ABOUT MAGNETIC SUSCEPTIBILITY OF DENSE SUPERFLUID NEUTRON MATTER WITH SPIN-TRIPLET p-WAVE PAIRING

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Pure neutron matter with the spin-triplet p-wave pairing is studied in the framework of the non-relativistic generalized Fermi-liquid theory at subnuclear and supranuclear densities (in the range $0.7n_0 \leq n < n_C(\text{Skyrme}) < 2n_0$, where $n_0 = 0.17$ fm^{-3} is the saturation density of the symmetric nuclear matter) at zero temperature and in the presence of a strong magnetic field. The Skyrme effective forces are used as interactions between neutrons. As a result, the general expression (valid for an arbitrary parametrization of the Skyrme forces) is obtained for the magnetic susceptibility of superfluid neutron matter, and it is specified then for three types of the Skyrme interaction with different power dependences on the density n. In particular, it is found for neutron matter with the so-called RATP, Gs, and SLy2 parametrizations of the Skyrme forces that the magnetic susceptibility diverges at the densities $n_C(\text{RATP}) \approx 1.03n_0$, $n_C(\text{Gs}) \approx 1.33n_0$ and $n_C(\text{SLy2}) \approx 1.72n_0$. These critical densities correspond to phase transitions from the superfluid paramagnetic state of neutron matter with triplet pairing to the ferromagnetic state which coexists with triplet superfluidity at densities higher than $n_C(\text{Skyrme})$. Such phase transitions might occur in the liquid outer cores of pulsars and the so-called magnetars.

1. Introduction

Already fifty years have past since the idea of possible manifestations of the pairing (which was originally introduced in theory by J. Bardeen, L.N. Cooper, and J.R. Schrieffer [3]) and the superfluidity phenomenon in infinite nuclear matter and in finite nuclei and also in dense neutron stars (A.B. Migdal [4]) was proposed for the first time by N.N. Bogolyubov [1] and A. Bohr, B. Mottelson, D. Pines [2]. Note that pulsars were discovered later in 1967 by S.J. Bell and A. Hewish [5] (see also [6]), and then they were identified by T. Gold [7] as rapidly rotating neutron stars (NS). Pairing and superfluidity play an important role in modeling the structure and properties of atomic nuclei and pulsars. But, in spite of the enormous efforts of many investigators (see, e.g., monographs [9–14] and reviews [8,15–18] and references therein), there are still many unsolved questions concerning the superfluidity in neutron stars.

As is commonly accepted, neutron stars consist of the crust (with subnuclear densities $n \lesssim n_0/2$, where $n_0 = 0.17$ fm^{-3} is the saturation density of symmetric nuclear matter), and the core which are composed of the so-called outer crust and inner crust and of the outer core and inner core, respectively. They are distinguished from each other by composition and by densities. The deeper the layer in the interior of a neutron star, the denser it is. Note that neutrons are the prime constituent in the outer core of NS with a small fraction of protons and electrons.

Here, we will restrict ourselves by studying the equilibrium properties of the superfluid phases of infinite pure neutron matter (SPNM) with spin-triplet pairing existing inside a liquid outer core of neutron stars at subnuclear $0.7n_0 \lesssim n \lesssim n_0$ and supranuclear $n > n_0$ densities of neutrons. These superfluid phases of pure neutron matter are examples of superfluid Fermi liquids (SFLs) with spin-triplet pairing similar to ^{3}He (see, e.g., [15,19,20] and references therein). Here, we have investigated theoretically dense SPNM with p-wave pairing of the $^3\text{He} - A_{1,2}$ type in a stationary homogeneous magnetic field H and have used the generalized non-relativistic Fermi-liquid approach [21] to derive nonlinear integral equations for the order parameter (OP) and the effective magnetic field (EMF) $\mathbf{H}_{\text{eff}}$ inside SPNM [22–24] which are valid at arbitrary temperatures from the interval $0 \leq T \leq T_c$ (T_c is the normal-superfluid phase transition (PT) temperature). The effective Skyrme interaction between neutrons depending on the neutron density (see reviews [25, 26]) has been used.

