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ON THE FW ABSORPTION IN MULTI-ION COMPONENT PLASMAS AT THE FUNDAMENTAL ICR HARMONIC IN THE CASE OF QUASIPERPENDICULAR PROPAGATION

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1. Introduction

In the work [1], devoted to the study of influence of the strong space dispersion upon the absorption of fast wave (FW) in the range of fundamental harmonic of ICR, it was shown that the formulae for the calculation of the absorption that were obtained during the 60s' are not exact enough in the case of the quasiperpendicular propagation of FW. The formulae were obtained in the frame of the finite Larmor radius approximation up to second order (FLR_2-approximation) [2]. The calculations of the exact absorption coefficient for any angle of FW propagation requires to take into account more strong space dispersion effects, viz. the finite Larmor radius approximation up to 4th order (FLR_4-approximation).

Such calculations were made and they gave corrections to the absorption coefficient and the propagating properties of FW in single-ion component plasmas in the case of quasitransverse propagation.

In the present report the results of the work [1] are developed with higher exactness and generalized to the case of two-ion component plasmas.

2. Calculation of the FW absorption taking into account the strong space dispersion.

We shall look for the absorption of FW by means of the solution of dispersion equation, that connects components of dielectric tensor with refractive index. The full dispersion equation describing FW, SW, and IBW has the form:

$$\varepsilon_{11}N_\perp^4+2N_{//}\varepsilon_{13}N_\perp^3+[-\varepsilon_{11}(\varepsilon_{22}-N_{//}^2)-\varepsilon_{33}(\varepsilon_{11}-N_{//}^2)+\varepsilon_{13}^2-\varepsilon_{12}^2]N_\perp^2+2N_{//}[\varepsilon_{12}\varepsilon_{23}-(\varepsilon_{22}-N_{//}^2)\varepsilon_{13}]N_\perp+\varepsilon_{33}[(\varepsilon_{11}-N_{//}^2)(\varepsilon_{22}-N_{//}^2)+\varepsilon_{12}^2]+2\varepsilon_{12}\varepsilon_{23}\varepsilon_{13}-(\varepsilon_{22}-N_{//}^2)\varepsilon_{13}^2+(\varepsilon_{11}-N_{//}^2)\varepsilon_{23}^2=0, \quad (1)$$

where ε_{ik} are the components of dielectric tensor that can be written as expansions in the parameter $(k_\perp\rho_\alpha)^2$ (index ,α, denotes resonant sort of ions). The equation (1) can be transformed into a double quadratic equation by taking into account that ε_{13} and ε_{23} are proportional to $N_\perp$. It is easy to show that the terms with ε_{13} and ε_{23} are mutually cancelled. The FW and SW branches are easily detached in the ICRF range because of the relation $\varepsilon_{33} >> \varepsilon_{11}, \varepsilon_{12}, \varepsilon_{22}$. As a consequence, the usual equation for this branch in the ICR frequency range can be used to calculate the FW absorption:

$$(\varepsilon_{11}-N_{//}^2)N_\perp^2-(\varepsilon_{11}-N_{//}^2)(\varepsilon_{22}-N_{//}^2)-\varepsilon_{12}^2=0, \quad (2)$$

The solution of this equation can be found, of course, numerically but, moreover, one can deal with it analytically. Considering the size of the several terms that appear in (2) and doing some approximations, as keeping only the main terms, we obtain the following dispersion equation for FW which is valid for any value of $N_{//} \geq (k_\perp\rho_\alpha)^2$:

$$N_\perp^4\left(-\tfrac{1}{4}i\frac{\omega_{p\alpha}^2}{\omega^2}\sqrt{\pi}\,Z_{0\alpha}\rho_\alpha^4\frac{\omega^4}{C^4W^2}W_{-1\alpha}\right)+N_\perp^2-(N_F^0)^2=0, \quad (3)$$

