APPLICATION OF KRYLOV-BOGOLYUBOV AVERAGING METHOD TO THE PROBLEM OF PERIODIC PERTURBATIONS IN THE LAYERED MEDIUM

K.Yu. Kramarenko, N.A. Khizhnyak
National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

Influence of small periodic perturbation in periodic dielectric medium on propagation of TM–polarized waves is investigated by the Krylov–Bogolyubov averaging method. As a result of periodic perturbation inside the allowed bands of unperturbed medium forbidden bands appear. The appearance of forbidden bands is conditioned by a parametric resonance between spatial harmonics of perturbation and plane waves, on which the solution of the wave equation in periodic medium is decomposed. The location, number and width of these bands are determined.
PACS 41.20.

1. INTRODUCTION

Periodic structures with varying parameters can prove useful in many applications. In paper [1] the theory of wave propagation in periodic structures with smoothly varying parameters is developed with the help of Wannier-function expansion. In particular, it is shown that the double periodic structure possesses a miniband structure. Such a spectrum has been observed in direct numerical simulations, described in paper [2].

The Krylov–Bogolyubov averaging method was developed for the investigation of non–linear differential equations. However, it is successfully applied for solving linear equations with small parameter in the theory of cyclic accelerators [3]. It is also possible to apply such method for investigation of TM–polarized wave propagation in a periodic medium with small periodic perturbation at an arbitrary period.

The purpose of this paper is to describe the effects occurring when an electromagnetic wave propagates in a periodic medium with small periodic perturbation.

2. KRYLOV–BOGOLYUBOV AVERAGING METHOD

At first, we consider the propagation of a TM polarized wave in the periodic medium. The periodicity exists only in one direction, and propagation of the electromagnetic wave in that direction is governed by the equation:

$$\varepsilon \frac{d}{dz}\left(\frac{1}{\varepsilon}\frac{dD}{dz}\right) + L^2\left(k^2\varepsilon(z) - k_\perp^2\right)D = 0, \qquad (1)$$

where $k = \frac{2\pi}{\lambda}$ is the wave number, $k_\perp$ - the transverse component of wave vector, L is the period of undisturbed medium, z - the undimensional variable and $\varepsilon(z+1) = \varepsilon(z)$. By replacing $U1 = D, U2 = \frac{1}{\varepsilon}\frac{dD}{dz}$ from the linear differential equation of the second order we come to the system of two equations of the first order:

$$\begin{cases} \dfrac{dU1}{dz} - \varepsilon(z)U2 = 0, \\ \dfrac{dU2}{dz} + \Omega(z)U1 = 0, \end{cases} \qquad (2)$$

where $\Omega(z) = \frac{L^2}{\varepsilon(z)}\left(k^2\varepsilon(z) - k_\perp^2\right)$. The solution of the above system of linear homogeneous differential equations with periodic coefficients can be written in the Floquet form:

$$\begin{aligned} U1 &= Cf_1\exp(i\psi z) + C^* f_1^* \exp(-i\psi z), \\ U2 &= Cf_2\exp(i\psi z) + C^* f_2^* \exp(-i\psi z), \end{aligned} \qquad (3)$$

where f_1, f_2, ψ are determined by the fundamental solutions of the equation (1). Functions f_1, f_2 are periodic ($f_1(z+1) = f_1(z)$, $f_2(z+1) = f_2(z)$), normalized by the following way: $f_1 f_2^* - f_2 f_1^* = -2i$.

In the case of periodic perturbation $\varepsilon(z) \to \varepsilon(z) + \Delta\varepsilon(z), \Delta\varepsilon(z+\Lambda) = \Delta\varepsilon(z)$ the set of equations (2) can be considered as inhomogeneous:

$$\begin{cases} \dfrac{dU1}{dz} - \varepsilon(z)U2 = \Delta\varepsilon(z)U2, \\ \dfrac{dU2}{dz} + \Omega(z)U1 = -\Delta\Omega\, U1, \end{cases} \qquad (4)$$

where $\Delta\Omega = \frac{\Delta\varepsilon}{\varepsilon(\varepsilon + \Delta\varepsilon)}k_\perp^2$. The solution of inhomogeneous set of equations is as follows:

$$\begin{aligned} U1 &= C(z)f_1\exp(i\psi z) + C^*(z)f_1^* \exp(-i\psi z), \\ U2 &= C(z)f_2\exp(i\psi z) + C^*(z)f_2^* \exp(-i\psi z), \end{aligned} \qquad (5)$$

PROBLEMS OF ATOMIC SCIENCE AND TECHNOLOGY. 2001, № 1.
Series: Nuclear Physics Investigations (37), p. 94-95.

where $C(z), C^*(z)$ are the solutions of the system of differential equations:

$$\frac{dC}{dz} = \frac{i}{2}\Big[C\Big(\Delta\varepsilon\,|f_2|^2 + \Delta\Omega\,|f_1|^2\Big) + \qquad (6a)$$

$$+ C^*\Big(\Delta\varepsilon\,\big(f_2^*\big)^2 + \Delta\Omega\,\big(f_1^*\big)^2\Big)\exp(-2i\psi\, z)\Big],$$

$$\frac{dC^*}{dz} = -\frac{i}{2}\Big[C^*\Big(\Delta\varepsilon\,|f_2|^2 + \Delta\Omega\,|f_1|^2\Big) +$$

$$+ C\Big(\Delta\varepsilon\, f_2^2 + \Delta\Omega\, f_1^2\Big)\exp(2i\psi\, z)\Big]. \qquad (6b)$$

