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## Piecewise uniform switched vector quantization of the memoryless two-dimensional Laplacian source

*A simple and complete asymptotical analysis of an optimal piecewise uniform quantization of two-dimensional memoryless Laplacian source with the respect to distortion ( $D$ ) i.e. the mean-square error (MSE) is presented. Piecewise uniform quantization consists of  $L$  different uniform vector quantizers. Uniform quantizer optimality conditions and all main equations for optimal number of output points and levels for each partition are presented (using rectangular cells). The optimal granular distortion  $D_g^{opt}(i)$  for each partition in a closed form is derived. Switched quantization is used in order to give higher quality by increasing signal-to-quantization noise ratio (SQNR) in a wide range of signal volumes (variances) or to decrease necessary sample rate.*

**Key words:** piecewise uniform quantization, switched quantization, distortion

### Introduction

The use of digital representation for audio, speech, images and video is rapidly growing with the extending use of computers and multimedia computer applications. To provide a more efficient representation of data, many compression algorithms have been developed, and in the basis of all these algorithms is quantization. The concept of quantization is a mapping of a large set of amplitudes of infinite precision to a smaller finite set of values, as shown in the Fig. 1(a). Vector quantization is simply an extension of the scalar quantization to multidimensional spaces; that is, a vector quantizer operates on vectors (blocks of samples) instead on scalars.

The quantizers play an important role in the theory and practice of modern signal processing. The asymptotic optimal quantization problem, even for the simplest case — uniform scalar quantization, is very actual nowadays [1, 2]. They do consider the problem of finding the optimal maximum amplitude, so-called, support region for scalar quantizers by minimization of the total distortion  $D$ , which is a combination of granular

( $D_g$ ) and overload ( $D_o$ ) distortion,  $D = D_g + D_o$ . Extensive results have been obtained on scalar quantization but more on vector quantization. The simplest vector quantization is two-dimensional vector quantization.

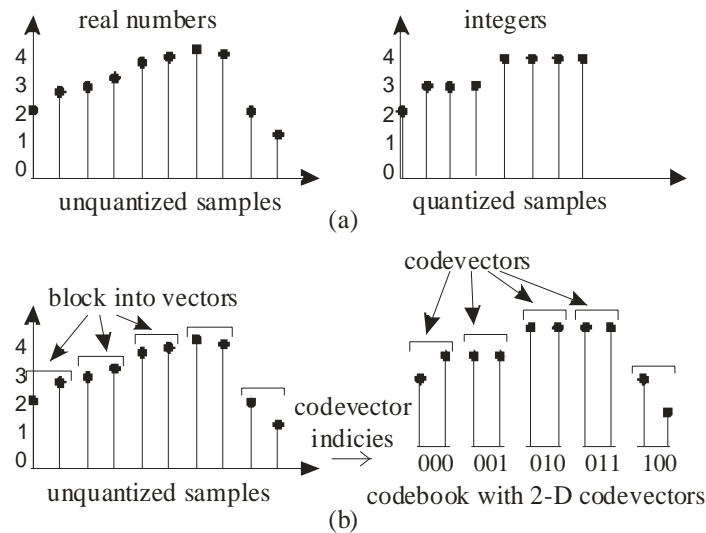


Fig. 1. Illustration of (a) scalar and (b) vector quantization

The analysis of vector quantizer for arbitrary distribution of the source signal is given in paper [3]. The authors derived the expression for the optimum granular distortion and optimum number of output points. However, they did not prove the optimality of the proposed solutions. Also, they did not define the partition of the multidimensional space into subregions. In paper [4], the expressions for the optimum number of output points are derived, however the proposed partitioning of the multidimensional space for memoryless Laplacian source does not consider the geometry of the multidimensional source. In paper [5], vector quantizers of Laplacian and Gaussian sources are analyzed. The proposed solution for the quantization of memoryless Laplacian source, unlike in [5], takes into consideration the geometry of the source, however, the proposed vector quantizer design procedure is too complicated and unpractical.