where $(N_F^0)^2=(\varepsilon_1^0-\varepsilon_2^0-N_{//}^2)(\varepsilon_1^0+\varepsilon_2^0-N_{//}^2)/(\varepsilon_1^0-N_{//}^2)$ is the square of refractive index for FW in the "cold" approximation, $Z_{0\alpha}=\omega/\sqrt{2}\,k_{//}V_{T\alpha}$, $W_{-1\alpha}=W(Z_{-1\alpha})$ is the plasma dispersion function of the argument $Z_{-1\alpha}=(\omega-\omega_{C\alpha})/\sqrt{2}\,k_{//}V_{T\alpha}$, $V_{T\alpha}=\sqrt{T_\alpha/m_\alpha}$ is the thermal velocity of resonant ions, $W=\omega/\omega_{C\alpha}$. The roots of equation (3) describe FW properties (exactly for $N_{//} \geq (k_\perp\rho_\alpha)^2$) and any Bernstein wave (with less level of accuracy). For FW we will have:

$$\mathrm{Im}\,N_\perp^2=\frac{(X-1)\mathrm{Re}W_{-1\alpha}-Y\,\mathrm{Im}\,W_{-1\alpha}}{d\,|W_{-1\alpha}|^2}$$

$$\mathrm{Re}N_\perp^2=\frac{(X-1)\mathrm{Im}\,W_{-1\alpha}+Y\,\mathrm{Re}W_{-1\alpha}}{d\,|W_{-1\alpha}|^2}, \quad (4)$$

where we have defined the quantities $d=\sqrt{\pi}\,N_{A\alpha}^2 Z_{0\alpha}\omega^4\rho_\alpha^4/C^4$; $N_{A\alpha}^2=\omega_{p\alpha}^2/\omega_{C\alpha}^2$;

$X=[\frac{1}{2}([1+d^2|(N_F^0)|^2|W_{-1\alpha}|^2+2d(\mathrm{Im}(N_F^0)^2\mathrm{Re}W_{-1\alpha}+\mathrm{Re}(N_F^0)^2\mathrm{Im}W_{-1\alpha})]^{1/2}+1+d(\mathrm{Im}(N_F^0)^2\mathrm{Re}W_{-1\alpha}+\mathrm{Re}(N_F^0)^2\mathrm{Im}\,W_{-1\alpha}))]^{1/2}$,

$Y=[\frac{1}{2}([1+d^2|(N_F^0)|^2|W_{-1\alpha}|^2+2d(\mathrm{Im}(N_F^0)^2\mathrm{Re}W_{-1\alpha}+\mathrm{Re}(N_F^0)^2\mathrm{Im}W_{-1\alpha})]^{1/2}-1-d(\mathrm{Im}(N_F^0)^2\mathrm{Re}W_{-1\alpha}+\mathrm{Re}(N_F^0)^2\mathrm{Im}\,W_{-1\alpha}))]^{1/2}$,

These expressions describe the dispersion and absorption of| FW for any value of $N_{//} \geq (k_\perp\rho_\alpha)^2$ near the resonance $\omega=\omega_{C\alpha}$. For instance, we can consider exactly the resonance case, where $\mathrm{Re}W_{-1\alpha}=1$ and $\mathrm{Im}W_{-1\alpha}=0$. We shall have the following possibilities:

1) $N_{//}>>1$ ($d<<1$) $\mathrm{Im}N_\perp^2=(\mathrm{Im}(N_F^0)^2+\frac{1}{4}|(N_F^0)^2|^2\sqrt{\pi}\,N_{A\alpha}^2 Z_{0\alpha}\omega^4\rho_\alpha^4/C^4$, $\mathrm{Re}N_\perp^2=\mathrm{Re}(N_F^0)^2$

