UDC 539.4

Modeling of Damage Interaction in Fatigue Relaxation for Long-Term Life Prediction. Case of Alloy 800 Grade 2 Study at 550°C

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УДК 539.4

Моделирование разрушающего взаимодействия при усталостной релаксации при прогнозировании длительной долговечности для сплава 800 (тип 2) при 550°C

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Прогнозирование усталостной долговечности в условиях ползучести при температуре 550°C выполнено для жаропрочного и жаростойкого сплава на никелевой основе (Inconel). Представлены модели эволюции повреждения Шабоша (Chaboche) и Леваяна (Levaillant), а также новая модель релаксации при совместном действии ползучести и усталости. Предложенная модель позволяет осуществлять долгосрочное прогнозирование без использования большого количества постоянных.

Ключевые слова: сплав 800, ползучесть, усталость, релаксация, долгосрочное прогнозирование, малые деформации.

1. **Introduction**.

1.1. **Problem Formulation**. Subject of our study is an alloy with 35% nickel, 20% chrome and an addition of titan and aluminum, which is further referred to as alloy 800 at the state of grade 1 [1–3]. When construction components made of this alloy are assembled by welding, thermal gradients are established in the nearest zones to the welded joints, while welding modes causing metallurgical changes produce the material structure which is similar to the state of grade 2. Fatigue relaxation tests carried out on the alloy 800 grade 2 at 500°C have revealed the following [3]:

– An apparition of intergranular failure and its dominance for a long period of time.

– Fatigue life reduction (in numbers of cycles), which is observed as soon as a hold time is introduced into a creep fatigue cycle. This is related to the failure mode changeover from transgranular to intergranular one that occur during creep-fatigue-relaxation tests.

Low cycle fatigue damage is different from that of creep [3]. It seems to be necessary to identify separately each damage mode, for taking them into consideration, when they are simultaneously present during fatigue relaxation tests, in a unique law, in order to predict a long life duration under long-term and small-scale deformation conditions.

1.2. **Methodology**. In this study, two models among the most recent ones are briefly reviewed. These deal with the evolution of creep, fatigue and creep/fatigue and fatigue/relaxation damage. The results of the long life prediction under small-scale deformations and long term conditions, which are obtained by the two models in the case of alloy 800 grade 2 to 550°C, are also presented [3]. Next the following issues are discussed:

1. The definition of our proper model, treating the evolution of damage in the fatigue relaxation.

2. The model validation on the basis of the same results as the two previous models.

3. Prediction of the life duration (in cycles) for long hold time periods and small-scale deformations (e.g., $t_{mt} = 1000$ hours and $\Delta\varepsilon_t = 0.6\%$).

4. Comparison of our results with the experimental data and with those obtained by the two previous models.

2. **The Chaboche Model of Creep-Fatigue Interaction**.

2.1. **Fatigue Model**. Chaboche [3, 4, 6] has proposed to describe the evolution of fatigue damage by the relation (1). This relation depends on the damage itself, on the maximal stress in tension, and on the average stress of the cycle and the number of cycles.

$$dD_F = [1-(1-D)^{1+\beta}]^{\alpha} \left(\frac{\sigma_{t\max} - \sigma_{av}}{M(\sigma_{av})(1-D_f)} \right)^{\beta} dN, \qquad (1)$$

where $\sigma_{t\max} = \sigma_a$ is the stress amplitude of the stabilized cycle; $\sigma_1 = \sigma_{10}$ is the inferior limit of failure in fatigue (endurance limit); σ_u is the final limit of the curve (stress of static load failure: a failure in the first quarter of the cycle); β, b, and M are constants.

Here

$$(1-\alpha) = a\sigma_{t\max} - \sigma_{10}(\sigma_{av})/(\sigma_u - \sigma_{t\max}),$$

$$\sigma_1(\sigma_{av}) = \sigma_{av} + \sigma_{10}(1-\sigma_{av}),$$

$$M(\sigma_{av}) = M_0(1-b\sigma_{av}).$$

Hypothesis: The relation (2) has been used to define an approximate value for the ultimate stress σ_u, using the results obtained during the tensile test, by (2).

$$\sigma_u = R_m[1 + A(\%)]. \qquad (2)$$

Chaboche [3, 4, 6] has then proposed the relation (3) to describe the evolution of the fatigue damage as a function of the number of cycles:

$$D_{fat} = 1 - \left[1 - \left(\frac{N}{N_R}\right)^{1/(1-\alpha)}\right]^{1/(1+\beta)}. \tag{3}$$

2.2. **Creep Model**. Chaboche [3, 4, 6] has proposed the relation (4) to describe the creep damage

$$D_c = (\sigma/A)^r (1 - D_c)^{-k} dt, \tag{4}$$

where σ is the stress applied to the element of the volume, k is a constant, independent on the stress σ, r is the slope of the creep curve ($\log t_R - \log \sigma$), and A is the material constant.

2.2.1. *Calculation of the Time to Failure*. Chaboche has introduced in his study [3, 4, 6] the relation (5) to calculate the time of breaking t_R by integrating the expression (4), between $D = 0$ and $D = D_{failure}$ and between $t = 0$ and t_R (time to failure).

$$t_R = \frac{1}{1+k}\left(\frac{\sigma^*}{A}\right)^{-r}, \tag{5}$$

where σ^* is a constant stress, which is a characteristic of the creep test and can have one of the 3 values of the described in [3]: (1) the nominal stress σ_0, (2) the stress at the end of loading σ_c, and the average actual stress σ_{vav}.

2.2.2. *Creep Damage Curve*. Chaboche [3, 4, 6] has proposed the expression (6), which gives the evolution of damage according to the ratio (t/t_R) integrating the relation (4) for $t = 0$ to $t = t$ and $D = 0$ to $D = D_c$.

$$D_{flu} = 1 - \left[1 - \left(\frac{t}{t_R}\right)\right]^{1+k}. \tag{6}$$

2.3. **Creep-Fatigue Interaction Model**. Under fatigue relaxation conditions, the fatigue and creep damages are simultaneously present. A nonlinear interaction proposed in [4, 6, 7] is represented by the elementary sum of fatigue and creep damage.

$$dD = \left[\left(\frac{\sigma}{A}\right)^r (1 - D)^{-k} dt\right] + \left[[1 - (1 - D)^{1+\beta}]^{\alpha}\left(\frac{\Delta\sigma}{2M(1-D)}\right)^{\beta} dN\right]. \tag{7}$$

In order to calculate the stress during relaxation, a viscoplastic relaxation rule named "law of CEA-M" [3, 4] described by Eq. (8) is used:

$$\frac{\sigma}{\sigma_{\max}} = [1+(n-1)EA(t)^p \sigma_{\max}^{(n-1)}]^{1/(1-n)}, \tag{8}$$

where σ is the current stress during relaxation, E is Young's modulus (MPa), t is the time from the beginning of the hold period, and n, A, and p are constants that are determined by a nonlinear regression, when fitting the relation (8) to the experimental curves.