Here, we have found analytically the approximate solution of the obtained integral equations at zero temperature $T = 0$ for SPNM with triplet p-wave pairing in a strong magnetic field H and have obtained the general approximate expression for the effective magnetic field H_{eff} (at $T = 0$) on the Fermi surface to the first order in the small parameter $h_{\rm ext} \equiv |\mu_n|H/\varepsilon_{\rm F} \ll 1$ (for an arbitrary parametrization of the Skyrme interaction and with the self-consistent accounting of the dependence of the neutron effective mass m_n^* on the NM density $n \equiv yn_0$). The function $H_{\rm eff}$ (at $T = 0$) is linear in H up to sufficiently high magnetic fields (but $H \ll \varepsilon_{\rm F}/|\mu_n|$, where $\varepsilon_{\rm F}(n)$ is the Fermi energy of NM, and $\mu_n < 0$ is the magnetic dipole moment of a neutron) and a nonlinear function of n. As a result, the general expression (valid for an arbitrary parametrization of the Skyrme forces) is obtained for the paramagnetic susceptibility $\chi_{\rm Skyrme}$ of superfluid neutron matter (at $T = 0$), and it is specified then for three types of the Skyrme interaction with different power dependences on the density n. In particular, it was found for neutron matter with the Sly2, Gs, and RATP parametrizations [27–29] of the Skyrme forces that the paramagnetic susceptibility is a monotonically increasing function of the neutron density (the corresponding figures were plotted), and it diverges at the critical densities n_C(Skyrme) in the range of densities $0.7n_0 \lesssim n < n_C(\text{Skyrme}) < 2n_0$ under consideration (where the non-relativistic Fermi-liquid theory is still valid). These critical densities n_C(Skyrme) correspond to phase transitions from the superfluid paramagnetic state of neutron matter with triplet pairing to the ferromagnetic state which coexists with triplet superfluidity at densities higher than n_C(Skyrme). Such phase transitions might occur in the liquid outer core of neutron stars.

Note that other authors have previously investigated the existence (or absence) of phase transitions of NM from the normal (nonsuperfluid) state to the ferromagnetic state in the absence of a magnetic field (see, e.g., [30–37] and references therein) and with the effects of a strong magnetic field (see, e.g., [38–40]) within other approaches and using different nucleon-nucleon effective and so-called realistic interactions in NM.

This paper is organized as follows. In the second section, we outline the main steps and assumptions made for the derivation of general equations for the order parameter and the effective magnetic field for SPNM with the Skyrme forces and spin-triplet pairing of the $^3\text{He} - A_{1,2}$ type between neutrons. The third section is devoted to the derivation of a general formula for the paramagnetic susceptibility in SPNM (valid for an arbitrary parametrization of the Skyrme forces) in a strong magnetic field at zero temperature, which is specified then for Sly2, Gs, and RATP parametrizations. In Conclusion, the general and particular results for SPNM with triplet pairing in a high magnetic field are briefly discussed and compared with those from some other works.

2. General Equations for the Order Parameter and the Effective Magnetic Field for SPNM with the Skyrme Forces and Triplet Pairing

The order parameter (OP) for the so-called non-unitary phase (NU) of $^3\text{He} - A_2$ type with spin-triplet p-wave pairing has the form [20]

$$\Delta_\alpha{}^{A_2}(\mathbf{p}) \equiv (\mathbf{\Delta}_+\hat{\mathbf{d}}_\alpha + \imath\mathbf{\Delta}_-\hat{\mathbf{e}}_\alpha)\psi(\hat{\mathbf{p}}),$$

$$\psi(\hat{\mathbf{p}}) \equiv (\hat{m}_j + \imath\hat{n}_j)\hat{p}_j, \quad \hat{\mathbf{p}} \equiv \frac{\mathbf{p}}{p}. \tag{1}$$

Here, $\Delta_\pm(T) \equiv (\Delta_\uparrow(T) \pm \Delta_\downarrow(T))/2$; $\hat{\mathbf{d}}$ and $\hat{\mathbf{e}}$ are mutually orthogonal real unit vectors in the spin space, $\hat{\mathbf{d}}\cdot\hat{\mathbf{e}} = 0$, $\hat{\mathbf{d}}^2 = \hat{\mathbf{e}}^2 = 1$; $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ are mutually orthogonal real unit vectors in the orbital space, $\hat{\mathbf{m}}\cdot\hat{\mathbf{n}} = 0$, $\hat{\mathbf{m}}^2 = \hat{\mathbf{n}}^2 = 1$. The value $\eta(\mathbf{p}) \equiv |\mathbf{\Delta}(\mathbf{p}) \times \mathbf{\Delta}^*(\mathbf{p})| \neq \mathbf{0}$ is non-zero for the NU superfluid phases of pure neutron matter with spin-triplet pairing, in particular. Note also that the superfluid phase of $^3\text{He} - A_1$ type is realized under the condition, when $\Delta_\downarrow = 0$, $\Delta_\uparrow \neq 0$.

