(\eta,\xi)+F_y(\eta,\xi)\right],$$

where i is the electron beam current, e is an electron charge, α is the fine structure constant, γ is the relativistic factor, $\xi = \lambda_c/2\lambda$, $\lambda_c = 4\pi R/3\gamma^3$, R is the orbit radius, ψ is the vertical observation angle, $\eta = \gamma\psi$ and

$$F_x(\eta,\xi) = \left(1+\eta^2\right)^2 K_{2/3}^2(\zeta),$$
$$F_y(\eta,\xi) = \left(1+\eta^2\right)\eta^2 K_{1/3}^2(\zeta).$$

Here $\zeta = \xi\left(1+\eta^2\right)^{3/2}$ and $K_\nu(\zeta)$ are the modified Bessel functions order of $\nu = 1/3, 2/3$.

The degree of circular polarization of the radiated photons is defined as:

$$P_c = 2\sqrt{F_x F_y}\Big/\left(F_x+F_y\right).$$

Let us now consider the radiation from elliptical multipole wiggler with N periods. Its magnetic field is described by the Eq. with the following conditions: K_y>>1 and $K_x \le 1$. The photons with highest energy are generated at the part of electron trajectory with the vertical field close to $B_y^{\max}$ and the horizontal field close to zero. Since K_y>>1, the radiation that is emitted from these regions may be described by the standard expressions for synchrotron radiation. In this case, the bending radius corresponding to $B_y^{\max}$ is equal to $R_u = \gamma\lambda_u\big/\left(2\pi K_y\right)$, and corresponding critical wavelength is equal to $\lambda_c = 4\pi R_u/(3\gamma^3)$. In addition, the respective parts of the electron's trajectory, which can be regarded as the arc of the circular orbit, is tilted up or down with the angle $\psi = K_x/\gamma$ by the alternating horizontal magnetic field. As a consequence, the radiation generated along the undulator axis from the $2N$ parts of the electron's trajectory, will be of the same intensity and equally polarized. As a result, the spectrum of EMPW can be obtained from Eq. for the off-plane synchrotron radiation spectra. Thus, the right - side part of the expression should be multiplied by the number of the poles $2N$ (i.e. the total number of the radiation sources), and use the following variable definitions: $\eta = K_x$ and $\lambda_c = 2\lambda_u\big/\left(3\gamma^2 K_y\right)$ [5,6].

In further analysis, the optimisation of the radiation parameters based on two different criteria: to maximize the photon flux density and to maximize the performance of BM and EMPW as circularly polarized radiation sources will be considered.

OPTIMIZATION OF THE RADIATION INTENSITY

The maximum of the flux density may be found by differentiating Eq. with respect to the parameters ξ and η with equating these derivatives to zero. As a result, one can get:

$$\begin{cases} q^2\left(1+\eta^2\right) - 3\zeta q\left(1+2\eta^2\right) + 2\eta^2 = 0,\\ \eta\left[3\zeta q\left(1+2\eta^2\right) - \left(1+\eta^2\right)\right] = 0,\end{cases}$$

where $q = K_{2/3}(\zeta)/K_{1/3}(\zeta)$. Numerical solution of Eq. gives absolute maximum at the point $\xi = 0.417$, $\eta = 0$. In the case of HiSOR sources, it corresponds to the photon energy of about 155 eV for the undulator at linear mode (since $\eta = K_x = 0$), and to the photon energy of 730 eV on the BM median plane.

Taking into consideration only the second equation of , which arises from the η -derivative, the optimum value of parameter η for every value of ξ can be found. In the case of BM it means to find such a vertical angle of observation, where flux density has a maximum. In case of EMPW, the problem comes to finding of the best value for $\eta = K_x$, i.e. of the best value for the array's shift d. It is obvious that one of the solutions of this equation is $\eta = 0$. More detail analysis shows that the point of $\xi_0 = 0.244$ separates the energy axis in two parts. For $\xi \ge \xi_0$ intensity has a maximum at $\eta = 0$, which means the operation at the BM median plane, or at the linear mode of EMPW. For $\xi < \xi_0$, $\eta = 0$ is the point of local minimum of radiation, and maximum of intensity is located at $\eta \ne 0$. In the case of the BM, it corresponds to the non-zero observation angle $\psi = \eta/\gamma$ above or below the bending plane. The numerical value of ξ_0 can be obtained as a solution of the equation $K_{2/3}(\xi)/K_{1/3}(\xi) = 1/3\xi$.