The application of this model requires determination of 17 coefficients. These have been determined within the framework of a previous study [3] in the case of alloy 800 grade 2 at 550°C. These results are compared with the experimental data, with results, corresponding to the Levaillant model, and with those obtained by our model.

3. **The Levaillant Model of Creep-Fatigue Interaction**. Levaillant [5] has proposed a model of creep-fatigue interaction where the intergranular cracks propagate according to preferential directions from a broken grain joint to another due to creep at the front of the fatigue crack.

3.1. **Fatigue**. The author [3, 5] postulated that the damage in continuous fatigue be assimilated at the depth of crack a. The propagation ratio of transgranular cracks with fatigue striations per cycle is obtained by measuring the distance between two successive striations (which is called "interstriation distance" i). By integrating the crack propagation rules of type (9):

$$da/dN = i = f(a). \tag{9}$$

The number of the cycles corresponding to the fatigue crack N_p can be calculated as

$$N_p = \int_{a_0}^{a_f} \frac{da}{f(a)}, \tag{10}$$

where $a_0 = 20$ μm and $a_f = 3$ mm.

Levaillant proposed the following relation (11) to calculate the number of propagation cycles in continuous fatigue N^{FC} according to the number of cycles to failure N_R^{FC}.

$$N_p^{FC} = \alpha_1 (N_R^{FC})^{\beta_1} - \alpha_2 (N_R^{FC})^{\beta_2}, \tag{11}$$

where β_1, β_2, α_2, and α_1 are constants to be determined for each particular material.

Thus, two phases are distinguished. The first one is the crack initiation phase N_a, and the second is the crack propagation phase N_p. Then, it can be written as

$$N_R = N_a + N_p. \tag{12}$$

The main assumptions made by Levaillant [3, 5] were the following:

Crack propagation phase in fatigue relaxation N_p^{FR} is reduced in relation to the continuous fatigue: $N_p^{FR} << N_p^{FC}$.

Crack initiation phase in fatigue relaxation N_a^{FR} is shorter than in continuous fatigue $N_a^{FR} << N_a^{FC}$. For very long maintain period, Levaillant has set this simplifying hypothesis: $N_a^{FR} = 0$.

The reduction of the crack propagation phase in fatigue relaxation in relation with the continuous fatigue is defined by the coefficient R^{FR}:

$$R^{FR} = \frac{N_p^{FC} - N_p^{FR}}{N_p^{FC}}. \tag{13}$$

3.2. **Intergranular Creep Damage Measurement**. In order to give a value to the intergranular creep damage, which takes place during fatigue relaxation tests, the intergranular damage coefficient D_m is measured as described in [3, 5] by Eq. (14)

$$D_m = L_f / L_a, \tag{14}$$

where L_f represents the accumulated length of cracked grain joints (except for the isolated cavities), while L_a represents the total length of the measured grain joints on the same micrographic field to a 200 magnification, a scope of 0.55–0.4 mm^2.

3.3. *Intergranular Damage per Cycle*. The evolution of D_m is a sensitive function of the number of cycles, which has allowed the author [3, 5] to attribute to every cycle an elementary damage coefficient D_c defined by Eq. (15)

$$D_c = D_m / N_r. \tag{15}$$

In the same way that the coefficient D_c has been defined, author [5] defined the reduction coefficient of the propagation phase by cycle R_c, which is expressed as $R_c = R^{FR} / N_R^{FR}$. Assuming that $N_a^{FR} = 0$ and

$$N_R^{FR} = N_a^{FR} + N_p^{FR}, \tag{16}$$

the life duration coefficient R_c has been obtained in [5] from Eq. (17):

$$R_c = \frac{N_p^{FC} - N_R^{FR}}{N_p^F N_R^F}, \tag{17}$$

This relation is very important because it allows to calculate the number of cycles to failure in fatigue relaxation, knowing only N_p^{FC}, which can be calculated by Eq. (11) if the number of cycles to failure N_R^{FC} is known. The latter one is calculated by using the Coffin–Manson rule.

4. **Experiment**.

4.1. **Material [1, 2, 3]**. Our study has been performed on the alloy 800, at the state of grade 1 (980°C). This alloy presents a structure, which is entirely austenitic. Its chemical composition and its mechanical properties at the delivery state are given in Tables 1 and 2.

Table 1

Material Chemical Composition [2, 3]

C	S	P	Si	Mn	Ni	Cr
0.03 – 0.06 / 0.03	0.015 / 0.004	0.015 / 0.008	0.07 / 0.50	1.00 / 0.070	32.0 – 32.5 / 33.30	19.0 – 23.0 / 21.0
Mo	**Co**	**Cu**	**Ti**	**Al**	**Ti+Al**	**N**
– / 0.05	0.25 / 0.02	0.75 / 0.02	0.30 – 0.50 / 0.41	0.10 – 0.25 / 0.13	0.45 – 0.75 / 0.54	0.030 / 0.014

Note. Here and in Table 2: over the line are given the data obtained according specification Novatome-NIRA* and under the line – EDF analysis.

Table 2

Mechanical Characteristics in As-Recieved State [2, 3]

20°C				400°C			
$R_{0.002}$, MPa	R_m, MPa	A, %	Z, %	$R_{0.002}$, MPa	R_m, MPa	A, %	Z, %
210 – 350 / 187 – 193	520 – 700 / 542 – 544	30 / 52 – 53	– / 74 – 75	160 / 108 – 113	455 / 452 – 456	– / 50	– / 69 – 71

4.2. **Experimental Method [3]**. The test was carried out to estimate the interaction fatigue creep have lead to specimen tests treated at state of grade 2, obtained after a heat treatment at 1100°C during 30 min [3].

The fatigue tests consisted in subjecting specimens heated to 550°C to cyclic deformation comprised between two limits of deformations up to the failure. In the case of fatigue relaxation, the deformation level is maintained constant during an interval of variable relaxation (Fig. 1). All the tests have been conducted using test machines with electromechanic servocontrol [3].

5. **Proposed Model**.

