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MAGNETOHYDRODYNAMIC WAVE SPECTRA IN LARGE TOKAMAKS WITH NON-CIRCULAR CROSS SECTION OF MAGNETIC SURFACES

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There is considered the problem on the spectra of magnetohydrodynamic (MHD) waves with the frequencies of the order of the ion cyclotron frequency propagating almost along the toroidal magnetic field in the large-size tokamaks with a non-circular cross section of magnetic surfaces. In this case the waves can propagate in the small vicinity of the extremum point for the square of the refractive index of the MHD wave travelling along the torus.

Developing the dielectric permittivity tensor of the "cold" plasma in a Taylor series around this point enables one to separate the variables and to express the natural functions of the boundary problem for Maxwell's equations through Hermite functions and to determine the natural frequencies of MHD waves. The solution obtained is a generalization of the previous result for arbitrary radial and azimuth numbers. On the ground of the perturbation theory the corrections to the natural functions and natural values are found which take into account the rotational transform.

1. Introduction

The flows of fast ions formed under injection of fast neutral particles in a reactor-tokamak may provide for the induction-free current and plasma equilibrium. But the strongly nonequilibrium distribution of fast ions may lead to the excitation of unstable Alfven and fast magnetosonic waves when the current density exceeds the threshold value determined by the wave damping due to electron Cherenkov and ion cyclotron damping.

The consideration of this problem for the ITER tokamak has shown that the threshold value is determined by the excitation of first radial modes in a simplified plasma model of a hollow cylinder. It has appeared that the threshold value is highly sensitive to the parameters of the ion beam and of the waves [1,2].

In order to solve the problem on the excitation of such waves by the beam of fast ions, it is necessary to know the natural frequencies of MHD waves for a more realistic model. This problem is solved in the present paper where the dispersion equation is obtained for the MHD waves propagating almost along the magnetic field in a tokamak with a non-circular cross section of magnetic surfaces.

2. Dispersion equation

From the Maxwell's equations for the plasma with small β for the MHD waves $\propto \exp(im\varphi - i\omega t)$ where m is the toroidal mode number, with neglect of the longitudinal inertia of electrons $(|\varepsilon_3| \to \infty)$ there follows the set of two equations for the left- and right-hand polarized components of the wave field $E_\pm = E_R \pm iE_Z$:

$$\left(\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R}+\frac{\partial^2}{\partial Z^2}-\frac{i}{R}\frac{\partial}{\partial R}\right)E_+ + k_+^2 E_+ =$$
$$\left(\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R}-\frac{\partial^2}{\partial Z^2}+2i\frac{\partial^2}{\partial R\partial Z}+\frac{i}{R}\frac{\partial}{\partial R}\right)E_-,$$

$$\left(\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R}+\frac{\partial^2}{\partial Z^2}+\frac{i}{R}\frac{\partial}{\partial R}\right)E_- + k_-^2 E_- =$$
$$\left(\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R}-\frac{\partial^2}{\partial Z^2}-2i\frac{\partial^2}{\partial R\partial Z}-\frac{i}{R}\frac{\partial}{\partial R}\right)E_+ \quad (1)$$

where

$$k_\pm^2 = 2\left[\frac{\omega^2}{c^2}(\varepsilon_1 \pm \varepsilon_2) - \frac{m^2}{R^2}\right], \quad (2)$$

$$\varepsilon_1 \approx \sum_i \frac{\omega_{pi}^2}{\omega_{Ci}^2 - \omega^2}, \ \varepsilon_2 \approx \sum_i \frac{\omega_{pi}^2 \omega}{\omega_{Ci}(\omega_{Ci}^2 - \omega^2)}, \quad (3)$$

R, φ and Z are the cylindrical coordinates, $\omega_{Ci} = eB_\varphi / m_i c$, $B_\varphi = B_0(R_c/R)$ is the toroidal magnetic field, R_c is the large radius of the toroidal chamber, $\omega_{pi}(R,Z)$ is the ion Langmuir frequency, the subscript i denotes the summation over ion species. In the equations (1) we neglect the effect of a small poloidal field $B_p = \sqrt{B_R^2 + B_Z^2}$ compared with the toroidal field B_φ.

As we consider the waves propagating almost along the torus, then within the region of wave localization $R \approx R_0, Z \approx Z_0$ it is necessary to meet one of the approximate equalities

$$\varepsilon_1 \pm \varepsilon_2 = \sum_i \frac{\omega_{pi}^2}{\omega_{Ci}(\omega_{Ci} \mp \omega)} \approx (mc/R\omega)^2 \quad (4)$$

For certainty we consider further the left-hand-polarized waves (Alfven waves) for which $\varepsilon_1 + \varepsilon_2 \approx (mc/R\omega)^2$. In this case $k_\pm^2 << k_-^2$ and one can neglect the right-hand side of the first equation of the set (1) proportional to E_- and obtain the following equation

$$\left(\frac{\partial^2}{\partial R^2}+\frac{1}{R}\frac{\partial}{\partial R}\right)E_+ + \left(\frac{\partial^2}{\partial Z^2}-\frac{i}{R}\frac{\partial}{\partial Z}\right)E_+ + k_+^2 E_+ = 0 \quad (5)$$

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which is valid for an arbitrary 2-D nonuniformity across the plasma column $k_+^2 = k_+^2(R,Z)$. Just this equation (5) will form the ground for studying the spectra of natural MHD oscillations of the toroidal plasma column. Assuming that the quantity k_+^2 approaches its maximum at the point $R = R_0, Z = Z_0$, we develop the function k_+^2 into the Taylor series in powers of small deviations $r = R - R_0, z = Z - Z_0$. Then we obtain