For HiSOR's bending magnet and linear / helical undulator in EMPW mode intensities at these optimum angles versus photon energy are shown in Fig. 2. Numerical calculations were performed for electron beam energy 0.7 GeV and current 300mA. For the helical / linear undulator in EMPW mode all results were obtained at 30mm gap. Due to summation of the

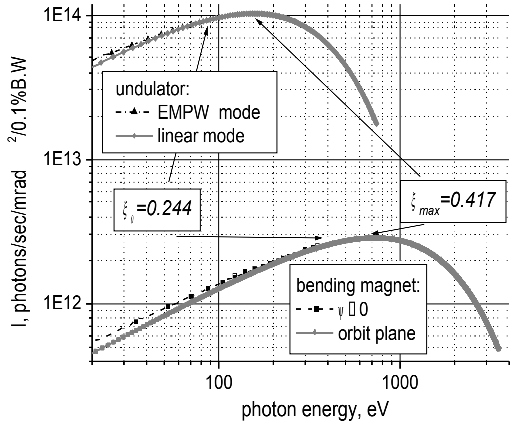

Fig. 2. *Maximum available radiation intensity vs photon energy. ξ_{max} is the point of absolute maximum of radiation. For $\xi<\xi_0$ maximum of radiation occurs at $\eta \neq 0$, for $\xi \geq \xi_0$ at $\eta=0$*

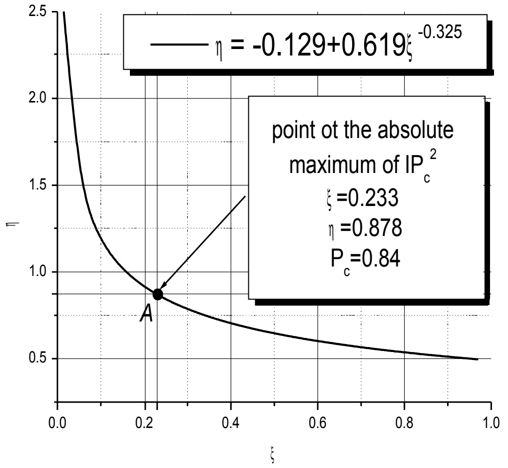

Fig. 3. *Functional dependence of parameter η vs ξ, being solution of the equation $\partial f / \partial \eta = 0$*

radiation from N periods, the radiation intensity from EMPW stands out above the BM intensity by the factor $2N$, but at the same time operating energy region for BM is wider than that of EMPW.

OPTIMIZATION OF THE CIRCULAR POLARIZED RADIATION

The optimization of the spectral output for BM and EMPW involves a trade-off between flux density and degree of circular polarization. As it was mentioned above, at $\xi \geq \xi_0$ the radiation with maximum available intensity is situated at $\eta = 0$ and is linearly polarized. With the increasing of η the degree of circular polarization P_c increases while the intensity decreases. For $\xi < \xi_0$, optimized radiation is elliptically polarized, but again if one wants to obtain higher degree of circular polarization the intensity will noticeably decreases. In order to find the best compromise between intensity and degree of polarization, it seems natural to maximize the product of circular polarized light intensity $\left(IP_c\right)$ and the degree of circular polarization $\left(P_c\right)$:

$$f\left(\eta,\xi\right) = IP_c^2 : \frac{\xi^2 F_x F_y}{F_x + F_y}.$$

This figure of merit is commonly accepted [11,12] in virtue of the fact that it is proportional to square of the signal-to-noise ratio in circular-dichroism experiments.

Similar to the previous section, by differentiating Eq. with respect to ξ and η and equating these derivatives to zero, the following relations can be derived:

$$\begin{cases} q^4 - \dfrac{2q^3}{3\zeta} - \dfrac{\eta^2 q}{3\zeta\left(1+\eta^2\right)} + \dfrac{\eta^2}{1+\eta^2} = 0, \\ q^4 - \dfrac{1+\eta^2}{3\zeta\eta^2} q^3 + \dfrac{\eta^2}{1+\eta^2} = 0. \end{cases}$$

The root of the second equation of the system , corresponding to $\partial f / \partial \eta = 0$, gives the value of η such that the function f reaches its maximum for any predetermined ξ and seems to be most interesting from the physical point of view. The solution of this equation is plotted in Fig. 3, allowing the easy choice of the devices operation parameters. For simulation reasons, it is more convenient to use the following approximation of this solution, which is valid within the interval $\xi \in \left(0.01,1\right)$:

$$\eta = -0.129 + 0.619\xi^{-0.325}.$$

It is important to note that Eq. is generally valid for radiation from any kind of bending magnet as well as elliptical wiggler. By solving of both equations of system , absolute maximum of f is obtained at $\xi = 0.223$, $\eta = 0.878$, as shown at the point A in Fig. 3. The degree of polarization P_c at that is about 0.84.