5.1. **Introduction and Objectives**. In order to describe the evolution of damage in continuous fatigue, a mechanical law proposed by Chaboche [3, 6, 7] has been considered. To describe the evolution of damage during fatigue-relaxation tests, the intergranular damage measurement by cycle has been used. It is characterized by the coefficient D_c deduced from metallurgic observations and obtained according to the Levaillant model described above. The following issues have to be considered:

1. Changing this metallurgic damage D_c into a mechanic damage.
2. Combining the fatigue damage and that of creep so as to have a law describing the interaction between the fatigue and creep damages, which happens simultaneously when the fatigue relaxation is present.

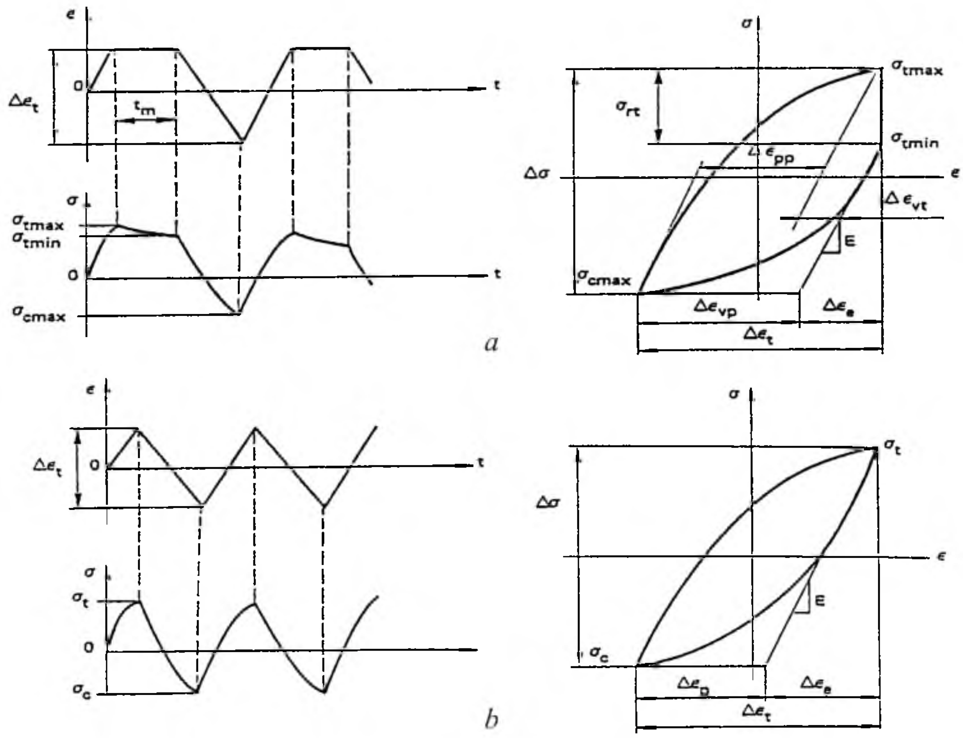

Fig. 1. Loading modes and cycle type obtained during continuous fatigue (*a*) and fatigue-relaxation tests (*b*).

3. Comparing the results of prediction obtained by the present model to these obtained by experiment and the two previous models.

5.2. **Model Description**. Our model is based on a simple idea, which assuming that there are tow damage mechanisms in fatigue relaxation.

a) *The first mechanism*: It characterizes the continuous fatigue damage, where the material degradation is considered as being directly related to the applied stress during the continuous fatigue tests as focused in [4, 6, 7]. Here the model of failure is transgranular and presents more often faces marked by fatigue striations witnessing that there is crack propagation from one or two sites of the crack initiation [3, 4].

b) *The second mechanism*: It characterizes the creep damage and stresses giving, thus, the influence of the hold time, which corresponds to the presence of intergranular microcracks in the test tube mass. This damage is generated by the importance of the viscoplastic distortion ε_{VP} or the relaxed stress σ_{rt}. This second mechanism is based on metallographic observations of the broken tube tests [3, 5].

5.2.1. *Fatigue Case*. The damage in fatigue is considered as a function, which depends on the stress amplitude σ_a, the average stress σ_{av} and the damage itself. This function has the following form (18).

$$dD_{fat} = h(\sigma_a, \sigma_{av}, D_{fat})dN, \tag{18}$$

where σ_a is the stress amplitude of a stabilized cycle, σ_{av} is the average stress, and D_{fat} is the fatigue damage.

5.2.2. *Creep during Relaxation*. Here, the creep damage is a function which depends on the relaxed stress σ_{rt} (or $\sigma_{t\max}$), the applied stress σ_{vav} and of the damage itself. This function has the form (19).

$$dD_{flu} = g[\sigma_{rt} \text{ (or } \sigma_{t\max}), \sigma_{vav}, D_c]dt, \tag{19}$$

where D_{flu} is creep damage, σ_{rt} is relaxation stress, $\sigma_{t\max}$ is the mplitude of maximum stress, σ_{vav} is the actual average stress, and D_c is the intergranular damage coefficient by cycle.

5.2.3. *Damage Interaction in Fatigue Relaxation*. The interaction of the two factors is accounted for by adding two elementary damages defined by Eqs. (18) and (19):

$$dD_t = dD_{fat} + dD_{flu},$$

$$dD_t = h(\sigma_a, \sigma_{av}, D_{fat})dN + g[\sigma_{rt} \text{ (or } \sigma_{t\max}), \sigma_{vav}, D_c]dt. \tag{20}$$

It is noteworthy that the two functions h and g can be determined independently, respectively either by pure fatigue test for the function h or by creep test with an imposed loading for the function g.

5.3. **Proposed Equation for Description of Damage Evolution in Fatigue Relaxation**. We propose a new damage law in fatigue relaxation. The particularity of this law is that it allows the interactive coupling, that is, the multiplying coupling of the damage. The global law is not a simple sum of the elementary damages. It is assumed that in fatigue relaxation with hold under tension, the fatigue intervenes accelerating of the main crack propagation. The proposed equation describing the evolution of fatigue relaxation damage is obtained by multiplying the coefficient representing the intergranular damage with the number of cycles N, in power $\gamma(\Delta\varepsilon_t)$, that is a function of the total imposed strained $N^{\gamma(\Delta\varepsilon_t)}D_c^*$. The creep intervenes with the term D_c^*, introducing the creep damage during fatigue relaxation tests. For the homogenisation of this equation, the coefficient D_c^* has been considered as the ratio between the intergranular damage coefficient per cycle D_c and $N_R^{\gamma-1}$ that yields:

$$D_c^* = \frac{D_c}{N_R^{\gamma-1}}. \tag{21}$$

In order to characterize the fatigue influence, some parameters have been taken into account, such as the number of cycles N, the ratio $1/N_R^{\gamma-1}$ and parameter ξ, which is a correction factor defined as $\xi = k\dfrac{N}{N_R}$. The term k is a constant to be determined.