In this paper we give a systematic analysis of piecewise uniform vector quantizer (PUQ) of Laplacian memoryless source. We give a general and simple way to design a piecewise uniform vector quantizer. We derive the optimum number of output points and the optimality of the proposed solutions is proved. The goal of this paper is to solve a quantization problem in a case of PUQ and to find corresponding support region. It is done by analytical optimization of the granular distortion and numerical optimization of the total distortion. If the distortion is measured by a squared error,  $D$  becomes the mean squared error (MSE). The distortion mean-squared error (MSE i.e. quantization noise) is used as a criterion for optimization.

The MSE of a two-dimensional vector source  $x = (x_1, x_2)$ , where  $x_i$  are zero-mean statistically independent Laplacian random variables of variance  $\sigma^2$ , is commonly used for the transform coefficients of speech or imagery. The first approximation to the long-time-averaged probability density function (pdf) of amplitudes is provided by Laplacian

model [7, p. 32]. The waveforms are sometimes represented in terms of adjacent-sample differences. The pdf of the difference signal for an image waveform follows the Laplacian function [7, p. 33]. The Laplace source is a model for speech [8, p. 384]. Consider two independent identically distributed Laplace random variables  $(x_1, x_2)$  with the zero mean and unity variance. To simplify the vector quantizer, the Helmert transformation is applied on the source vector giving contours with constant probability densities. The transformation is defined as:  $r = \frac{1}{\sqrt{2}}(|x_1| + |x_2|)$ ,  $u = \frac{1}{\sqrt{2}}(|x_1| - |x_2|)$ . In this paper, quantizers are designed and analysed under additional constraint — each scalar quantizer is a uniform one.

PUQ consists of  $L$  optimal uniform vector quantizers. More precisely, our quantizer divides the input plane into  $L$  partitions and every partition is further subdivided into  $L_i$  ( $1 \leq i \leq L$ ) subpartitions. Every concentric subpartition can be subdivided in four equivalent regions, i.e. The  $j$ -th subpartition in signal plane is allowed to have  $p_{ij}$  ( $1 \leq i \leq L, 1 \leq j \leq L_i$ ) cells. We perform two-step optimization: 1) distortion optimization ( $D_i$ ) in every partition under the constraint  $4 \sum_{j=1}^{L_i} p_{ij} = N_i$  and 2) optimization of the

total granular distortion  $D_g = \sum_{i=1}^L D_i$  which achieves the optimal number of points  $N_i$

on each partition under the constraint  $\sum_{i=1}^L N_i = N$ .

In this work we design a piecewise uniform vector quantizer for optimal compression function. We perform analytical optimisation of the granular distortion and numerical optimization of the total distortion using rectangular cells.

The switching quantization aims are to improve the quality of the signal-to-noise ratio in the wide range of the signal average power (i.e. variance) or to decrease the sample rate. The switching quantization is adaptive quantization for memoryless sources and it is applicable only if adaptation is performed on the basis of the signal average power, what was just done in this paper. As an input source, we will consider memoryless Laplacian source.

## Basic notes on VQ

The conceptual notion of VQ is illustrated in Fig. 1(b). These blocks of samples are represented by code vectors and stored in a codebook — a process called encoding. A block diagram of the encoder is shown in Fig. 2. The encoder  $\varepsilon$  performs a mapping from  $k$ -dimensional space  $R^k$  to the index set  $I$ , and the decoder  $D$  maps the index set  $I$  into the finite subset  $C$ , which is the codebook. The codebook has a positive integer number of code vectors (denoted by  $y_i$ ) that defines the codebook size, denoted by  $N$ . The bit rate  $R$  associated with the VQ depends on  $N$  and the vector dimension  $k$ . Since the bit rate is the number of bits per sample,

$$R = (\log_2 N) / k. \quad (1)$$

In contravention to basic scalar quantization which is fixed-rate, for VQ is natural to have fractional bit rates such as  $\frac{1}{2}$ ,  $\frac{3}{4}$ , etc. The decoding process is very simple and requires only a table (codebook) lookup, but the encoding procedure is complex and involves finding a best matching code vector, using a distortion measure as a criterion. The most common distortion measure is *mean squared error*, given by:

$$d(x, y) = (x - y)^t (x - y) = \sum_{l=1}^k (x[l] - y[l])^2, \quad (2)$$

where  $x[l]$  and  $y[l]$  are the elements of the vector  $x$  and  $y$ , respectively.