The results of this optimisation have been applied to the both HiSOR radiation sources, BM and linear/helical undulator. In the case of BM (Fig. 4a), the corresponding vertical angle ψ was found for any photon energy by using the Eq. . After that $f = IP_c^2$ value was calculated using Eq. and Eq. (Fig. 4b). One can see that the function f peaks at the photon energy of 389 eV and at vertical observation angle $\psi \approx 0.64$ mrad, corresponding to the point A of the general solution. As was mentioned above, for this photon energy the degree of polarization will be about 0.84. It is possible to obtain higher P_c at larger observation angles ψ, but photon flux will be lower.

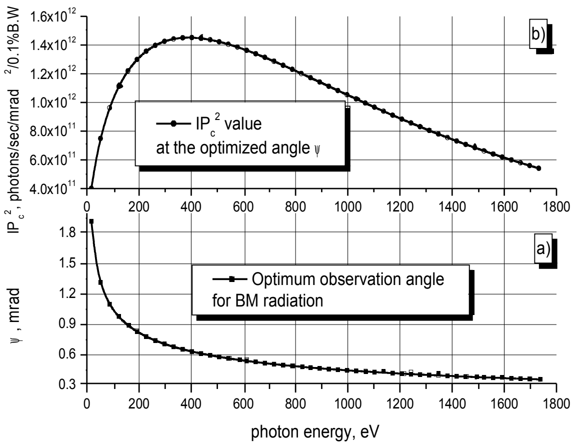

Fig 4. *a) The operation parameters for HiSOR's bending magnet. b) Corresponding "figure of merit"*

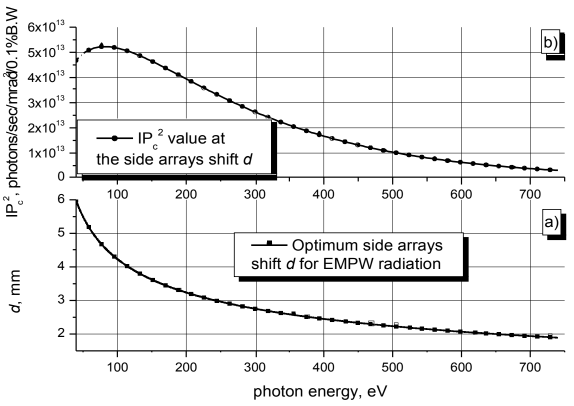

Fig. 5. *a) The operation parameters for HiSOR's linear/helical undulator. b) Corresponding "figure of merit"*

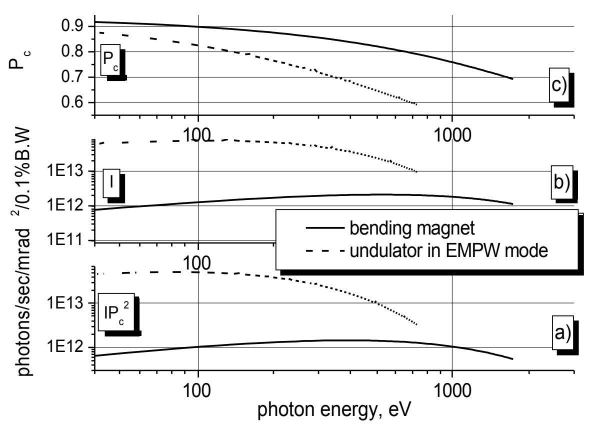

Fig. 6. *BM and EMPW radiation properties, calculated for optimum operation parameters. (a) "Figure of merit"* $f = IP_c^2$*; (b)* Flux *density of radiation* I*; (c) Degree of circular polarization P*

In the case of EMPW, the situation is much more complicated since the variation of the side arrays shift d change both vertical and horizontal magnetic fields. At first, using Eq. , $\eta = K_x$ has to be found as the function of ξ , then d and K_y values have to be found using corresponding expressions for $B_{x,y}^{\max}$, after that ξ has to be converted to the photon energy using relation $\lambda = \lambda_u/\left(3\gamma^2 K_y \xi\right)$. The result of this calculation is shown in Fig. 5a. Figure 5b shows the value of f as a function of the photon energy. One can see that f peaks at 82 eV, which corresponds to $\xi = 0.223$. In this case, the horizontal deflection parameter K_x is equal to 0.878, the relative displacement of the side magnetic arrays (*d*) is equal to 4.6 mm and accordingly $K_y = 5.252$.