The final rule for the evolution of total damage D_{total} in fatigue relaxation, is described by Eq. (22):

$$D_{total} = 1 - \left[1 - k \frac{N}{N_R} \left(N^{\gamma(\Delta\varepsilon_t)} \frac{D_c}{N_R^{\gamma-1}} \right)^{\lambda} \right]^m , \tag{22}$$

where γ is a coefficient depending on the material and the imposed strain amplitude $\gamma = f(\Delta\varepsilon_t)$, λ is a coefficient depending on the material, m is a coefficient allowing to adjust the concavity of the curves of damage evolution as a function of life duration, and ξ is a correction factor introducing the influence of the fatigue damage evolution.

The damage criteria applied in this study are:

a) if $N = 0$, then $D = 0$;

b) if $N = N_R$, then $D = 1$.

5.4. **Calculation of the Number of Cycles to Failure N_R**. At failure, the total damage is equal to the unit:

$$k[N_R^{\gamma(\Delta\varepsilon_t)} D_c^*]^{\lambda} = 1,$$

where

$$D_c^* = \frac{D_c}{N_R^{\gamma-1}}.$$

Equation (14) can be obtained as

$$N_R = \frac{1}{D_c} \left(\frac{1}{k} \right)^{1/\lambda}. \tag{23}$$

This relation allows computing of the number of cycles to failure N_R in fatigue relaxation, only knowing the intergranular damage coefficient measured per cycle D_c and the two coefficients k and λ.

5.5. **Determination of Coefficients of the Model for the Life Prediction**. In order to identify the parameters λ, k, and m considered to be positives, it is assumed that the equation giving the evolution of the total damage D_{total} is positive. The evolution of the intergranular damage D_c is studied first.

5.5.1. *Evolution of Intergranular Damage D_c.*

Evolution of D_c as a function of the relaxation stress (σ_{rt}). The coefficient D_c given by Eq. (21), is in good correlation with the relaxation stress σ_{rt} [3], as shown in Fig. 2. It is considered that the responsible parameter of the grain joint damaging is the viscoplastic deformation ε_{vp} provoked by the relaxation stress σ_{rt} that is generated while holding under relaxation. The linear regression performed on the couples (D_c, σ_{rt}) for $\Delta\varepsilon_t = 1.5\%$ yields

$$D_c(1.5\%) = 18 \cdot 10^{-4} (\sigma_{rt})^{2.61}. \tag{24}$$

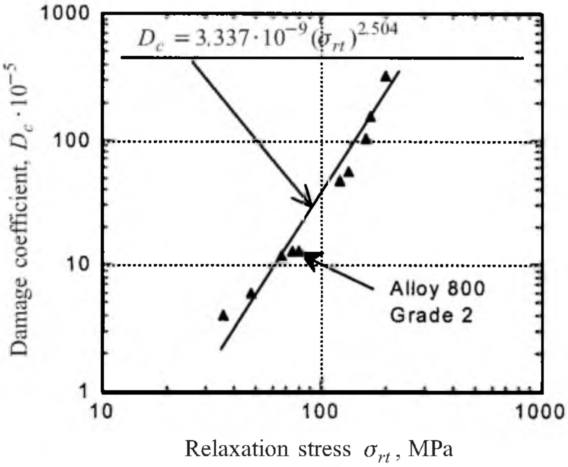

Fig. 2. Correlation between the damage coefficient D_c and the relaxation stress σ_{rt} [3].

Evolution of D_c as a function of hold time (t_{mt}). Figure 3 shows the evolution of the intergranular damage per cycle D_c as a function of the hold time under tension t_{mt} for $\Delta\varepsilon_t = 1.5\%$ [3].

The relation found between D_c and t_{mt} has the form (25):

$$D_c(1.5\%) = p(t_{mt})^{\mu(1.5\%)}. \tag{25}$$

The linear regression allows the identification of the two coefficients: $\mu = 0.4468$ and $p = 5.2033 \cdot 10^{-5}$.

For long-term predictions with hold time t_{mt} = 60,000 min and $\Delta\varepsilon_t = 1.5\%$, $\Delta\varepsilon_t = 0.8\%$, and $\Delta\varepsilon_t = 0.6\%$, Eqs. (25)–(27) can be used as extrapolation rules to determine the coefficient D_c.

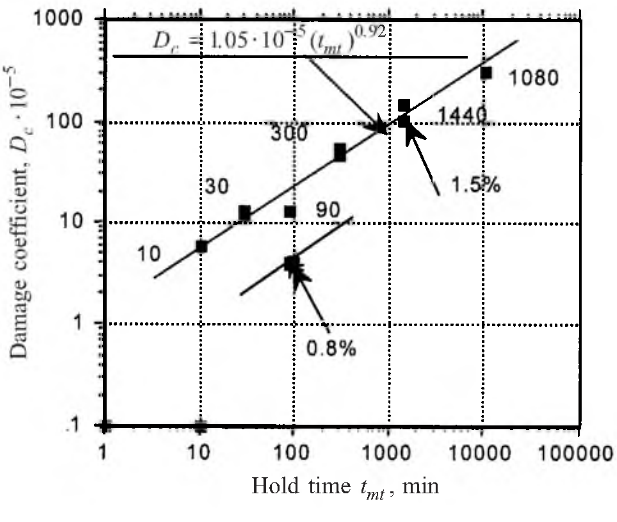

Fig. 3. Correlation between the intergranular damage coefficient D_c and hold time t_{mt} [3].

5.5.2. *Method of Determination of the Coefficient D_c for Small-Scale Deformations*. The method is based on the use of Eq. (25) that allows computing of the coefficient D_c for long-term hold duration and large-scale deformations, for example: t_{mt} = 10,080 min and $\Delta\varepsilon_t$ = 1.5%. However, the evolution of D_c has to be known for the other strain amplitudes $\Delta\varepsilon_t$ = 0.8% and $\Delta\varepsilon_t$ = 0.6%.