2) $N_{//}<<1$ ($d>>1$) $\mathrm{Im}N_\perp^2=\mathrm{Re}N_\perp^2 \cong$

$$\frac{1}{\sqrt{2}}\frac{|(N_F^0)^2|^{1/2}}{d^{1/2}}, \quad (5)$$

For the typical plasma parameters of "T-10" tokamak (B_0=3T, $n_e=7*10^{13}cm^{-3}$, $T_\alpha=T_\beta=T_e$=3KeV), one obtains that the absorption is enhanced about 2 times as compared with "cold" case at $N_{//}=3$. When the value of $N_{//}$ decreases the absorption sharply increases and at about $N_{//}=0.1$ one has $\mathrm{Im}N_\perp^2 \sim \mathrm{Re}N_\perp^2$. Furthermore, when the value of $N_{//}$ goes to 0, then $\mathrm{Im}N_\perp^2$ goes to 0 too, in accordance with expression (5). From the expression (5) it follows that additional absorption appears when $1+2n_\beta z_\beta/n_\alpha z_\alpha(1+\xi)<\sqrt{\pi}\, Z_{0\alpha}(k_\perp\rho_I)^2$ (here $\xi=z_\beta m_\alpha/z_\alpha m_\beta$), since $\mathrm{Im}(N_F^0)^2=\mathrm{Re}(N_F^0)^2(1+\ 2n_\beta z_\beta/n_\alpha z_\alpha(1+\xi)/(2\sqrt{\pi}Z_{0\alpha})$. Thus, if the concentration of resonant ions, n_α, is more or order of the concentration of nonresonant ions, n_β, then additional dissipation appears when $Z_{0\alpha}(k_\perp\rho_I)^2>1$ or $N_{//}<(k_\perp\rho_I)^2C/V_{T\alpha}$. This condition coincides with analogical one for single-ion plasmas [1].

To evaluate the increasing of absorption that this calculations provide in the case of n_α is larger or of the order than of n_β, it is useful to compute the optical thickness $\tau=\int \mathrm{Im}\, k_\perp dR$. This quantity is estimated assuming that the magnetic field B_0 changes as 1/R. ICR absorption is proved to be strong only in a narrow domain $\Delta R=R_0/Z_\alpha$ in this case. For the case of cold plasma, we have :

$$\tau=\mathrm{Im}(k_\perp\Delta R)=\mathrm{Re}k_\perp\Delta R/(4\sqrt{\pi}Z_0)=k_A R_0/(4\sqrt{\pi}Z_{0\alpha}^2), \quad (6)$$

For the case of "hot" plasma it is necessary to use the expression (4) to calculate $\mathrm{Im}k_\perp$. Then we have

$$\tau=\mathrm{Im}k_\perp\Delta R=\frac{1}{2}\frac{\mathrm{Im}\,N_\perp^2}{\mathrm{Re}\,N_\perp^2}K_A\frac{R_0}{Z_{0a}}=\frac{1}{2}\frac{X-1}{2Y}K_A\frac{R_0}{Z_{0\alpha}}, \quad (7)$$

This formula is simpler for the extreme cases:

a) $N_{//}>>1$ (d<<1), $\tau=k_A(3\sqrt{\pi}/8)R_0k^4\rho_\alpha^4+k_A\dfrac{R_0}{4\sqrt{\pi}Z_{0\alpha}^2}$, (8)

b) $N_{//}<<1$ (d>>1), $\tau=k_AR_0/(2Z_{0\alpha})+k_A\dfrac{R_0}{4\sqrt{\pi}Z_{0\alpha}^2}$, (9)

Thus when $N_{//}>>1$, the optical thickness does not depend on $N_{//}$ and therefore, it may be estimated using formula (6) for "cold" plasma at the value of $N_{//}$ for which the full absorption and "cold" one are approximately the same. This value of $N_{//}$ is easy to obtain from the equation $Z_{0\alpha}(k_\perp\rho_I)^2=1$. The calculations for the same plasma parameters as above give for a middle device of the type of tokamak "T-10" a small value of optical thickness $\tau=10^{-4}$. However for large devices of the type of "ITER" tokamak one has $\tau=0.5$ and therefore, RF heating by FW at first harmonic resonance becomes rather interesting as a heating regime. Note also the fact that the optical thickness does not depend on $N_{//}$, except for very small values of $N_{//}$, makes easier the task of injecting high frequency power into plasma due to the possibility of using smaller values of $N_{//}$.