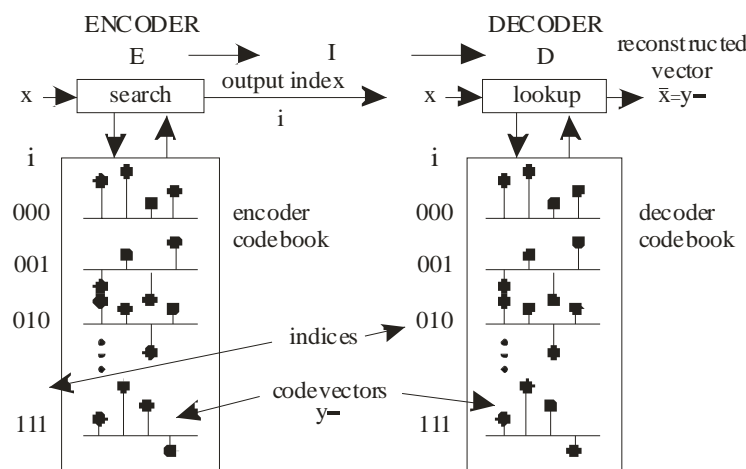


Fig. 2. Block diagram of a VQ encoder and decoder

It is convenient to view the operation of a vector quantizer geometrically, using our intuition for the case of two- or three-dimensional space. Thus, a 2-dimensional quantizer assigns any input point in the plane to one of a particular set of  $N$  points or locations in the plane. The plane is divided into  $N$  partition cells, as shown in the Fig. 3(a), and the dots represent code vectors, one in each cell. A unique partitioning of the space is defined by the encoding procedure, and optimized for a given input source, to give the best performance. Now consider the quantizing of our fictional input source with scalar quantization at an equivalent bit rate. The cells implying the use of a scalar quantizer for the input source are shown in Fig. 3(b). Notice that each cell is should to be rectangular, and some cells are forced to be placed in regions where the input source may not be significantly populated. These observations lead to two immediately recognizable advantages of VQ over scalar quantization. First, VQ provides greater freedom to control the shapes of the cells to achieve more efficient tilings of the space. This property is often called *cell shape gain*. Second, VQ allows a greater number of cells to be concentrated in the regions where the source has the greatest density, which reduces the average distortion.

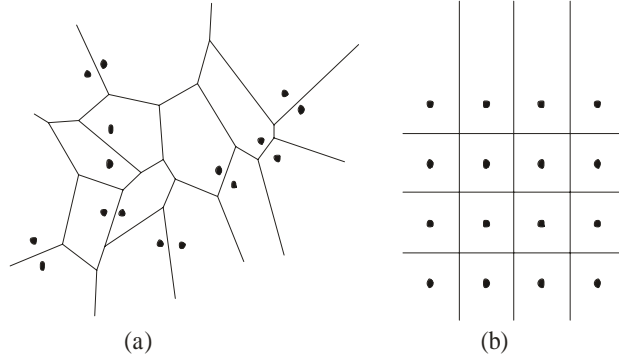


Fig. 3. Illustration of the partition cell associated with VQ and scalar quantization: a) partition cells for a 2D VQ; b) partition cells corresponding to scalar quantization

### Piecewise uniform vector quantizer design

Joint pdf function of two independent, identically distributed Laplace random variables  $(x_1, x_2)$  with zero mean is given with the following expression

$$f_{1,2}(x_1, x_2) = \frac{1}{2\sigma^2} e^{-\frac{\sqrt{2}(|x_1|+|x_2|)}{\sigma}}. \quad (3)$$