Assumption: In order to determine the two coefficients $D_c(0.8\%)$ and $D_c(0.6\%)$ for various hold periods, it is assumed that the evolution of these coefficients as a function of the hold time (t_{mt}), is similar to the one corresponding to $D_c(1.5\%)$, as shown in Fig. 3. As a matter of fact, the evolution of $D_c(0.8\%)$ and $D_c(0.6\%)$, as a function of t_{mt}, can be given by Eqs. (17) and (18).

$$D_c(0.8\%) = p_1(t_{mt})^{\mu(0.8\%)=\mu(1.5\%)}, \tag{26}$$

$$D_c(0.6\%) = p_2(t_{mt})^{\mu(0.6\%)=\mu(1.5\%)}. \tag{27}$$

In order to determine the two coefficients p_1 and p_2, measurements of D_c are available for the hold period $t_{mt} = 90$ min – $\Delta\varepsilon_t$ = 1.5% and $\Delta\varepsilon_t$ = 0.8%:

$$D_c = 28 \cdot 10^{-5},$$

$$D_c = 8 \cdot 10^{-5}.$$

However, measurements concerning $D_c(0.6\%)$ are not available and they have to be determined herewith.

5.5.3. *Determination of the Coefficient $D_c(0.6\%)$*. Figure 4 shows the evolution of the coefficient D_c as a function of total strain $\Delta\varepsilon_t(\%)$ for the hold time of 90 min, and this plot allows to obtain $D_c(0.6\%)$ for the hold time of 90 min. For this reason, it is assumed that $D_c(0.6\%)$ has a linear evolution as a function of the strain $\Delta\varepsilon_t(\%)$.

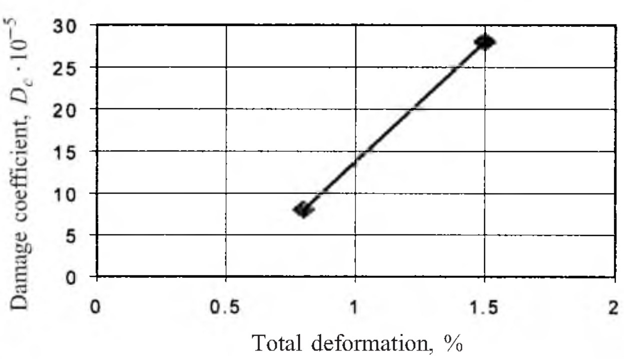

Fig. 4. Evolution of D_c as a function of $\Delta\varepsilon_t(\%)$.

The evolution of D_c as a function of total strain amplitude $\Delta\varepsilon_t(\%)$ is given by Eq. (19), which allows one to determine D_c for any total strain amplitude.

Particularly for $\Delta\varepsilon_t = 0.6\%$, the estimated result by this equation is $D_c(0.6\%) = 2.2856 \cdot 10^{-5}$.

$$D_c = 28.572(\Delta\varepsilon_t) - 14.857. \qquad (28)$$

As shown in Fig. 5, the evolution of the coefficient $D_c(1.5\%)$ is given as a function of hold duration (t_{mt}) in logarithmic coordinates. Two parallel lines to the evolution line of $D_c(1.5\%)$ have been drawn from the two points representing the two couples of values: (90 min – $8 \cdot 10^{-5}$) and (90 min – $2.2856 \cdot 10^{-5}$). These two lines represent the evolution of $D_c(0.8\%)$ and $D_c(0.6\%)$.

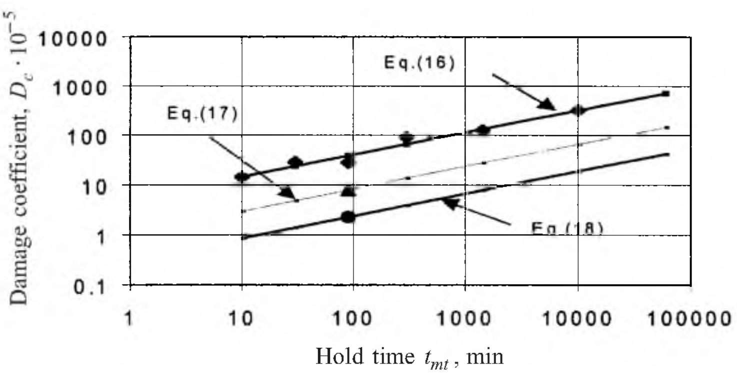

Fig. 5. Correlation between the coefficient D_c and hold time t_{mt} for three levels of deformation and the determination of the coefficients p_i and μ_i [3, 8].

We have established that:

1. Consequently to the equality of slopes, $\mu(1.5\%) = \mu(0.8\%) = \mu(0.6\%) = 0.4468$.

2. The linear regression allows to identify coefficients of Eqs. (17) and (18). Finally, the following relations are obtained:

$$D_c(1.5\%) = 5.2033 \cdot 10^{-5} (t_{mt})^{0.4468}, \qquad (29)$$

$$D_c(0.8\%) = 1.0713 \cdot 10^{-5} (t_{mt})^{0.4468}, \qquad (30)$$

$$D_c(0.6\%) = 0.3061 \cdot 10^{-5} (t_{mt})^{0.4468}. \qquad (31)$$

5.5.4. *Determination of Coefficient* $\gamma(\Delta\varepsilon_t)$. The fitting of the relation $N_R = (D_c)^{-1/\gamma}$ to the couples (N_R^{exp}, $D_c^{measured}$) in Fig. 6 for various hold periods at $\Delta\varepsilon_t = 1.5\%$ made it possible to obtain values of $\gamma(1.5\%)$ and $\gamma(0.8\%)$ for $t_{mt} = 90$ min (see Table 6).

It is found that values of $\gamma(1.5\%)$ vary between a maximum and a minimum. It is thought that is the same for the values of $\gamma(0.8\%)$ and $\gamma(0.6\%)$. It is thought that there is a maximum and a minimum around the points obtained respectively for $\Delta\varepsilon_t = 0.8\%$ and $\Delta\varepsilon_t = 0.6\%$ with $t_{mt} = 90$ min. The estimation of the values of $\gamma(0.8\%)$ and $\gamma(0.6\%)$ for various hold periods has been obtained using the relation $\gamma(\Delta\varepsilon_t) = 1.2118 + 0.1478\Delta\varepsilon_t$ (see Table 3). The evolution of $\gamma[\Delta\varepsilon_t(\%)]$, deduced this way, as a function of the total strain $\Delta\varepsilon_t(\%)$ is given in Fig. 6.