Application of the averaging method to equations (6) becomes possible if we assume the following parameters $\Delta\varepsilon, \Delta\Omega$ are small. The application of the above method is based on the idea that if the derivatives are small, than values of functions can be naturally seen as the superposition of slowly varying part $\overline{C}$ and small rapidly oscillating terms. Considering, that these terms cause only small oscillations of real function about its mean part, it can be neglected in zero-order approximation: $C = \overline{C}$. The right-hand part of equations (6) is averaged on explicitly contained variable z:

$$\frac{dC}{dz} = i\left\langle \frac{1}{2}\Big(\Delta\varepsilon|f_2|^2 + \Delta\Omega\,|f_1|^2\Big)\right\rangle C +$$

$$+ \left\langle \frac{i}{2}\Big(\Delta\varepsilon\big(f_2^*\big)^2 + \Delta\Omega\,\big(f_1^*\big)^2\Big)\exp(-2i\psi z)\right\rangle C^*,$$

$$\frac{dC^*}{dz} = -i\left\langle \frac{1}{2}\Big(\Delta\varepsilon|f_2|^2 + \Delta\Omega\,|f_1|^2\Big)\right\rangle C^* + \qquad (7)$$

$$+ \left\langle \frac{i}{2}\Big(\Delta\varepsilon f_2^2 + \Delta\Omega\, f_1^2\Big)\exp(2i\psi z)\right\rangle C,$$

where $\langle ...\rangle = \lim\limits_{T\to\infty}\frac{1}{T}\int\limits_0^T ...$. At first, consider the second items in the equations (7). The periodic functions can be decomposed in Fourier series. After the carrying out the averaging operation, only the items satisfying the following condition remain:

$$\psi = \pi\left(\frac{l}{\Lambda} + n\right) + p = \psi_{n,l} + p, \qquad (8)$$

where $l = 0, \pm 1, \pm 2, ...$, $n = 0, \pm 1, \pm 2, ...$, $p << 1$. By introducing the following descriptions

$$\delta\psi = \left\langle \frac{1}{2}\Big(\Delta\varepsilon|f_2|^2 + \Delta\Omega\,|f_1|^2\Big)\right\rangle,$$

$$\Delta\psi = \left\langle \frac{i}{2}\Big(\Delta\varepsilon\big(f_2^*\big)^2 + \Delta\Omega\,\big(f_1^*\big)^2\Big)\exp(-2i\psi_{n,l}z)\right\rangle$$

one can receive such equations:

$$\frac{dC}{dz} = i\delta\psi\, C + \Delta\psi\, C^*\exp(-2ipz), \qquad (9)$$

$$\frac{dC^*}{dz} = -i\delta\psi\, C^* + \Delta\psi^*\, C\exp(2ipz).$$

These equations (9) describe the phenomenon of a parametric resonance [3]. In the considered medium the resonance is conditioned by the interaction of spatial harmonics of periodic perturbation and plane waves, on which the solution of wave equation in periodic medium is decomposed. The resonance bands correspond to forbidden bands of electromagnetic waves. As all previous presumptions were made only for objective ψ, the forbidden bands occur inside the allowed bands of unperturbed periodic medium. The position of the forbidden bands is given by the formula:

$$\psi = \pi\left(\frac{l}{\Lambda} + n\right) + \delta\psi = \psi_{n,l} + \delta\psi, \qquad (10)$$

where $l = 0, \pm 1, \pm 2, ...$; n=0, ±1, ±2, ... The width of forbidden band is as follows:

$$2|\Delta\psi| = \left|\left\langle i\begin{pmatrix}\Delta\varepsilon\big(f_2^*\big)^2 + \\ \Delta\Omega\big(f_1^*\big)^2\end{pmatrix}\exp(-2i\psi_{n,l}z)\right\rangle\right|. \qquad (11)$$

The number of appearing bands depends on Λ. If Λ is integer ($\Lambda > 1$), $\Lambda - 1$ forbidden bands appear in each allowed band of undisturbed medium. m-1 forbidden bands appear if Λ is a rational number, $\Lambda = m/j$. Such a forbidden bands structure for layered medium with meander perturbation was observed in [4].

3. CONCLUSION

Influence of small periodic perturbation in periodic dielectric medium on propagation of TM−polarized waves is investigated by the Krylov−Bogolyubov averaging method. As a result of periodic perturbation inside the allowed bands of undisturbed medium forbidden bands appear. The location, number and width of these bands are determined.

By using the averaging method, it is possible to evaluate the influence of the sum of perturbations on periodic medium dispersion properties. For example, for two perturbations with periods $\Lambda_1 = m_1/j_1$, $\Lambda_2 = m_2/j_2$ the number of forbidden bands is determined according to the formula: $(M - 1)$, where M is the least aliquot for m_1 and m_2.

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