We have chosen the effective Skyrme forces as the interaction between neutrons for SPNM with spin-triplet p-wave pairing in a spatially uniform magnetic field $\mathbf{H}$. A system of coupled equations for the OP of the $^3\text{He}-A_2$ type and the effective magnetic field $\mathbf{H}_{\rm eff}$ inside SPNM is simplified (in comparison with an analogous superfluid phase of real helium-3) because, in the case of the Skyrme interaction, the normal Fermi-liquid Landau's exchange amplitudes $F_l^a \neq 0$ are non-zero only for $l = 0$ and $l = 1$. We also assumed that the quantization axes of spin and orbital moment of the Cooper pairs (i.e. the vectors $[\mathbf{d}\times\mathbf{e}]$ and $[\mathbf{m}\times\mathbf{n}]$) and the magnetic field $\mathbf{H}$ are collinear to one another as in the so-called $^3P_2$ superfluid state of a dense neutron liquid of neutron stars (where the strong spin-orbit coupling is taken into account). As a result, using general formulas (obtained by us previously [41,42]) for the anomalous and normal distribution functions of quasiparticles (neutrons) for SPNM in a magnetic field, we have derived a system of integral equations for $\xi(p)$, $\Delta_\uparrow^{A_2}$, and $\Delta_\downarrow^{A_2}$. In this case for SPNM, $\boldsymbol{\xi}(\mathbf{p}) = \xi(p)\mathbf{H}/H \equiv -\mu_n\mathbf{H}_{\rm eff}(p)$ ($\mu_n \approx -0.60308 \times 10^{-17}$ MeV/G is the magnetic dipole moment of a neutron [43]), and we have the equation

$$\xi(p) = -\mu_n H + (r + sp^2)K_2(\xi) + sK_4(\xi). \tag{2}$$

Here, $r = t_0' + (t_3'/6)n^\alpha$ and $s = (t_1' - t_2')/(4\hbar^2)$, $n \equiv yn_0$ is the density of neutron matter; $t_0' = t_0(1 - x_0)$, $t_1' =$

$t_1(1-x_1)$, $t_2' = t_2(1+x_2)$, $t_3' = t_3(1-x_3)$ and $1/6 \le \alpha \le 1/3$ are the parameters of the Skyrme interaction. The functionals $K_\beta(\xi)$ ($\beta = 2, 4$) in Eq. (2) have the form

$$K_\beta(\xi) = \frac{1}{8\pi^2\hbar^3} \int\limits_{p_{\min}}^{p_{\max}} dq q^\beta \int\limits_0^1 dx \kappa(q, x), \tag{3}$$

where

$$\kappa(q,x) = \frac{z(q)+\xi(q)}{E_+(q,x^2)} \tanh\left(\frac{E_+(q,x^2)}{2T}\right) -$$

$$-\frac{z(q)-\xi(q)}{E_-(q,x^2)} \tanh\left(\frac{E_-(q,x^2)}{2T}\right), \tag{4}$$

$$E_\pm^2 = q^2 \Delta_{\uparrow(\downarrow)}^2 (1-x^2) + (z(q) \pm \xi(q))^2, \tag{5}$$

$z(q) = q^2/2m_n^* - \mu$ (m_n^* is the effective mass of a neutron, and μ is the chemical potential). We have taken into account that, for SPNM with pairing of the $^3\mathrm{He}-A_2$ type, the OP can be written as $\Delta_{\uparrow(\downarrow)}^{A_2}(T,\xi,q) = q\Delta_{\uparrow(\downarrow)}(T,\xi)$, where the functions $\Delta_{\uparrow(\downarrow)}(T,\xi)$ obey the equations

$$\Delta_{\uparrow(\downarrow)}(T,\xi) = -\Delta_{\uparrow(\downarrow)}(T,\xi) \frac{c_3}{8\pi^2\hbar^3} \times$$

$$\times \int\limits_{p_{\min}}^{p_{\max}} dq q^4 \int\limits_0^1 dx (1-x^2) \frac{\tanh(E_\pm(q,x^2)/2T)}{E_\pm(q,x^2)}, \tag{6}$$

($p_{\max} \gtrsim p_{\mathrm{F}}$ and $(p_{\max} - p_{\min})/p_{\mathrm{F}} < 1$, where p_{F} is the Fermi momentum). Here, $c_3 \equiv t_2(1+x_2)/\hbar^2 < 0$ is the coupling constant leading to the spin-triplet p-wave pairing of neutrons which is expressed through the parameters t_2 and x_2 of the Skyrme interaction. We consider here a model of neutron Cooper pairing in a shell symmetric with respect to the Fermi sphere (i.e., $p_{\max} - p_{\mathrm{F}} = p_{\mathrm{F}} - p_{\min}$).

This system of nonlinear integral equations (2) and (6) for the EMF and OP gives us a possibility to describe the thermodynamics of superfluid non-unitary phases of the $^3\mathrm{He} - A_{1,2}$ type in dense SPNM with spin-triplet p-wave pairing in a static uniform high magnetic field at arbitrary temperatures from the interval $0 \le T \le T_c(H)$. In the general case, these equations cannot be solved analytically, and it is necessary to use numerical methods for their solving. But we can solve Eqs. (2) and (6), by using analytical methods in the limiting case, at zero temperature (T=0), and it is the theme of the next section.