After applying the Helmert transformation [9, 10]

$$r = \frac{1}{\sqrt{2}}(|x_1| + |x_2|), \quad u = \frac{1}{\sqrt{2}}(|x_1| - |x_2|) \quad (4)$$

we get the probability density function

$$f(r, u) = \frac{1}{2\sigma^2} e^{-\frac{2r}{\sigma}}. \quad (5)$$

In the two-dimensional  $ru$  system the pdf function given by equation (5) represents a square line. The square surface  $(0, r_{\max})$  representing dynamic range of a two-dimensional quantizer, can be partitioned into  $L$  concentric domains as shown in Fig. 4. In the case of nonuniform vector quantization, these concentric domains are of unequal width. The number of output points in each domain is denoted by  $N_i$ , where  $N = \sum_{i=1}^L N_i$  represents the total number of output points. Every concentric domain can be further partitioned into  $L_i$  concentric subdomains of equal width. Every subdomain is divided into four regions each containing  $p_{i,j}$  rectangular cells. An output point is placed in the centre of each cell. Coordinates of the  $k$ -th output point in  $j$ -th subregion of the  $i$ -th region in the  $ru$  coordinate system are  $(m_{i,j}, \mu_{i,j,k})$ .

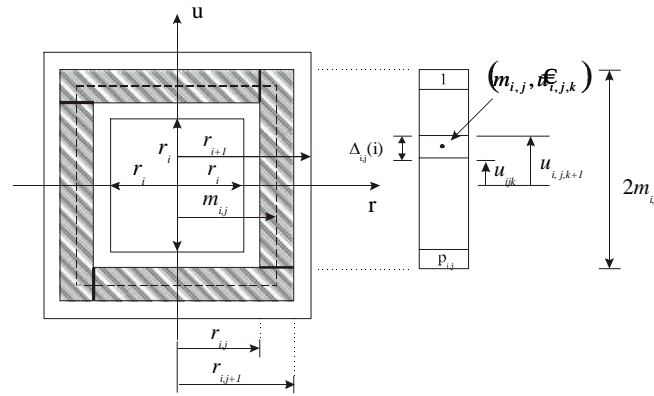


Fig. 4. Two-dimensional space partitioning

The initial expression for granular distortion is

$$D_g = 4 \sum_{i=1}^L \sum_{j=1}^{L_i} \sum_{k=1}^{p_{i,j}} \int_{r_{i,j}}^{r_{i,j+1}} \int_{u_{i,j,k}}^{u_{i,j,k+1}} [(r - m_{i,j})^2 + (u - u_{i,j,k})^2] \cdot \frac{1}{2\sigma^2} e^{-\frac{2r}{\sigma}} dr du. \quad (6)$$

The output point coordinates are given by the equations

$$m_{i,j} = \frac{r_{i,j+1} + r_{i,j}}{2} \quad \text{and} \quad u_{i,j,k} = \frac{u_{i,j,k} + u_{i,j,k+1}}{2}. \quad (7)$$

Rectangular cell dimensions are:

$$\Delta_i = r_{i+1} - r_i, \quad \Delta'_i = \frac{\Delta_i}{L_i} \quad \text{and} \quad \Delta_{ij} = \frac{r_{i,j} + r_{i,j+1}}{p_{i,j}}; \quad (8)$$

$$r_{i,j} = r_i + j \cdot \Delta_i, \quad i = 0, \dots, L, \quad j = 0, \dots, L_i. \quad (9)$$

The range of the quantizer is  $r_{\max}$ . To determine the boundary values of every concentric domain, denoted as  $r_i$ , for the case of nonuniform vector quantization we perform the segmentation and linearization of the optimal compression function, given by the following expression

$$h(r) = r_{\max} \frac{e^{-\frac{1}{2\sigma}r} - 1}{e^{-\frac{1}{2\sigma}r_{\max}} - 1}. \quad (10)$$

The method for linearization of compression function, named *the first derivat segmentation*, was selected based on the analysis performed in [11]. The principle used

in this method is to do a uniform segmentation of the first derivate of compression function, and find corresponding  $r_i$  points by substituting uniformly distributed  $h'$  values in the inverse first derivate function.

