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TO THE THEORY OF PLASMA WAVES IN PERIODIC PLASMA WAVEGUIDES

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Forbidden bands for plasma waves in periodical plasma waveguides are predicted and correctly studied. It was shown that the periodicity influence over frequencies far from the forbidden bands is neglectfully small even for the comparatively large corrugation depth. At the same time while frequency is approaching to any of forbidden bands periodicity influence over dispersion properties and field distribution increases. A group velocity decreases down to zero on the boundaries of forbidden bands. Plasma wave fields concentrates in small space domains and concentration power increases by approaching to the cut off frequencies. This is caused by higher space harmonics contribution which unlike electromagnetic waves in periodical vacuum waveguides have spatial nature and their calculation is of principle.

1. Introduction

Periodic plasma-filled waveguide structures are widely used in plasma microwave electronics for development of efficient methods of microwave generation and for charged particles acceleration. Also they are used in plasma heating systems.

However, despite of a great number of practical applications, dispersion properties of plasma-filled periodic structures at frequencies below plasma one have not been studed yet even qualitatively.

As it is well known, plasma filled periodic waveguides support two sets of modes: electromagnetic and plasma ones. In the simplest case of transversally homogeneous plasma electromagnetic waves occure at frequencies more then plasma one do not overlaping with plasma waves which exist at frequencies below plasma one. In contrast with electromagnetic modes, which can be successfully analyzed by conventional approaches, plasma modes form so-called “dense” spectrum [1,2]. Such spectral behavior being quite different from usual one can not be described on the basis of conventional approaches which are widely used for the analysis of electromagnetic modes in periodic structures [3].

In this article the plasma “dense” spectrum is investigated basing on a new approach [4,5] according to which, the corresponding spectral problem is formulated in the kind of some homogeneous integral equation. It allowed us to correctly calculate the dispersion curves for plasma modes revealing some new features. Particularly, forbidden bands for plasma waves in periodical plasma waveguides are predicted and correctly studied. It was shown that the periodicity influence at frequencies far from the forbidden bands is neglectfully small even for the comparatively large corrugation depth. At the same time while the mode frequency is approaching to any of forbidden bands periodicity influence on dispersion properties and field distribution significantly increases. A group velocity decreases down to zero on the boundaries of forbidden bands (cut off frequencies). Meanwhile, plasma wave fields concentrates in small space domains and the field concentration increases at approaching to the cut off frequencies. This is caused by higher space harmonics contribution which unlike electromagnetic waves in periodical vacuum waveguides have spatial nature and their calculation is of principle.

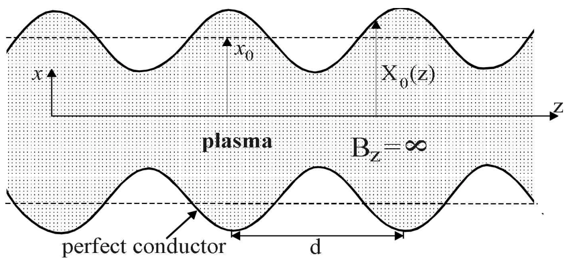

Fig.1. *Geometry of the problem*

II. Formulation of the Problem

Let’s consider a planar waveguide with periodically varying thickness loaded with cold homogenous collisionless plasma with a longitudinally applied infinite magnetic field (see, Fig.1). We suppose that all perturbations are of TM – polarization $\left(E_x, H_y, E_z\right) \propto e^{-i\omega t}$ symmetric with respect of z-axis: $E_z(-x,z)=E_z(x,z)$.

In this case Maxwell equations can be reduced to only equation for the transverse magnetic field:

$$\left(\frac{\partial^2}{\partial x^2}+\varepsilon\frac{\partial^2}{\partial z^2}+\varepsilon k^2\right)H_y(x,z)=0 \qquad (1)$$

In the region of plasma wave existence $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} \prec 0$. So, making transition to a new variable $|\varepsilon|^{1/2} x = \tilde{x}$, from (1) we come to the telegrafist's equation

$$\left(\frac{\partial^2}{\partial \tilde{x}^2} - \frac{\partial^2}{\partial z^2} + k^2 \right) \tilde{H}_y(\tilde{x}, z) = 0 \qquad (2)$$

where $\tilde{H}_y(\tilde{x}, z) = H_y(x, z)$.