3. Solutions of Equations for EMF and OP for Dense SPNM at $T = 0$

Let us consider SPNM at $T = 0$. In this case, we have solved analytically the integral equation (2) (with regard for Eq. (6)) for EMF on the Fermi surface using perturbation theory on the small parameter $h_{\mathrm{ext}} \equiv |\mu_n|H/\varepsilon_{\mathrm{F}} \ll a < 1$. Here, $a \equiv \varepsilon_{\max}/\varepsilon_{\mathrm{F}} - 1 = 1 - \varepsilon_{\min}/\varepsilon_{\mathrm{F}}$ is the cutoff parameter which is connected to the maximal $\varepsilon_{\max} = {p_{\max}}^2/2m_n^*$ and minimal $\varepsilon_{\min} = {p_{\min}}^2/2m_n^*$ energies of quasiparticles (neutrons) (where $p_{\max}$ and $p_{\min}$ have been introduced above in (3) and (6)). Note that the maximal energy is somewhat larger than the Fermi energy, $\varepsilon_{\max} > \varepsilon_{\mathrm{F}}$, so that $0 < a < 1$. Thus, we have obtained the following solution of Eqs. (2) and (6) to the first order in the small parameter h_{ext}:

$$\gamma(H,y) \equiv \frac{|\mu_n| H_{\mathrm{eff}}(p_{\mathrm{F}}, H)}{\varepsilon_{\mathrm{F}}(y)} = \frac{h_{\mathrm{ext}}(H,y)}{1 - (r + 2sp_{\mathrm{F}}^2)\nu_{\mathrm{F}}/2}, \tag{7}$$

where r and s (see the text after (2)) are combinations of the Skyrme parameters, and the density of states ν_{F} at the Fermi surface is

$$\nu_{\mathrm{F}}(y) = (m_n^* p_{\mathrm{F}})/(\pi^2\hbar^3) \approx$$

$$\approx 0.00419 \frac{m_n^*(y)}{m_n} y^{1/3} \quad \mathrm{MeV}^{-1}\mathrm{fm}^{-3}. \tag{8}$$

Formulas (4)–(8) contain a free neutron mass $m_n \approx 939.56563$ MeV/c^2 [43] and the effective neutron mass m_n^* which depends on the density of NM $n = yn_0$ according to the formula

$$\frac{m}{m_n^*} = 1 + \frac{myn_0}{4\hbar^2}[t_1(1-x_1) + 3t_2(1+x_2)], \tag{9}$$

where $m \equiv (m_p + m_n)/2 \approx 938.91897$MeV/$c^2$ is the mean value of a free nucleon mass [27]; the parameters t_1, t_2, x_1, and x_2 have specific numerical values for each Skyrme parametrization. Note also that the Fermi energy $\varepsilon_{\mathrm{F}} \equiv p_{\mathrm{F}}^2/2m_n^*$ of a pure NM with density $n = yn_0$ is defined by the formula

$$\varepsilon_{\mathrm{F}} = (3\pi^2 yn_0)^{2/3} \frac{\hbar^2}{2m_n^*} \approx 60.8601 y^{2/3} \frac{m_n}{m_n^*} \quad \mathrm{MeV}. \tag{10}$$

It should be emphasized that the general approximate formula (7) for $H_{\mathrm{eff}}(p_{\mathrm{F}}, H)$ is valid for all parametrizations of the Skyrme forces admissible for an NM, and H_{eff} is independent of the cutoff parameter $a < 1$ and the energy gap (with accuracy of the first order) in the energy spectrum of neutrons in SPNM.

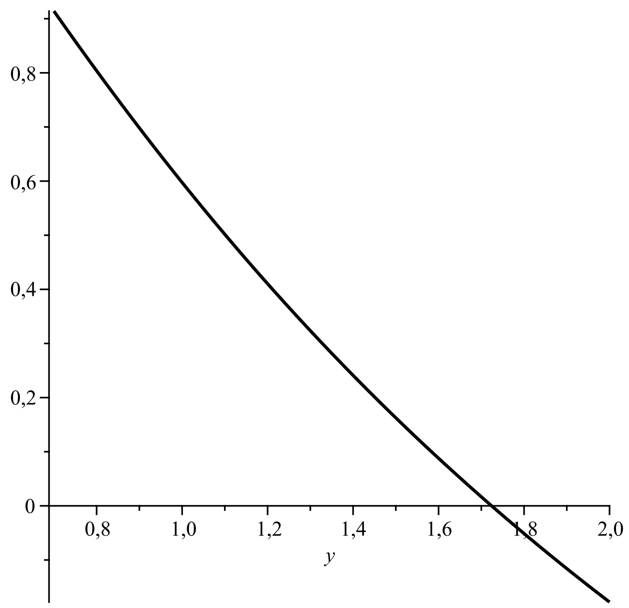

Fig. 1. Ratio $\chi_{\rm Free}/\chi_{\rm SLy2}(y)$ (see (12)) as a function of the reduced density $y = n/n_0$ for superfluid NM with Sly2 parametrization of the Skyrme forces and spin-triplet p-wave pairing of the $^3{\rm He}-A$ type in a magnetic field at $T=0$