Table 3

Values of k for Various Hold Durations

t_{mt}, min	10	30	300	1440	10080
$(N_R D_c)^{-1}$	8.7870	7.9926	7.1537	6.015	5.2247

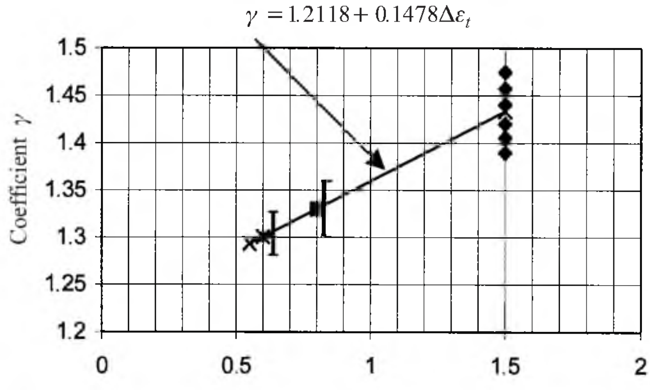

Fig. 6. Determination of γ for strain amplitudes 0.8 and 0.6% [8].

5.5.5. *Method of Identification of Coefficients k, λ, m, and γ.*

a) *Determination of the value of the coefficient k.* In order to calculate the value of k, it should be verified that $D_{total} \le 1$. For this reason, it is necessary that the expression (20) be positive.

$$\left[1 - \left(k\frac{N}{N_R}\right)(N^{\gamma(\Delta\varepsilon_t)} D_c^*)^\lambda\right] \ge 0. \tag{32}$$

Since

$$\left(k\frac{N}{N_R}\right)(N^{\gamma(\Delta\varepsilon_t)} D_c^*)^\lambda \le 1. \tag{33}$$

Therefore, it is necessary that

$$k \le \frac{N_R}{N}(N^{\gamma(\Delta\varepsilon_t)} D_c^*)^{-\lambda}.$$

If the function $g(N)$ is considered to be defined by (27):

$$g(N) = \frac{N_R}{N}(N^{\gamma(\Delta\varepsilon_t)} D_c^*)^{-\lambda}. \tag{34}$$

The function (34) is an increasing function, as its derivative $g'(N)$ is <0. As a matter of fact,

$$g'(N) = [N_R (D_c^*)^{-\lambda}][-(\gamma\lambda) - 1] N^{-(\gamma\lambda)-2}. \qquad (35)$$

It is assumed that

$$k = g(N_R) = \min\{g(N),\ N > 0\}. \qquad (36)$$

This allows to obtain k by the following relation:

$$k = (N_R^{\gamma(\Delta\varepsilon_t)} D_c^*)^{-\lambda} = (N_R D_c)^{-\lambda}. \qquad (37)$$

A value of k is obtained for each test.

For $D_{total} \le 1$ for all the tests, the condition $k \le \inf(N_R D_c)^{-\lambda}$ is necessary.

Assumption: If it is assumed that $\lambda \ge 1$, then $k \le \inf(N_R D_c)^{-1}$.

The five values obtained for $(N_R D_c)^{-1}$, for five hold periods, respectively are given in Table 3.

It is accepted that $k = 5.224$.

b) *Determination of the value of the coefficient* λ. The coefficient λ can be determined from Eq. (14), with $\lambda = -\dfrac{\log k}{\log(N_R D_c)}$ and $k = 5.224$. For the five different hold times t_{mt}, respectively: 10, 30, 300, 1440, and 10080 min, the five following values are obtained (Table 4).

Table 4

Values of λ for Various Hold Durations

t_{mt}, min	10	30	300	1440	10080
λ	0.7607	0.7954	0.8402	0.9214	0.9999

Choice: $\lambda \ge 1$. In our case, the five computed values are <1. It is accepted $\lambda = 1$.

c) *Determination of the value of the coefficient m*. In order to determine the value of coefficient *m*, the conditions of increasing and of concavity D_{total} have been used.

(i) D_{total} is increasing as its derivative $(D_{total})'$ is positive.

$$(D_{total})' = \frac{1}{N_R} m(1+\gamma\lambda)(N_R D_c)^{\lambda} \left[1 - \left(k \frac{N}{N_R}\right)(N^{\gamma(\Delta\varepsilon_t)} D_c^*)^{\lambda}\right]^{m-1}. \qquad (38)$$

(ii) For D_{total} be concave, it is necessary that the second derivative $(D_{total})''$ be positive.

$$(D_{total})'' = \frac{1}{N_R} mk(1+\gamma\lambda)(D_c^*)^\lambda N^{\gamma\lambda-1}\left[1-\left(k\frac{N}{N_R}\right)(N^{\gamma(\Delta\varepsilon_t)}D_c^*)^\lambda\right]^{m-2}, \quad (39)$$

This yields

$$\left[\gamma\lambda - k\frac{N^{\gamma\lambda+1}}{N_R}(D_c^*)^\lambda[m(1+\gamma\lambda)-1]\right] \geq 0. \quad (40)$$

or

$$m \leq \left[\frac{\gamma\lambda N_R}{kN^{\lambda\gamma+1}(D_c^*)^\lambda}+1\right]\frac{1}{\lambda\gamma+1}. \quad (41)$$

As the term $(\lambda\gamma+1)$ is positive, it is taken that $N = N_R$:

$$m \leq \left(1+\frac{\lambda\gamma}{k(N_R D_c)^\lambda}\right)\frac{1}{\lambda\gamma+1}.$$

The five values obtained for the following expression

$$\left(1+\frac{\lambda\gamma}{k(N_R D_c)^\lambda}\right)\frac{1}{\lambda\gamma+1}$$

are given in Table 5.

Table 5

Values m for Various Hold Durations

t_{mt}, min	10	30	300	1440	10080
m	1.4045	1.3158	1.2184	1.0885	1.0001

Let's take: $m = 1.0001$.

5.5.6. *Damage Evolution*. The application of Eq. (13) is given in Figs. 7 and 8:

Figure 7 shows the damage evolution D_{total} as a function of the ratio N/N_R for the fatigue-relaxation tests under $\Delta\varepsilon_t = 1.5\%$ and various hold intervals. It is found that total damage increases since the first cycles and becomes preponderant while N/N_R tends towards 1.

It is found that the life duration is decreasing while:

– The total damage D_{total} increases sensitively since the first cycles et tends towards its maximal value assigning its failure.

– The holding interval increases.

– The coefficient of intergranular damage per cycle D_c increases.

Table 6 indicates the results obtained by applying different models.

Figures 9 to 12 show the life duration predictions N_R (cycles) given by the model of El Gharad compared to the experimental results [3, 8].