The total number of output points is

$$N = \sum_{i=1}^L N_i, \quad (11)$$

where  $N_i$  is the number of output points in the  $i$ -th domain. We can also write:

$$\sum_{j=1}^{L_i} p_{i,j} = \frac{N_i}{4} \quad (12)$$

Equation (12) can be written as

$$D_g = \sum_{i=1}^L D_g(i), \quad (13)$$

where  $D_g(i)$  is

$$D_g(i) = 4 \sum_{j=1}^{L_i} \sum_{k=1}^{p_{i,j}} \int_{r_{i,j}}^{r_{i,j+1}} \int_{u_{i,j,k}}^{u_{i,j,k+1}} [(r - m_{i,j})^2 + (u - \mu_{i,j,k})^2] \cdot \frac{1}{2\sigma^2} e^{-\frac{2r}{\sigma}} dr du. \quad (14)$$

After integration over  $u$  and reordering equation (14) becomes

$$D_g(i) = \sum_{j=1}^{L_i} \left[ \int_{r_{i,j}}^{r_{i,j+1}} (r - m_{i,j})^2 (r_{i,j+1} + r_{i,j}) \frac{2}{\sigma^2} e^{-\frac{2r}{\sigma}} dr + \int_{r_{i,j}}^{r_{i,j+1}} \frac{(r_{i,j+1} + r_{i,j})^3}{12p_{i,j}^2} \frac{2}{\sigma^2} e^{-\frac{2r}{\sigma}} dr \right]. \quad (15)$$

From equation (7) it follows that  $r_{i,j+1} + r_{i,j} = 2m_{i,j}$ . When we substitute this in equation (15) we get

$$D_g(i) = \sum_{j=1}^{L_i} \left[ \int_{r_{i,j}}^{r_{i,j+1}} (r - m_{i,j})^2 4m_{i,j} \frac{1}{\sigma^2} e^{-\frac{2r}{\sigma}} dr + \int_{r_{i,j}}^{r_{i,j+1}} \frac{m_{i,j}^2}{3p_{i,j}^2} 4m_{i,j} \frac{1}{\sigma^2} e^{-\frac{2r}{\sigma}} dr \right]. \quad (16)$$

We will now assume that  $\exp(-2r/\sigma)$  is constant over  $\Delta_i$ . In that case we can substitute  $\exp(-2r/\sigma)$  with  $\exp(-2m_{i,j}/\sigma)$ . Equation (16) can be now written as:

$$\begin{aligned}
 D_g(i) &= \sum_{j=1}^{L_i} 4m_{i,j} \frac{1}{\sigma^2} e^{-\frac{2m_{i,j}}{\sigma}} \left[ \int_{r_{i,j}}^{r_{i,j+1}} (r - m_{i,j})^2 dr + \int_{r_{i,j}}^{r_{i,j+1}} \frac{m_{i,j}^2}{3p_{i,j}^2} dr \right] = \\
 &= \sum_{j=1}^{L_i} 4m_{i,j} \frac{1}{\sigma^2} e^{-\frac{2m_{i,j}}{\sigma}} \left[ \frac{\Delta_i^3}{12} + \frac{m_{i,j}^2}{3p_{i,j}^2} \Delta_i \right] = \\
 &= \sum_{j=1}^{L_i} \left[ 2m_{i,j} \frac{\Delta_i^2}{12} + \frac{2m_{i,j}^3}{3p_{i,j}^2} \right] \cdot P(m_{i,j}).
 \end{aligned} \tag{17}$$

where  $P(m_{i,j})$  denotes the probability

$$P(m_{i,j}) = \Delta_i f(m_{i,j}) = \Delta_i \frac{2}{\sigma^2} e^{-\frac{2m_{i,j}}{\sigma}}. \tag{18}$$

Function  $f(m_{i,j})$  is defined as

$$f(m_{i,j}) = \frac{2}{\sigma^2} e^{-\frac{2m_{i,j}}{\sigma}}. \tag{19}$$

By using the Lagrangian multipliers we can obtain the optimum number of cells in one region  $p_{i,j}$ , which yields the minimum granular distortion defined by the equation (17). Because we are designing an optimal quantizer for one value of variance  $\sigma_0$ , in calculating  $p_{i,j}$  we will use  $\sigma_0$  instead of  $\sigma$ . We will start from the following equation

$$J = D_g(i) + \lambda \sum_{j=1}^{L_i} p_{i,j} \tag{20}$$

After differentiating  $J$  with respect to  $p_{i,j}$ , and equalizing the derivative with zero we get

$$\frac{\partial J}{\partial p_{i,j}} = 0 \Rightarrow -\frac{4m_{i,j}^3}{3p_{i,j}^3} P_0(m_{i,j}) + \lambda = 0. \tag{21}$$