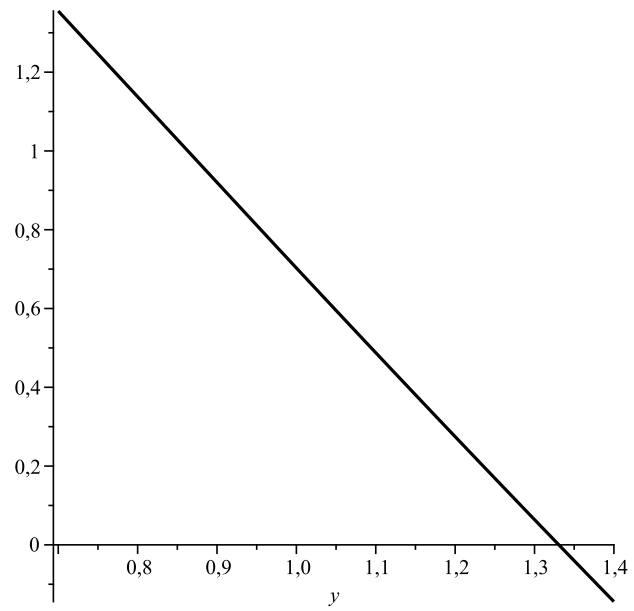

Fig. 2. Function $\chi_{\rm Free}/\chi_{\rm Gs}(y)$ (see (13)) for superfluid NM with the Gs parametrization of the Skyrme forces and spin-triplet p-wave pairing of the $^3{\rm He}-A$ type in a magnetic field at $T=0$

Now let us consider the Sly2, Gs, and RATP parametrizations of the Skyrme forces (see [27–29]). This specification gives us a possibility to plot the figures for the ratio of the Pauli susceptibility of the free neutron gas $\chi_{\rm Free}$ and the paramagnetic susceptibility of SPNM with the Skyrme interaction $\chi_{\rm Skyrme}(y)$. Note that the inverse ratio of these functions (see (7))

$$\frac{\chi_{\rm Skyrme}(y)}{\chi_{\rm Free}} = \frac{1}{1-(r+2sp_{\rm F}^2)\nu_{\rm F}/2} \qquad (11)$$

describes a renormalization of the magnetic field inside SPNM with triplet p-wave pairing of the $^3{\rm He}-A_{1,2}$ type.

Here, we represent, for the Sly2, Gs and RATP-variants of the Skyrme interaction, the power indices $\alpha_{\rm SLy2}=1/6$, $\alpha_{\rm Gs}=0.30$, and $\alpha_{\rm RATP}=0.20$ in their density dependence. Then, for the SLy2 parametrization of the Skyrme forces, we have obtained the required expression for the ratio of $\chi_{\rm Free}$ and $\chi_{\rm Skyrme}(y)$ from (11):

$$\frac{\chi_{\rm Free}}{\chi_{SLy2}(y)} = 1+\frac{2y^{1/3}(0.5004y^{1/6}+0.5461)}{(1+0.659y)}-$$

$$-\frac{2.7608y}{(1+0.659y)}. \qquad (12)$$

In a similar way for the Gs parametrization of the Skyrme forces [28], we have found from (11) that

$$\frac{\chi_{\rm Free}}{\chi_{Gs}(y)} = 1-\frac{2y^{1/3}(2.31247y^{3/10}-2.80053)}{(1+0.0810y)}-$$

$$-\frac{1.29732y}{(1+0.0810y)}. \qquad (13)$$

Finally for the RATP parametrization of the Skyrme forces [29], we have obtained the expression

$$\frac{\chi_{\rm Free}}{\chi_{RATP}(y)} = 1-\frac{y^{1/3}(1.1757y^{1/5}-2.6318)}{(1+0.235y)}-$$

$$-\frac{2.6248y}{(1+0.235y)}. \qquad (14)$$

Now, on the basis of formulae (12)–(14), we can represent Figs. 1–3 for the functions $\chi_{\rm Free}/\chi_{\rm Skyrme}(y)$.

Note that the points of intersection of these three lines (12)–(14) with the abscissa axis correspond to the critical densities $n_C({\rm SLy2}) \approx 1.72{\rm n}_0$, $n_C({\rm Gs}) \approx 1.33{\rm n}_0$, and $n_C({\rm RATP}) \approx 1.03n_0$, respectively. These critical densities correspond to phase transitions from the superfluid