T a b l e 6

Results of Life Predictions Obtained by the Model of El Gharad [8], Compared to the Ones Obtained by the Two Models of Chaboche [3, 8] and Levaillant [3, 8]

$\Delta\varepsilon_t$, %	Holding times t_{mt}, min	Life duration N_R (cycles) tests	$D_c^{1)} \cdot 10^{-5}$	γ	$D_c^{2)} \cdot 10^{-5}$	Life prediction N_R (cycles) by the models		
						El Gharad [8]	Chaboche [3, 8]	Levaillant [3, 8]
1.5	10	794	14.333	1.4573	14.5572	1335[3)]	744	634
	30	439	28.500	1.4745	23.7824	671[3)]	408	514
	90	360	28.000	1.2515	38.8538	736[3)]	–	–
	300	198	90.600	1.4467	66.5359	211[3)]	158	278
	1440	133	125.000	1.4048	134.1019	153[3)]	92	170
	10080	60	319.000	1.4056	319.9079	60[3)]	52	85
	60000	–	–	1.3998	709.8340	27[4)]	37	48
0.8	90	1975	8	1.3311	7.9988	2393[3)]	1229	2048
	300			1.3311	13.6977	1397[4)]	–	–
	1440			1.3311	27.6074	693[4)]	784	1029
	10080			1.3311	65.8592	290[4)]	384	530
	60000			1.3311	146.1330	130[4)]	190	253
0.6	90			1.3124	8.7202	205[4)]	9976	3849
	300			1.3124	17.5755	2195[4)]	–	–
	1440			1.3124	17.5755	1089[4)]	930	3122
	10080			1.3124	41.9274	456[4)]	191	2782
	60000			1.3124	93.0316	205[4)]	39	2386

Notes. [1)] Values that are slightly higher than the ones obtained by tests in [3, 8]. [2)] Values are computed with Eqs. (16)–(18). [3)] Predictions obtained using $D_c^{1)}$. [4)] Predictions obtained using $D_c^{2)}$.

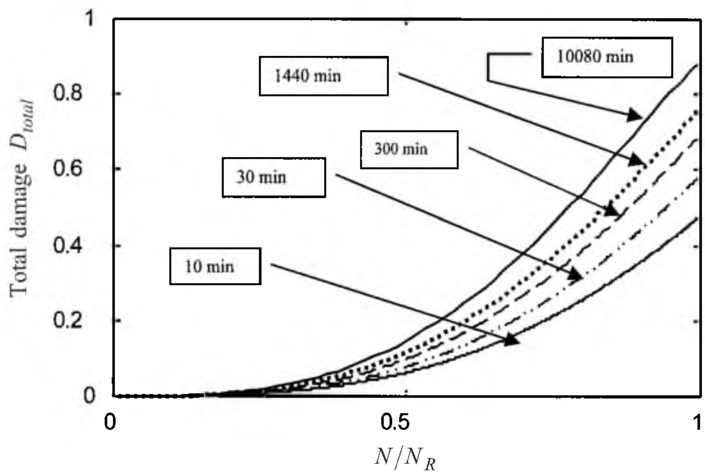

Fig. 7. Damage evolution as a function of N/N_R in fatigue relaxation for hold intervals from 10 to 10,080 min, by means of the model [8].

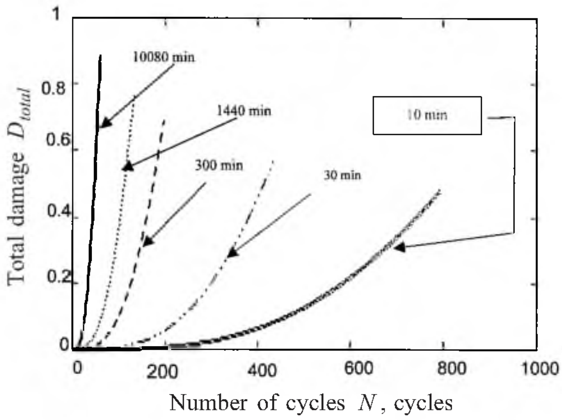

Fig. 8. Comparison of evolutions of total damage D_{total} as a function of number of cycles N from various fatigue relaxation tests by the model [8].

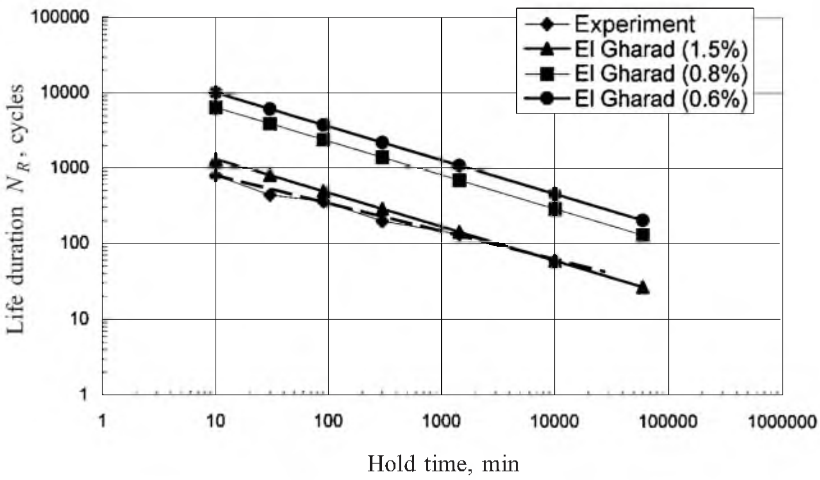

Fig. 9. Life duration prediction by the model of El Gharad and comparison of results to the tests [3, 8].

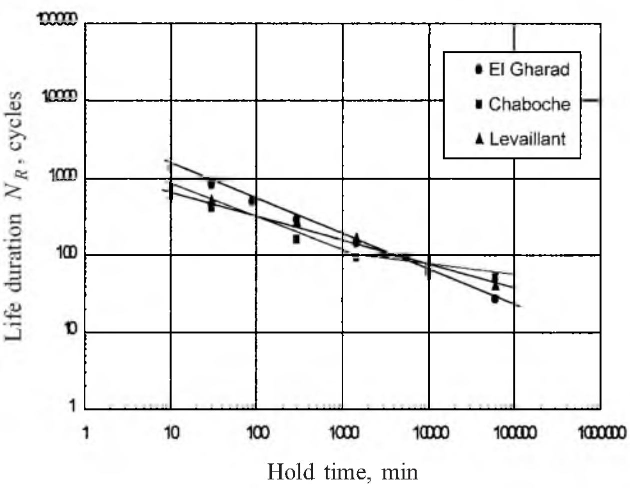

Fig. 10. Comparison of life duration prediction obtained by the three fatigue relaxation damage models for $\Delta\varepsilon_t = 1.5\%$ [3, 8].