From the preceding equation we can write the following:

$$p_{i,j} = \sqrt[3]{\frac{4m_{i,j}^3}{3\lambda} P_0(m_{i,j})} = m_{i,j} \sqrt[3]{\frac{4f_0(m_{i,j})\Delta_i}{3\lambda}}. \tag{22}$$



If we substitute  $p_{i,j}$  from equation (22) in equation (12)

$$\sum_{j=1}^{L_i} m_{i,j} \sqrt[3]{\frac{4f_0(m_{i,j})\Delta_i}{3\lambda}} = \frac{N_i}{4}, \quad (23)$$

we can eliminate  $\lambda$  by substituting

$$\sqrt{3\lambda} = \frac{4}{N_i} \sum_{j=1}^{L_i} m_{i,j} \sqrt[3]{4f_0(m_{i,j})\Delta_i} \quad (24)$$

in equation (22):

$$p_{i,j} = \frac{N_i}{4} \frac{m_{i,j} \sqrt[3]{g_0(m_{i,j})}}{\sum_{k=1}^{L_i} m_{i,k} \sqrt[3]{g_0(m_{i,k})}}, \quad (25)$$

where  $g_0(m_{i,j})$  denotes the function

$$g_0(m_{i,j}) = \frac{1}{\sigma_0^2} e^{-\frac{2m_{i,j}}{\sigma_0}}. \quad (26)$$

If we multiply numerator and denominator with  $\Delta_i$ , we can approximate the sum by the integral

$$p_{i,j} = \frac{N_i}{4} \frac{m_{i,j} \sqrt[3]{g_0(m_{i,j})} \cdot \Delta_i}{\int_{r_i}^{r_{i+1}} r \cdot \sqrt[3]{g_0(r)} dr}. \quad (27)$$

By substituting  $p_{i,j}$  from equation (27) in equation (17) we get

$$D_g(i) = \frac{\Delta_i^2}{3L_i^2} \sum_{j=1}^{L_i} m_{i,j} g_0(m_{i,j}) \Delta_i' + \frac{64L_i^2}{3N_i^2 \Delta_i^2} I_0'(i)^2 \sum_{j=1}^{L_i} m_{i,j} \frac{g(m_{i,j})}{g_0(m_{i,j})^{2/3}} \Delta_i'. \quad (28)$$

After approximating the sum by the integral, we can rewrite (28) as

$$D_g(i) = \frac{\Delta_i^2}{3L_i^2} I(i) + \frac{64L_i^2}{3N_i^2 \Delta_i^2} I_0'(i)^2 I'(i). \quad (29)$$

The functions  $I'_0(i)$ ,  $I'(i)$  and  $I(i)$  are defined as:

$$\begin{aligned} I'_0(i) &= \int_{r_i}^{r_{i+1}} r \cdot \sqrt[3]{g_0(r)} dr; \\ I'(i) &= \int_{r_i}^{r_{i+1}} r \cdot \frac{g(r)}{g_0(r)^{2/3}} dr; \\ I(i) &= \int_{r_i}^{r_{i+1}} r \cdot g(r) dr. \end{aligned} \quad (30)$$

After differentiating  $D_g$  from equation (29) with respect to  $L_i$ , and for  $\sigma_0$ , we obtain the optimum number subdomains in  $i$ -th domain

$$L_{iopt} = \Delta^4 \sqrt[4]{\frac{I_0(i)N_i^2}{64I'_0(i)^3}}, \quad (31)$$

where  $I_0(i)$  is defined as

$$I_0(i) = \int_{r_i}^{r_{i+1}} r \cdot g_0(r) dr. \quad (32)$$

Substituting the expression for  $L_i$  from equation (31) in equation (29),  $D_g(i)$  becomes

$$D_g(i) = \frac{8}{N_i} I'_0(i) \left( \sqrt{\frac{I'_0(i)}{I_0(i)}} \cdot I(i) + \sqrt{\frac{I_0(i)}{I'_0(i)}} \cdot I'(i) \right). \quad (33)$$