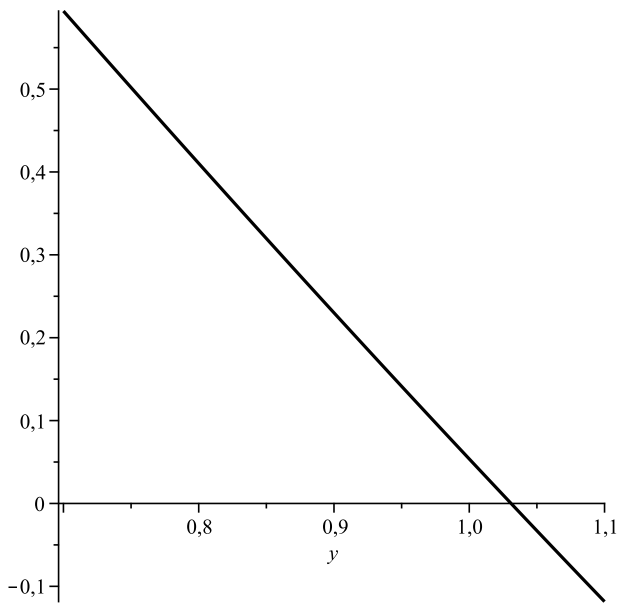

Fig. 3. Function $\chi_{\rm Free}/\chi_{\rm RATP}(y)$ (see (14)) for superfluid NM with the RATP parametrization of the Skyrme forces and spin-triplet p-wave pairing of the $^3{\rm He}-A$ type in a magnetic field at $T=0$

paramagnetic state of neutron matter with triplet pairing to the ferromagnetic state which coexists, quite possibly, with the triplet superfluidity at densities higher than n_C(Skyrme). But this problem of coexistence of spin-triplet superfluidity and ferromagnetism in a dense pure neutron matter should be examined in more details in a separate investigation. Such phase transitions might occur in the liquid outer core of neutron stars.

We have also solved Eq. (6) for the OP $\Delta_0(a;y)=\Delta_\uparrow(a;y,H=0)=\Delta_\downarrow(a;y,H=0)$ of the SPNM at $H=0$ and $T=0$ with a pairing of the $^3{\rm He}-A$ type and have derived the ratio of the maximal value of the reduced anisotropic energy gap $g(a;y)\equiv p_{\rm F}(y)\Delta_0(a;y)/\varepsilon_{\rm F}(y)$ to the PT temperature $t_{c0}\equiv T_{c0}/\varepsilon_{\rm F}$ (T_{c0} is the temperature of PT for NM to the superfluid state with triplet p-wave pairing without magnetic field) which is valid for an arbitrary parametrization of the Skyrme forces (see [24]) in the form

$$\frac{g(a;y)}{t_{c0}(a;y)}=2\exp\left(\frac{5}{6}-\frac{b_0}{2}\right)\approx 2.0174, \tag{15}$$

where

$$t_{c0}=\frac{a}{2}\exp\left(\frac{\ell(a)}{2}+\frac{2}{c_3 n_0 y m_n^*}\right), \tag{16}$$

$$\ell(a)\approx b_0+\frac{3a^2}{8}+\frac{3a^4}{256}, \tag{17}$$

$$b_0=2\left(1-\frac{1}{9}+\frac{2}{75}\right)+4\sum_{k=1}^{\infty}(-1)^{k+1}Ei(-2k)\approx$$

$$\approx 1.64932, \tag{18}$$

and

$$Ei(-x)=\int_{-\infty}^{-x}\frac{e^t}{t}dt \tag{19}$$

[formulas (15) and (16) are valid for all Skyrme parametrizations]. Here, $c_3 n_0 m_n^*<0$ is the dimensionless value depending on the Skyrme parameters t_2, x_2 and t_1, x_1 (see the text after Eq. (6) and Eq. (9)).

Note that, at $T=0$ in a sufficiently strong EMF such that

$$\gamma(H,y)\ll a<1 \tag{20}$$

(see (7)), the approximate analytic expressions for $g_\uparrow\neq g_\downarrow$ (which are, by definition, $g_{\uparrow(\downarrow)}(a;y,H)\equiv p_{\rm F}(y)\Delta_{\uparrow(\downarrow)}(a;y,H)/\varepsilon_{\rm F}(y)$) can be found from the integral equations (6) for the OP. But here we only remark that, according to our numerical estimates for the SLy2, Gs, and RATP parametrizations [26–29] (proposed for astrophysical purposes to describe NM properties in the core of a neutron star at high densities), the values $|g_{\uparrow(\downarrow)}-g|/g\lesssim 0.01$ are small (in the $0.7\lesssim y<2.0$ interval studied here) even in sufficiently strong magnetic fields $H_{\rm eff,Skyrme}<10^{17}$ G (see (20) and (7)) which are realized very likely in the so-called magnetars [6,44,45], i.e., strongly magnetized neutron stars. It is possible (as it was argued in [46]) that magnetars constitute about 10% of the neutron star population.