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{R_0}\frac{\partial}{\partial r}\right)E_+ + \cdot \tag{6}$$
$$\left(\frac{\partial^2}{\partial z^2} - \frac{i}{R_0}\frac{\partial}{\partial z}\right)E_+ + \left[k_0^2 - ar^2 - bz^2 - crz\right]E_+ = 0,$$

where $k_0^2 = k_+^2(R_0, Z_0)$ and

$$a = -\frac{1}{2}\frac{\partial^2 k_+^2(R_0,Z_0)}{\partial R_0^2},\ b = -\frac{1}{2}\frac{\partial^2 k_+^2(R_0,Z_0)}{\partial Z_0^2},$$
$$c = -\frac{\partial^2 k_+^2(R_0,Z_0)}{\partial R_0 \partial Z_0}. \tag{7}$$

For a symmetric location of the plasma column with respect to the plane *Z=0 c=0*. Then the variables *r,z* can be separated straightforward. To perform the separation of variables in the equation (6) with arbitrary location of the column, we will make, as in [3], the change of variables:

$$r' = \frac{\alpha r + z}{1+\alpha^2},\ z' = \frac{r - \alpha z}{1+\alpha^2}, \tag{8}$$

where $\alpha = \alpha_{1,2}$, $\alpha_{1,2} = \frac{a-b}{c} \pm \sqrt{\left(\frac{a-b}{c}\right)^2 + 1}$. (9)

This change not only makes diagonal the expression k_+^2 in (6), but also provides for the absence of mixed derivatives with respect to new variables. After this change we obtain for E_+ the equation (6) in which one should make the following substitutions:

$r \to r',\ z \to z'$,

$1/R_0 \to (\alpha - i)/R_0,\ -i/R_0 \to (1 + i\alpha)/R_0$,

$k_0 \to k_0',\ a \to p, b \to q,\ c \to 0$,

where $p = (1+\alpha^2)(a\alpha^2 + b + c\alpha)$,

$$q = (1+\alpha^2)(a + b\alpha^2 - c\alpha), \tag{10}$$
$$k_0'^2 = (1+\alpha^2)k_0^2.$$

The solution of the modified equation (6) may be found by separating the variables:

$$E_+ \propto H_\ell(\xi)H_n(\eta)\exp[-(\mu\xi + \nu\eta)/2], \tag{11}$$

where H_ℓ, H_n are the Hermite functions,

$$\xi = \sqrt[4]{p}\,r',\ \eta = \sqrt[4]{q}\,z',$$
$$\mu = \frac{\alpha - i}{\sqrt[4]{p}R_0},\ \nu = \frac{1 + i\alpha}{\sqrt[4]{p}R_0}. \tag{12}$$

The frequencies of natural MHD oscillations are found from the dispersion equation

$$k_0'^2 = \sqrt{p}(2\ell + 1) + \sqrt{q}(2n + 1). \tag{13}$$

3. Analytic solution

Consider the simplest solution of the equation (13) for a single ion species at $\omega \le \omega_{Ci}$. Then in the zeroth approximation we will have for the frequency of natural oscillations the formula

$$\omega_0 = \omega_\pm = -\frac{1}{2}\frac{m^2 v_A^2}{R_0^2\omega_{Ci}} \pm \sqrt{\left(\frac{m^2 v_A^2}{2R_0^2\omega_{Ci}}\right)^2 + \frac{m^2 v_A^2}{R_0^2}} \tag{14}$$

where $v_A(R_0,Z_0) = c\,\omega_{Ci}(R_0)/\omega_{pi}(R_0,Z_0)$. To the order of magnitude we have

$$\omega_0 \sim \omega_{Ci} \quad \text{at} \quad \frac{mc}{\omega_{pi}R_0} \sim 1 \tag{15}$$

In this case the condition $k_\pm^2 << k_-^2$ holds if $k_F a_p \sim m a_p / R_0 >> 1$, where a_p is the characteristic distance over which the density varies, i.e., this condition takes place for tokamaks with large dimensions.

According to (13), the correction to the frequency (14) taking into account the finite values of the transverse wavenumbers is

$$\frac{\Delta\omega}{\omega_0} \approx \frac{v_A^2(1 - \omega_0/\omega_{Ci})}{\omega_0^2\left[2 + \omega_0/(\omega_{Ci} - \omega_0)\right]}\frac{1}{1+\alpha^2} \times$$
$$\times\left[\sqrt{p}(2\ell + 1) + \sqrt{q}(2n+1)\right]. \tag{16}$$

Using the expressions (10), we get for the correction (16) the order-of-magnitude estimate

$$\frac{\Delta\omega}{\omega_0} \sim \left[|2\ell + 1| + |2n + 1|\right]\frac{R_0}{m a_p} << 1. \tag{17}$$

Note that for the high number modes with $|\ell| + |n| >> 1$ the dispersion equation (13) was obtained by this method in the report [3], and for the first modes with $\ell, n \sim 1$ in the report [4].

Taking into account the poloidal magnetic field of the tokamak will lead to the correction to the MHD wave frequency that is less than the correction (16):

$$\frac{\delta\omega}{\omega_0} \sim \frac{R_0}{m a_p}\frac{B_{R,Z}}{B_\varphi}. \tag{18}$$

The consideration of the right-hand-polarized wave with this technique will lead to similar results.

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