The optimum number of output points in the  $i$ th subdomain is obtained using Lagrangian multipliers

$$J = D_g(N_i) + \lambda \sum_{i=1}^L N_i \quad (34)$$

After differentiating (33) with respect to  $N_i$ , and for  $\sigma_0$  we get:

$$N_i = \sqrt{\frac{16I'_0(i)\sqrt{I'_0(i)I_0(i)}}{3\lambda}}. \quad (35)$$

Using the condition (11) we can eliminate  $\lambda$  from the equation (35)

$$N_i = N \frac{[I'_0(i)^3 I_0(i)]^{1/4}}{\sum_{k=1}^L [I'_0(k)^3 I_0(k)]^{1/4}}. \quad (36)$$

Finally, after substituting the expression for optimum number of output points from equation (36) into equation (33) we can write

$$D_g(i) = \frac{8}{3N} \frac{\sum_{k=1}^L [I'_0(k)^3 I_0(k)]^{1/4}}{[I'_0(i)^3 I_0(i)]^{1/4}} I'_0(i) \cdot \left( \sqrt{\frac{I'_0(i)}{I_0(i)}} \cdot I(i) + \sqrt{\frac{I_0(i)}{I'_0(i)}} \cdot I'(i) \right). \quad (37)$$

Now, we can calculate the optimum granular distortion of uniform piecewise vector quantizer as

$$D_g = \sum_{i=1}^L D_g(i) = \frac{8}{3N} \sum_{i=1}^L [I'_0(i)^3 I_0(i)]^{1/4} \cdot \sum_{i=1}^L \left[ \left( \frac{I'_0(i)}{I_0(i)} \right)^{3/4} \cdot I(i) + \left( \frac{I_0(i)}{I'_0(i)} \right)^{1/4} \cdot I'(i) \right]. \quad (38)$$

We can calculate the overload distortion as

$$D_o = 4 \sum_{j=1}^{p_{L,L_L}} \int_{r_{\max}}^{\infty} \int_{u_{L,j}}^{u_{L,j}'} \left[ (r - m_{L,L_L})^2 + (u - \epsilon_{L,j})^2 \right] \frac{1}{2\sigma^2} e^{-\frac{2r}{\sigma}} dr du. \quad (39)$$

After some calculation, we get

$$D_o = \frac{m_{L,L_L}}{\sigma^2} e^{-\frac{2r_{\max}}{\sigma}} \left[ 2\sigma r_{\max}^2 + r_{\max} (2\sigma^2 - 4\sigma m_{L,L_L}) + \sigma^3 - 2m_{L,L_L} \sigma + 2m_{L,L_L}^2 \sigma + \sigma \frac{2m_{L,L_L}^2}{3p_{L,L_L}^2} \right]. \quad (40)$$

From equations (31) and (33), we can calculate the total distortion for one dimension as

$$D = \frac{1}{2} (D_g + D_o). \quad (41)$$

## Numerical results

The results are shown in Fig. 5 for two values of  $\sigma_0$  (two different quantizers): 0dB and -10dB, and for bit rate of  $R = 7.5$ . For comparison, dash-dotted lines show SQNR for scalar quantization for  $R = 8$ , for a compression function given with

$$f(r) = r_{\max} \frac{1 - e^{-\frac{r}{r_{\max}}}}{1 - e^{-v}}, \quad (42)$$

where  $v$  is a compression factor, with a critical value for  $\sigma_0$

$$v_c = \frac{\sqrt{2} r_{\max}}{3 \sigma_0}. \quad (43)$$

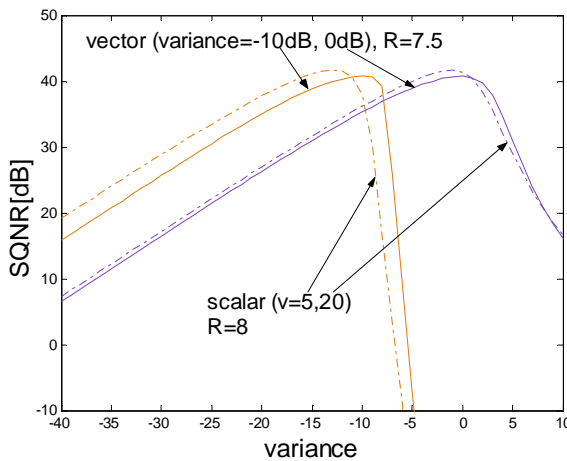


Fig. 5. SQNR for scalar (bit rate = 8) and vector (bit rate = 7,5) quantization with two optimal quantizers designed for different values of  $\sigma_0$