4. Conclusion

Having solved the integral equations (2) and (6), we have obtained the general analytic formulas (7) (see also (11)) and (15) for the EMF and OP valid at zero temperature $T=0$ for arbitrary parametrizations of the Skyrme forces in a dense SPNM (at subnuclear and supranuclear densities in the range $0.7n_0\lesssim n<n_C$(Skyrme) $<2n_0$) with anisotropic OP similar to those of $^3{\rm He}-A$. We have specified Eq. (11) for the specific parametrizations SLy2, Gs, and RATP [26–29] of the Skyrme forces and obtained formulae (12), (13) and (14) for the ratio of paramagnetic susceptibilities $\chi_{\rm Free}/\chi_{\rm Skyrme}(y)$ which are valid even for sufficiently strong magnetic fields H (but $H \ll H_{\rm max}$). For upper limit $H_{\rm max,Skyrme}(y)$ of magnetic fields, we have (see (9) and (10))

$$H_{\rm max,Skyrme}(y) \approx 1.0098 \times 10^{19} y^{2/3}(1+\beta_{\rm Skyrme}\, y)\ {\rm G}, \tag{21}$$

where $\beta_{\rm SLy2} \approx 0.659$, $\beta_{\rm Gs} \approx 0.081$, and $\beta_{\rm RATP} \approx 0.235$. Ultra-strong magnetic fields may approach 10^{18} G (see, e.g., [47,48]) in the core region of magnetars.

Note that the critical densities for the onset of ferromagnetism n_C(RATP) ≈ 0.17 fm^{-3} and n_C(SLy2) ≈ 0.29 fm^{-3} (which are very close to our results for n_C(Skyrme)) have been obtained also in [30] and [31], but for the case of phase transitions in pure NM from the normal (nonsuperfluid) state to the ferromagnetic state. Such a proximity with our results (accounting the triplet superfluidity) for n_C(Skyrme) can be explained by a very small value of superfluid corrections (which are of the second order, i.e. they are proportional to $(\Delta/\varepsilon_{\rm F})^2 \ll 1$, where Δ is the maximal anisotropic energy gap in the spectrum of quasiparticles (neutrons) in the SPNM considered here) to the paramagnetic susceptibility in superfluid NM with spin-triplet pairing. Similarly, in the case of the phase transition between normal liquid ^{3}He and superfluid $^3{\rm He}-A$ [19, 20] in a magnetic field, their paramagnetic susceptibilities are almost coincide with each other (the difference is less than 1%, see also [49]).

We note finally that the phenomena of superfluidity and magnetism in NM at high densities $n > 2n_0$ (inside the fluid cores of pulsars and magnetars [6,44,45]) should be studied in the framework of a relativistic approach and with various interpretations of the hadron matter structure (including mesons, hyperons, quarks, and other possible constituents).

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1. N.N. Bogolyubov, DAN SSSR **119**, 52 (1958); see also N.N. Bogolyubov, *Selected Proceedings in Statistical Physics* (Moscow Univer., Moscow, 1979) (in Russian).
2. A. Bohr, B.R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958).
3. J. Bardeen, L.N. Cooper, and J.R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
4. A.B. Migdal, Zh. Eksp. Teor. Fiz. **37**, 249 (1959).
5. A. Hewish, S.J. Bell, J.D.H. Pilkingston, P.F. Scott, and R.A. Collins, Nature **217**, 709 (1968).
6. AIP Conf. Proc. **983**, (2008), *40 Years of Pulsars: Millisecond Pulsars, Magnetars and More*, edited by C.G. Bassa, Z. Wang, A. Cumming, and V.M. Kaspi.
7. T. Gold, Nature **218**, 731 (1968).
8. S.T. Belyaev, Mat.-Phys. Medd. Danske-Vid. Selskab. **31**, No.11 (1959).
9. V.G. Soloviev, in *Selected Topics in Nuclear Theory* (IAEA, Vienna, 1963), p. 223.
10. V.G. Soloviev, *Theory of Complex Nuclei* (Pergamon Press, Oxford, 1976).
11. J.M. Eisenberg and W. Greiner, *Nuclear Theory, Microscopic Theory of the Nucleus* (North-Holland, Amsterdam, 1972), Vol. 3.
12. A. Bohr and B.R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2.
13. S.L. Shapiro and S.A. Teukolski, *Black Holes, White Dwarfs, and Neutron Stars* (Wiley, New York, 1983).
14. D.M. Brink and R.A. Broglia, *Nuclear Superfluidity* (Cambridge Univ. Press, Cambridge, 2005).
15. T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. Suppl. **112**, 27 (1993).
16. D.G. Yakovlev, K.P. Levenfish, and Yu.A. Shibanov, Uspekhi Fiz. Nauk **169**, 825 (1999).
17. U. Lombardo and H.-J. Schulze, in *Physics of Neutron Star Interiors*, edited by D. Blaschke *et al.* (Springer, New York, 2001), p. 30.
18. D.J. Dean and M. Hjorth-Jensen, Rev. Mod. Phys. **75**, 607 (2003).
19. A.J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).
20. D. Vollhardt and P. Wolfle, *The Superfluid Phases of Helium 3* (Taylor and Francis, London, 1990).
21. A.I. Akhiezer, V.V. Krasil'nikov, S.V. Peletminskii, and A.A. Yatsenko, Phys. Rep. **245**, 1 (1994).
22. A.N. Tarasov, Physica B **329-333**, Part 1, 100 (2003).
23. A.N. Tarasov, Europhys. Lett. **65**, 620 (2004).
24. A.N. Tarasov, AIP Conf. Proc. **850**, 109 (2006).
25. M. Brack, C. Guet, and H.-B. Hakansson, Phys. Rep. **123**, 275 (1985).
26. J. Rikovska Stone, J.C. Miller, R. Koncewicz, P.D. Stevenson, and M.R. Strayer, Phys. Rev. C **68**, 034324 (2003).
27. E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A **627**, 710 (1997).
28. J. Friedrich and P.-G. Reinhard, Phys. Rev. C **33**, 335 (1986).
29. M. Rayet, M. Arnould, F. Tondeur, and G. Paulus, Astron. Astrophys. **116**, 183 (1982).