Equation (2) is a hyperbolic type equation. Its solution can be expressed with integrals of the unknown function and its derivative on the z-axis:

$$\tilde{H}_y(\tilde{x}, z) = \frac{1}{2} \int_{z-\tilde{x}}^{z+\tilde{x}} J_0 (k \sqrt{(z-\zeta)^2 - \tilde{x}^2}) \times$$
$$\times g(\zeta) d\zeta \qquad (3)$$

where $g(\zeta) = \left. \frac{\partial \tilde{H}_y}{\partial \tilde{x}} \right|_{\tilde{x}=0}$

Making transition to the old variable we obtain

$$H_y(x, z) = \frac{1}{2} \int_{z-|\varepsilon|^{1/2} x}^{z+|\varepsilon|^{1/2} x} d\zeta g(\zeta) \times \qquad (4)$$
$$\times J_0 (k \sqrt{(z-\zeta)^2 - |\varepsilon| x^2})$$

Applying the boundary condition for the tangential component of the electric field on the waveguide wall $E_x \sin\alpha + E_z \cos\alpha|_{x=X(z)} = 0$, which is equivalent to the following condition for $H_y(x, z)$:

$$\left(X'(z) \frac{\partial H_y}{\partial z} - \frac{1}{\varepsilon} \frac{\partial H_y}{\partial x} \right) \Bigg|_{x=X(z)} = 0 \qquad (5)$$

we obtain the basic integral equation

$$g(z+\varphi(z))(1+\varphi'(z)) + g(z-\varphi(z)) \times$$
$$\times (1-\varphi'(z)) - k \int_{z-\varphi(z)}^{z+\varphi(z)} d\zeta [\varphi'(z)(z-\zeta) -$$
$$-\varphi(z)] \times \qquad (6)$$
$$\times \frac{J_1\left(k\sqrt{(z-\zeta)^2 - \varphi^2(z)}\right)}{\sqrt{(z-\zeta)^2 - \varphi^2(z)}} g(\zeta) = 0,$$

where $\varphi(z) = |\varepsilon|^{1/2} X(z)$.

Approximate integral value in the last equation can be derived by the spline functions. For this let's divide inergrating interval by the discrete sections and introduce following values $z_i = \frac{i}{N} 2d$, where N - is a number of discrete sections, and i - is a section number.

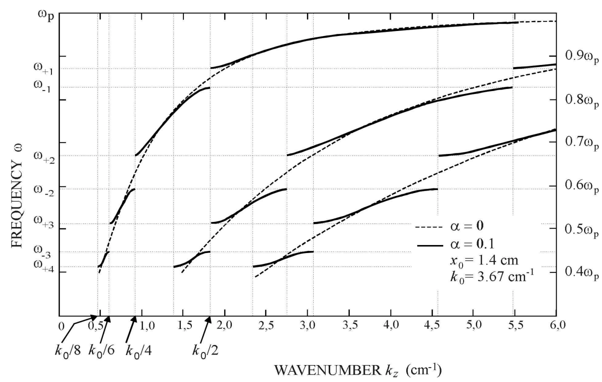

Fig.2. Dispersion diagrams for the first three plasma modes of the sinusoidal rippled plasma waveguide

$$M(i) = \left[\frac{z-\varphi(z)}{2d} \cdot N \right]$$
$$P(i) = \left[\frac{z+\varphi(z)}{2d} \cdot N \right] \qquad (7)$$

$M(i)$, $P(i)$ - are the numbers of the first and the last discrete sections correspondingly.