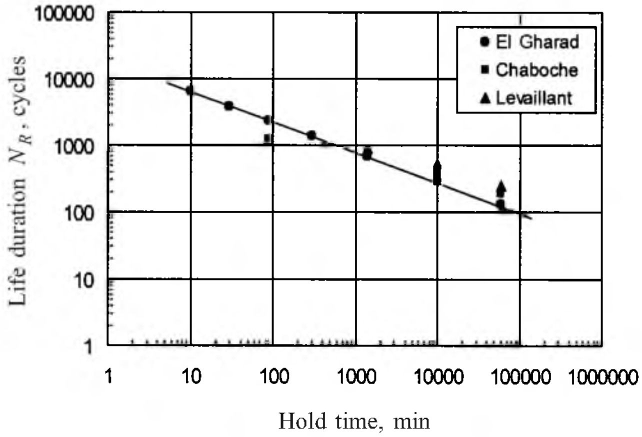

Fig. 11. Comparison of life duration predictions N_R (cycles) obtained by the three fatigue relaxation damage for $\Delta\varepsilon_t = 0.8\%$ [3, 8].

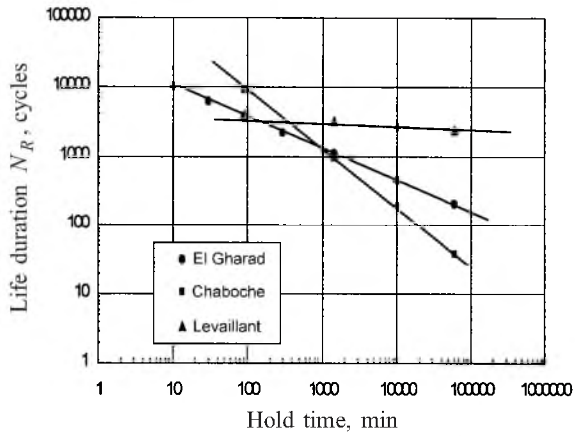

Fig. 12. Comparison of life duration N_R (cycles), obtained by the three fatigue relaxation damage models for $\Delta\varepsilon_t = 0.6\%$ [3, 8].

6. **Discussion**. The difference between the three models is characterized by the simplicity of use and the number of coefficients required to be determined for each model.

The three models proposed, respectively, by J. L. Chaboche, C. Levaillant, and A. El Gharad have been compared with the results of continuous fatigue tests, fatigue relaxation and creep, obtained on the alloy 800 grade 2 at 550°C.

6.1. **The Chaboche Model Specification**. In order to use this model in fatigue relaxation:

(i) It's necessary to identify 17 coefficients, which is very difficult task to perform.

(ii) It's also necessary to know $\sigma(t)$ during the relaxation cycle.

(iii) For a long term, $\sigma_{t\min}$ should be known. It can be determined preliminarily by knowing the stress $\sigma_{t\max}$ and the relaxation rule. Finally, the iterative calculation gives the number of cycles to failure in fatigue relaxation N_R^{FR}.

It is noteworthy that the long-term prediction result (for $\Delta\varepsilon_t = 1.5\%$) is in good accordance with the experiment $\sigma_{t\max}$ and it is also reasonable for the extrapolation to 1000 hours. Meanwhile, for $\Delta\varepsilon_t = 0.6$ and 0.8%, the calculation of σ_{rt} is obtained when the minimal stress $\sigma_{t\min}$ is determined by the relaxation rules, based on the hypothesis about $\sigma_{t\max}$.

6.2. **The Levaillant Model Specification**. It should be noted that this model is related to the stress σ_{rt}, and insofar as it is based on metallographic observations, an exhaustive metallographic analysis must be performed in order to apply this model to the case of fatigue relaxation.

In continuous fatigue: We use the fatigue curve to calculate N_R^{FC}, and Eq. (11) to calculate N_p^{CF}.

In fatigue relaxation: It is obligatory to know the relaxed stress σ_{rt} for the total deformation $\Delta\varepsilon_t$ and the hold time t_{mt}.

Within the framework of this model, it is necessary to interrupt tests to determine $\sigma_{t\min}$ or a relaxation rule to know σ_{rt}, from where the calculation of the number of cycles to failure in fatigue relaxation N_R^{FR} is possible.

It is noticed that the long-term prediction results (for $\Delta\varepsilon_t = 1.5\%$) are in good accordance with the experimental data on $\sigma_{t\max}$. Moreover, the obtained results are sensitive to the relaxation rule used to extrapolate the relaxed stress σ_{rt}, especially in the case of small-scale deformations ($\Delta\varepsilon_t = 0.6\%$).

Conclusions. This study has allowed us to give preference to the new model which permits to predict the life duration in fatigue relaxation with small-scale deformations and long-term hold periods. In fact, this model is based on a simple concept of interaction of various damage mechanisms.

The relaxation process is characterized both by fatigue and creep mechanisms which occur simultaneously when fatigue relaxation tests are carried out.

In order to apply the above model, the following steps are to be made:

It has been chosen to use the equation proposed by J. L. Chaboche to describe the fatigue damage [3, 4, 6, 7].

To describe the creep damage evolution, we strongly recommend to use the parameter D_c that characterizes the measure of the intergranular creep damage per cycle according to the Levaillant method [3, 5].

The relation describing the creep damage progress is characterised by parameter $\gamma(\Delta\varepsilon_t)$ which is a function of the amplitude of the applied deformation. It's application has allowed to notice that the damage progress is a function of the parameter D_c, i.e., of the hold time in relaxation.

For the fatigue relaxation, Eq. (3) has been applied. This shows that the fatigue damage becomes negligible compared to that of creep, which dominates especially for the tests, whose maintain time is very long. The identification of this model requires to know: a) 5 coefficients which are easily determined, and b) measures of intergranular damage of large-scale deformations.

The life prediction is given by Eq. (15), obtained by applying the damage criterion.

The results obtained are in good accordance with the experiment for large-scale deformations and satisfying for the small-scale ones.

The comparison of the results obtained by our model with those obtained by the model proposed by J. L. Chaboche [3, 4, 6, 7] has allowed us to show the advantage of this model, especially for small-scale deformations. Insofar as the estimation of the parameter $\gamma(\Delta\varepsilon_t)$ influences the obtained results, it is necessary to be careful while identifying the model.

Резюме

Прогнозування утомної довговічності в умовах повзучості при температурі 550°С виконано для жароміцного і жаростійкого сплаву на нікелевій основі (Inconel). Представлено методи еволюції пошкодження Шабоша (Chaboche) і Леваяна (Levaillant), а також нову модель релаксації при сумісній дії повзучості й утоми. За допомогою запропонованої моделі можна проводити довготермінове прогнозування без використання великої кількості сталих.

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Received 09. 10. 2001