30. A. Vidaurre, J. Navarro, and J. Bernabeu, Astron. Astrophys. **135**, 361 (1984).
31. A. Rios, A. Polls, and I. Vidana, Phys. Rev. C **71**, 055802 (2005).
32. M. Kutschera and W. Wojcik, Phys. Lett. B **325**, 271 (1994).
33. J. Margueron, J. Navarro, and N.V. Giai, Phys. Rev. C **66**, 014303 (2002).
34. S. Fantoni, A. Sarsa, and K.E. Schmidt, Phys. Rev. Lett. **87**, 181101 (2001).
35. A.A. Isayev and J. Yang, Phys. Rev. C **69**, 025801 (2004).
36. D. Lopez-Val, A. Rios, A. Polls, and I. Vidana, Phys. Rev. C **74**, 068801 (2006).
37. I. Bombaci, A. Polls, A. Ramos, A. Rios, and I. Vidana, Phys. Lett. B **632**, 638 (2006).
38. M.A. Perez-Garcia, Phys. Rev. C **77**, 065806 (2008).
39. M.A. Perez-Garcia, J. Navarro, and A. Polls, Phys. Rev. C **80**, 025802 (2009).
40. A.A. Isayev and J. Yang, e-print arXiv:0908.1368v1 [nucl-th] (2009).
41. A.N. Tarasov, Low Temp. Phys. **24**, 324 (1998); **26**, 785 (2000).
42. A.N. Tarasov, J. Probl. Atom. Sci. Techn., No. 6(2), 356 (2001).
43. *Review of Particle Properties*, Phys. Rev. D **50**, Part 1, 1233, 1673, 1680 (1994).
44. R.C. Duncan and Ch. Thompson, Astrophys. J. **392**, L9 (1992).
45. Ch. Thompson and R.C. Duncan, Astrophys. J. **408**, 194 (1993).
46. C. Kouveliotou *et al*, Nature **393**, 235 (1998).
47. D. Lai and S.L. Shapiro, Astrophys. J. **383**, 745 (1991).
48. A. Broderick, M. Prakash, and J.M. Lattimer, Astrophys. J. **537**, 351 (2000).
49. V.P. Mineev, Uspekhi Fiz. Nauk **139**, 303 (1983).

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ПРО МАГНІТНУ СПРИЙНЯТЛИВІСТЬ ГУСТОЇ НАДПЛИННОЇ НЕЙТРОННОЇ МАТЕРІЇ ЗІ СПІН-ТРИПЛЕТНИМ p-СПАРЮВАННЯМ

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Р е з ю м е

Суто нейтронна матерія зі спін-триплетним p-спарюванням вивчається у межах нерелятивістської узагальненої теорії фермі-рідини при суб'ядерних та над'ядерних густинах (у діапазоні густин $0,7n_0 \leq n < n_C(\mathrm{Skyrme}) < 2n_0$, де $n_0 = 0,17$ фм$^{-3}$ – це густина насичення симетричної ядерної матерії) при температурі, що дорівнює нулю, та за наявності сильного магнітного поля. Ефективні сили Скірма використовуються у ролі взаємодії між нейтронами. У результаті отримано загальний аналітичний вираз (справедливий для довільної параметризації сил Скірма) для парамагнітної сприйнятливості надплинної нейтронної матерії як функції від густини при нульовій температурі. Цей вираз далі конкретизовано для трьох типів взаємодії Скірма з різними степеневими залежностями від густини n. А саме, знайдено, що для випадків нейтронної матерії з так званими RATP, Gs та SLy2 параметризаціями сил Скірма у магнітної сприйнятливості виникає розбіжність при критичних густинах $n_C(\mathrm{RATP}) \approx 1,03n_0$, $n_C(\mathrm{Gs}) \approx 1,33n_0$ та $n_C(\mathrm{SLy2}) \approx 1,72n_0$. Ці критичні густини відповідають фазовим переходам з надплинного парамагнітного стану нейтронної матерії з триплетним спарюванням у феромагнітний стан, який може співіснувати з триплетною надплинністю при густинах, більших за $n_C(\mathrm{Skyrme})$. Такі фазові переходи можуть виникати у рідких зовнішніх ядрах пульсарів і так званих магнетарів.