SQNR is a signal-to-quantization noise ratio, given with:

$$SQNR[dB] = 10 \log \frac{\sigma^2}{D_{uk}}. \quad (44)$$

We can see from this figure that there is a 0,5 bits gain in the case of vector quantization. In Fig. 6 we have changed bitrate to  $R = 4$ .

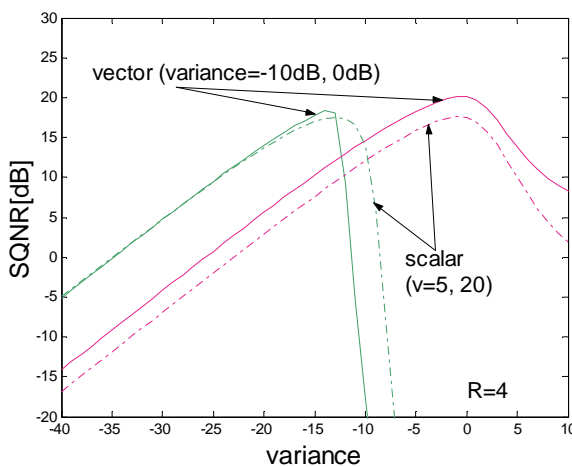


Fig. 6. SQNR for scalar and vector (bit rate = 4) quantization with two optimal quantizers designed for different values of  $\sigma_0$

In Table, as an illustration of the previous analysis, the number of rectangular cells in each of four regions in a particular subdomain, denoted as  $p_{i,j}$ , is given. The optimal number of subdomains in every domain ( $L_{i,opt}$ ) is calculated for  $L = 4$  concentric domains and bit rate  $R = 4$ , giving the value 2.

Number of rectangular cells for $R = 4$					
$p_{i,j}$		$L = 4$			
		1	2	3	4
1	1	1	6	9	<b>13</b>
	2	3	7	11	<b>13</b>

For nonstationary inputs a logical scheme is switched quantization; this consists in providing a bank of  $B$  fixed quantizers, and switching among them as appropriate, in response to changing input statistics. This scheme is used for two designed quantizers, as shown in Fig. 7.

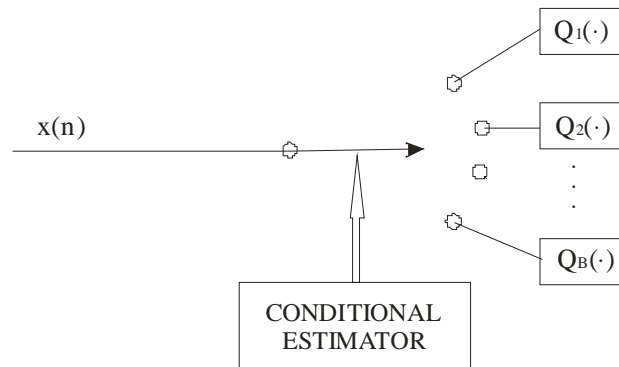


Fig. 7. Block diagram of switched quantization

## Conclusion

The optimization of two-dimensional Laplace source piecewise nonuniform vector quantization is carried out. A simple expression for granular distortion, a number of subdomains and a number of output points in closed form is obtained. The results obtained by using two vector quantizers optimized for two different values of  $\sigma$  (variance) demonstrate the significant performance gain over the uniform scalar quantization, giving a 0,5 bits/sample gain. Memoryless Laplacian source is used, considering the possible application of this quantizer. The transform coefficients of DCT (discrete cosine transform) encoding of speech or imagery are often modeled as Laplacian, except for the DC coefficient of imagery. By using switched quantization, with two quantizers in this case, necessary rate is decreased by 0,5 bits per sample, with compliance of

the appropriate standard. With a larger set of quantizers we could increase this sample rate even more, thus giving a better compression quality with higher SQNR.

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