Fig. 6 shows the dependence of EMPW and BM radiation parameters (f , flux density I , P_c) vs. photon energy. They were calculated in the same manner as for the Figs. 4,5 and, gathered together, enables us to compare performance of both radiation sources. Intensity of the radiation from bending magnet is 1-2 order weaker than that from elliptical wiggler. However, the degree of polarization is larger than 0.75 up to 1000eV. In the case of EMPW, for the energy region of 40-150 eV, the value f is high enough with P_c ¡ 0.8 , but the degree of circular polarization rapidly declines for photon energy above 150 eV.

It is important also to check area of application of the EMPW mode. To cover energy range above 40eV it is necessary to use the arrays shift $|d| < 6$ mm (Fig. 5a). At such small shifts EMPW approximation K_y ? K_x] 1 is valid since the ratio K_y/K_x ¡ 4.6 .

CONCLUSION

In this article the way to obtain maximum of two different parameters: radiation intensity and product of circular polarized light intensity and degree of circular polarization from EMPW and BM is shown. Some results were derived for the general case, which allows its application to other radiation sources.

It is shown that a spectral range of circularly polarized light, which is generated by a linear/helical multimode undulator at helical mode, may be considerably extended to high photon energy region by using EMPW mode. Both EMPW and BM are found to have a high degree of circular polarization in the spectral region above 40 eV. The polarization characteristics can be effectively controlled by varying of the horizontal magnetic field in case of EMPW, or by varying of the vertical angle of observation in case of BM.

The photon flux density of EMPW is obviously higher than that of BM due to the large number of radiation sources ($2N$). On the other hand, the BM radiation is bending magnet field is much stronger

(2.7 T) than the much more intense in the high-energy region since the EMPW field amplitude (0.597 T). From the present analysis it is possible to conclude that the elliptical mode of the multimode undulator has much potential for generating of circularly polarized light at high energy, up to 0.5 keV.

The presented results were calculated for ideal case, i.e. for zero emittance electron beam and infinitely small acceptance aperture. The inclusion of these real parameters into simulation may slightly change the spectral distributions. The effect of finite vertical aperture on the correspondent fluxes is shown in Fig. 7. The BM and EMPW radiations were integrated over the vertical slits bounding the vertical observation angles. For the BM, slit was positioned at the optimum angle $\psi = \eta / \gamma$, which was varied with changing of the photon energy according to Eq. . For the EMPW the center of the slit was positioned on-axis. Two types of slits were analysed. The first slit size corresponds to the angle $\Delta\psi = 1/(2\gamma)$, independent with the photon energy. The second slit uses the vertical size equal to the full width at half maximum (FWHM) of the vertical intensity distribution at corresponding photon energy.

For the relatively soft radiation with energies below the critical energy, the size of FWHM is larger than $1/(2\gamma)$ and corresponding value of IP_c^2 at this energy is higher. Fig. 7 demonstrates some effects of the real experiment parameters on the radiation characteristics, in comparison with Fig. 6a, which was calculated for infinitely small slit size. One can see that an ideal case approximation is adequately enough for the optimisation of radiation sources.

In conclusion it should be noted that using of EMPW mode of the linear/helical multimode undulator in addition to the usual, linear and helical modes, allows obtaining of the radiation with various kinds of polarization in the wide spectral region.

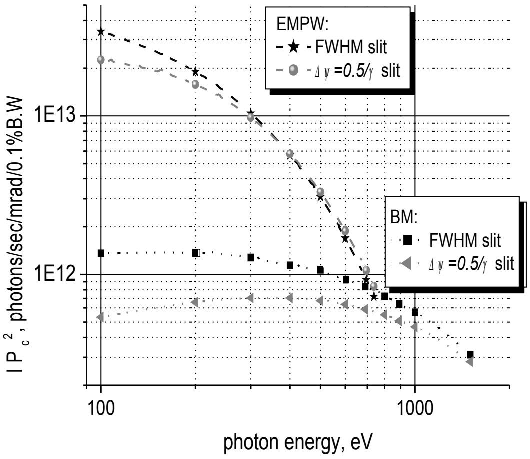

Fig. 7. *Integrated over 2 different slits* $f=IP_c^2$

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