3. The physical nature of the "additional" FW absorption

To find out the nature of appearing of the FW additional absorption in the case of quasiperpendicular propagation, let us calculate FW polarisation in "cold" and "hot" cases. In the "cold" case, it can be readily obtained from the wave equation:

$$\frac{E_X}{E_Y}=\frac{\varepsilon_1-N_{//}^2}{i\varepsilon_2}\approx i(-1+\frac{3}{2\pi Z_{0\alpha}^2})-\frac{1}{Z_{0\alpha}\sqrt{\pi}}\approx(-i)-\frac{[1+2n_\beta z_\beta/n_\alpha z_\alpha(1+\xi)]}{\sqrt{\pi}Z_{0\alpha}}$$

where the value of the imaginary part corresponds to circular polarisation of FW and its sign corresponds to the direction of rotation of the electrical field vector, against the direction of ion rotation. Since one has that $Z_{0\alpha}>>1$, the real part of the polarisation introduces a weak (if $n_\alpha\geq n_\beta$) ellipticity and is the responsible of the existence of weak absorption of order $\sim|\frac{E_Y}{E_X}|^2 \sim \frac{[1+2n_\beta z_\beta/n_\alpha z_\alpha(1+\xi)]^2}{\pi Z_{0\alpha}^2}$. In the "hot" case the polarisation is given by:

$$\frac{E_Y}{E_X}\approx i(-1-x+...)-\frac{[1+2n_\beta z_\beta/n_\alpha z_\beta(1+\xi)]}{2\sqrt{\pi}Z_{0\alpha}(\frac{1}{2}-x+...)}$$

where $x=(k_\perp\rho_i)^2$,. From this expression it follows that in the real part there are not practically changes in comparison with the "cold" case, because x<<1 for FW. However it appears a small addendum in the imaginary part which is proportional to x. This term introduces additional ellipticity in the polarisation and, consequently, enhances the FW absorption. This effect is due to the fact that the absorption is proportional to $\sim|\frac{E_Y}{E_X}|^2$, then the absorption strengthening must be ~ $x^2=(k_\perp\rho_i)^4$. Comparison of the additional "hot" absorption with "cold" one gives

$$\frac{D_{hot}}{D_{cold}}=\pi(k_\perp\rho_\alpha)^4Z_{0\alpha}^2/[1+2n_\beta z_\beta/n_\alpha z_\alpha(1+\xi)]^2$$

From that appreciation it follows that when $N_{//}$ decreases the additional "hot" absorption may become stronger than the "cold" one (if $n_\alpha\geq n_\beta$). Thus, if the value of $N_{//}$ is small enough the "hot" addition in the polarization will enhance the FW absorption. The calculation of this absorption using the more strict formula gives, in terms of the main parameter $x=(k_\perp\rho_\alpha)^2$, the following result

$$D=\frac{\omega}{8\pi}|E_X|^2\sqrt{\pi}N_{A\alpha}^2Z_{0\alpha}\exp(-Z_{-1\alpha}^2)\left[\frac{x^2}{4}+\frac{(1+2\frac{n_\beta z_\beta}{n_\alpha z_\alpha(1+\xi)})^2}{2\pi Z_{0\alpha}^2(\exp(-Z_{-1\alpha}^2))^2}\right]$$

where the first term in brackets is connected with the "hot" absorption and the second term with the "cold" one.

References

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2. Akhiezer A.I., Akhiezer I.A., Polovin A.V., Sitenko A.G., Stepanov K.N., Plasma Electrodynamic,Pergamon Press, Oxford (1975)