$$g(z+\varphi(z))(1+\varphi'(z)) + g(z-\varphi(z)) \times$$
$$\times (1-\varphi'(z)) + \frac{z_{M(i)+1} - (z-\varphi(z))}{2} \times$$
$$\left[G(z, z_{M(i)+1}) g(z_{M(i)+1}) - \frac{1}{2} k \cdot g(z-\varphi(z)) \right] +$$
$$+ \frac{z+\varphi(z) - z_{P(i)}}{2} \times \qquad (8)$$
$$\times \left[G(z, z_{P(i)}) g(z_{P(i)}) - \frac{1}{2} k \cdot g(z+\varphi(z)) \right] +$$
$$+ \frac{d}{2N} \sum_{k=M(i)+1}^{P(i)-1} [G(z, z_k) g(z_k) +$$
$$+ G(z, z_{k+1}) g(z_{k+1})] = 0$$

where $G(z, \zeta)$ is an integral kernel. Since the argument of the Bessel function is imaginary all over the inergrating interval we will make transition to the modified Bessel function

$$G(z,\zeta) = -k \cdot [\varphi'(z)(z-\zeta) - \varphi(z)] \times$$
$$\times \frac{J_1\left(k\sqrt{(z-\zeta)^2 - \varphi^2(z)}\right)}{\sqrt{(z-\zeta)^2 - \varphi^2(z)}} =$$
$$= -k \cdot [\varphi'(z)(z-\zeta) - \varphi(z)] \times \quad (9)$$
$$\times \frac{I_1\left(k\sqrt{\varphi^2(z) - (z-\zeta)^2}\right)}{\sqrt{\varphi^2(z) - (z-\zeta)^2}}$$

Computer simulations for the equation (8) can be carried out.

In the case when $c \to \infty$ the integral equation (6) coincides with the equation obtained in [4] based on expansion of fields in spatial harmonic series. Full mathematical identity of these approaches under the condition $\varphi'(z) \le 1$ was shown also in [5].

Now we consider the case of electrostatic waves ($c = \infty$). In this case the integral equation (6) transforms into the functional equation:

$$e^{ik_z\varphi(z)}\Psi(z+\varphi(z))(1+\varphi'(z)) + $$
$$+ e^{-ik_z\varphi(z)}\Psi(z-\varphi(z))(1-\varphi'(z)) = 0 \quad (10)$$

where the new function $\Psi(z) = E_z(0,z)e^{-ik_z z}$ has been introduced, k_z is a wavenumber of perturbations.

It was shown that increasing of exactness in numerical calculations of equation (10) does not lead to sufficient modification of numerical results obtained in [5].

III. Numerical Results

Numerical calculations of (13) at $n = 0$ have been performed by expansion of $\theta(z)$ into the series of the spline functions and into Fourier series. The results were identical for the large number of terms (~100) taken into account. Fig. 2 displays the dispersion curves for the first three plasma modes in the case of sinusoidally rippled waveguide: $X(z) = x_0(1+\alpha\cos(k_0 z))$ with parameters $\alpha = 0.1$, $k_0 = 3.67$ cm^{-1}, $x_0 = 1.4$ cm.

At these parameters plasma modes have four allowed bands:

$$\omega_p < \omega < \omega_{+1}, \omega_{-1} < \omega < \omega_{+2}, \omega_{-2} < \omega <$$
$$< \omega_{+3}, \omega_{-3} < \omega < \omega_{+4}$$

Where $\omega_{\pm q} = \omega_p / \left(1 + (\pi q / x_0 k_0 (1 \pm \alpha))^2\right)^{-2}$, $q = 1,2,3,4$.

Below ω_{+4} the equation (10) has no solution. Thus, it seems plasma modes in periodic plasma-filled structures have lower cut off frequency, which is determined mostly by the ripple height what has a clear physical meaning: at low frequencies a number of radial modes is very large and they are located very closely to each other. So, reflections from rippled walls of waveguide, which are provided by them become very efficient, blocking the propagation of any mode. However, we can not state this for sure since the lower cut off frequency usually lies out of region of validity of equation (10). The latter, in our case, is defined by the relation [5]: $\alpha k_0 x_0 |\varepsilon|^{1/2} \le 1$.

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