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Effect of the Static Strength of a Metal on Long-Rod Penetration at Low Velocities

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Влияние статической прочности металла на проникание длинного стержня с низкой скоростью

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Рассмотрено влияние статической прочности (предела текучести) материала на основные параметры процесса проникания – удельную работу образования каверны и давление на поверхности контакта. Получены аналитические выражения и количественные оценки удельной работы, затраченной на образование сферической и цилиндрической каверн, и давления на поверхности таких каверн при их статическом расширении для жестко-пластической и упругопластической моделей материала. Показана адекватность оценок удельной работы деформирования пластины при образовании сферической каверны и при образовании каверны в результате проникания жесткого стержня. Оценено влияние деформационного упрочнения материала на максимальное радиальное напряжение на поверхности каверны. Показано, что при низких скоростях проникания существенное влияние на сопротивление прониканию и расширение каверны оказывает упругая сжимаемость материала в области неупругих деформаций. Проведен анализ кинетики напряженно-деформированного состояния материала при проникании деформируемого стержня.

Introduction. The longitudinal force, affecting a long rod at its penetration in an elastoplastic medium, is usually represented as the sum of two components, one of them is associated with the hydrodynamic pressure at the contact surface of a rod head and the other with the resistance of a material to plastic deformation. Though the real process of penetration should account for the nonlinear interaction of those two components, their independent representation can greatly simplify the analysis of penetration.

The static resistance of a plastic medium to the penetration of a long rod is a parameter widely used for evaluating the strength effect on penetration at high velocities [1–7]. This component of resistance to penetration and its association with the characteristics of a material, determining its mechanical behavior, is examined in a number of publications [8–11]. However, various models of penetration, models of an elastoplastic material used for its simulation, and methods of solution of mathematical equations resulted in different values of resistance and dimensions of a plastic region [3, 5, 8, 11–14]. It will suffice to mention that the pressure at the interface at penetration is assumed to be equal to the pressure necessary for the expansion of a cylindrical or spherical cavity in the unlimited volume of a material, or to some intermediate value [13, 15, 16].

Simplified rigid-plastic or elastoplastic models of a material are often used for the analysis of penetration. The methods of calculation include computer simulations and various analytical methods, with some simplifications of real plastic flow [4, 11, 12]. The main calculated parameters are the pressure onto the contact surface and the sizes of an elastoplastic interface.

Intense deformations in real materials influence their resistance to deformation, which varies with strain hardening and/or softening (the latter is the result of structural changes, damage, and heating of a material at plastic flow). Therefore, the stress-strain kinetics and the distribution of stress, strain, and a strain rate near the rod head are important for the evaluation of strength at penetration, but those effects are not investigated thoroughly enough.

The resistance of a plastic medium to the static penetration of a rod is determined by its resistance to plastic deformation accounting for a real 3D stress-strain state in the material near the rod head. Modern models of homogeneous viscoplastic materials allow one to relate the stress intensity σ_i, to the plastic strain intensity ε_i, the plastic strain rate intensity ε'_i, the pressure p, and the temperature T, increasing at penetration as the result of thermal effects at plastic flow:

$$\sigma_i = \sigma(\varepsilon_i, \varepsilon'_i, p, T).$$

The longitudinal force, affecting the rod at penetration in a plastic material, is the integral characteristic of the pressure distribution near the contact surface of interacting bodies. Plastic flow criterion-based approaches [8, 10] are proposed to evaluate the magnitude of the pressure and its distribution. According to known findings, the pressure at the contact surface at long-rod penetration is connected with the strength of a plastic medium, and the pressure onto the surfaces of a spherical or cylindrical cavity at its static expansion can be taken as a good approximation. To get the more reliable value of resistance to penetration, one should account for the changes in the stress-strain state of the material at its flow along the rod head. From general representations, in the layer of the material located between the two planes perpendicular to the rod axis, the cylindrical cavity formed is caused by the development of axisymmetric flow, which includes the displacements of the material in radial and axial directions. The analysis of the flow and changes in the resistance to penetration is one of the main goals introduced below.

This paper is confined only to the analysis of certain penetration effects due to the static strength inherent in an elastoplastic medium (without strain hardening). The penetration is an essentially dynamic process, but at near-threshold velocities, at the last stage of high-velocity penetration, and at the inertial expansion of a cavity, the plastic flow is mainly controlled by the static strength of the medium.

1. Pressure onto the Surface of a Spherical Cavity at Its Expansion under Static Loading.

1.1. **Rigid-Plastic Material**. According to known approaches, the equation of static equilibrium for the elementary volume of a spherical layer in the ideally plastic region ($r < r_e$, r_e is the radius of the elastoplastic interface), using the condition of plasticity $(\sigma_r - \sigma_t) = 2\tau_Y$ (τ_Y is the yield stress in shear for a material without strain hardening), can be written in the form:

$$d\sigma_r / dr = -2(\sigma_r - \sigma_t)/r = -4\tau_Y / r. \tag{1}$$

Integrating this equation with the account of $\sigma_r = -(4/3)\tau_Y$ at $r = r_e$ for the elastoplastic interface, we obtain the distribution of radial stress components in the plastic region:

$$\sigma_r = -(4/3)\tau_Y[1 - \ln(r/r_e)]. \tag{2}$$

The stress and plastic strain components in spherical coordinates (Fig. 1)

$$\sigma_1 = \sigma_r;\ \sigma_2 = \sigma_3 = \sigma_t = \sigma_r + 2\tau_Y;\ \tau_{12} = \tau_{23} = \tau_{31} = 0;$$

$$\varepsilon_1 = \varepsilon_r;\ \varepsilon_2 = \varepsilon_3 = \varepsilon_t = -\varepsilon_r/2;\ \varepsilon_{12} = \varepsilon_{23} = \varepsilon_{31} = 0$$

correspond to the stress and plastic strain intensities in the plastic region:

$$\sigma_i^2 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + 3(\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2)]/2 = 4\tau_Y^2;$$

$$\varepsilon_i^2 = (2/9)[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2 + 3(\varepsilon_{12}^2 + \varepsilon_{23}^2 + \varepsilon_{31}^2)] = \varepsilon_r^2. \tag{3}$$

If we take into account that the stress intensity in a material at the initiation of plastic flow in a 1D stress state is equal to the yield strength σ_Y (at the strain ε_Y) according to the Mises criterion of plastic flow, it follows from Eq. (3): $\tau_Y = \sigma_Y / 2$.

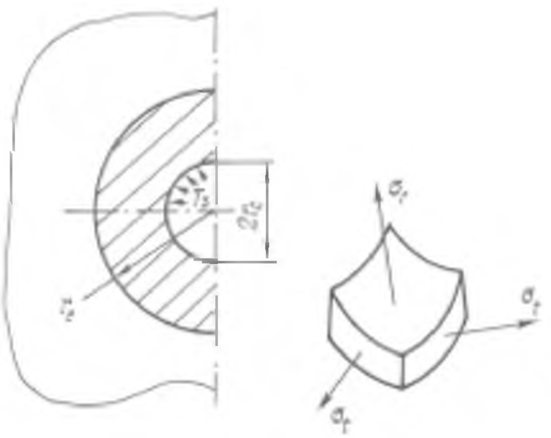

Fig. 1. Stress state near a spherical cavity at its expansion.

Neglecting the elastic volume compression in the plastic region, the expansion of a spherical cavity to a given radius r_c is accompanied with the expansion of an elastoplastic interface to the radius r_e. From the equality of the cavity volume $(4\pi/3)r_c^3$ in an incompressible (at plastic flow) material and the change of the volume inside the elastoplastic interface with the radius r_e under the pressure at this interface $p_e = (2/3)\sigma_Y$ (it causes the initiation of plastic flow at the tangential elastic strain ε_{te}), we get:

$$(r_c / r_e)^3 = (3/2)\varepsilon_Y. \qquad (4)$$

The pressure T_3 onto the surface of an expanding spherical cavity is obtained from Eqs. (2), (3), and (4) in the form:

$$T_3 = -\sigma_{rc} = (2/3)\sigma_Y\{1 - \ln[(3/2)\varepsilon_Y]\}. \qquad (5)$$

This value does not depend on the cavity radius owing to the constant ratio of the interface radius to the cavity radius.

If at a 1D stress state the conventional yield stress corresponds to the longitudinal plastic strain $\varepsilon_Y = 0.2\%$ (usually used for the evaluation of yield stress), from Eq. (5) we get the pressure onto the cavity surface at its expansion:

$$T_3 = (2/3)\sigma_Y[1 - \ln 0.003] = 4.55\sigma_Y.$$

The specific work (per unit volume of a cavity) spent for the formation of a cavity with the volume W is constant during its expansion and coincides in magnitude with the pressure T_3 necessary for the cavity expansion:

$$a_v = dA_3 / dW = 4\pi R^2 dR T_3 / 4\pi R^2 dR = T_3. \qquad (6)$$

1.2. **Elastoplastic Material.**

1.2.1. *Evaluation of the Radius of a Spherical Elastoplastic Interface*. The radial pressure at the elastoplastic interface $(r = r_e)$ under static loading of the surface of an expanding spherical cavity is calculated from the solution of an elastic problem, taking into account the compressibility of a material in the plastic region. The elastic compressibility in the plastic region influences the relative radius of the elastoplastic interface $\alpha = r_e / r_c$. The simplified evaluation of this effect is based on the equality of the mass within the spherical elastoplastic boundary with the radius r_e at the final stage (after the formation of a cavity with the radius r_c) and the same mass (before the cavity formation) bounded by the spherical surface of the radius

$$r_{e0} = r_e(1 - \varepsilon_{tY})$$

($\varepsilon_{tY} = u / r_e$ is the tangential strain induced by the radial displacement u of the material near the elastoplastic interface).

The spherical symmetry of flow results in the elastoplastic transition at the stresses $\sigma_r = -(2/3)\sigma_Y$; $\sigma_t = (1/3)\sigma_Y$. Thus,

$$\varepsilon_{tY} = [\sigma_t - \nu(\sigma_r + \sigma_t)]/E = (1+\nu)\varepsilon_Y.$$

To determine the distribution of the pressure p along the radius in the plastic region ($r_c < r < r_e$), we use the distribution of radial stresses derived for an incompressible plastic medium:

$$\sigma_r = -(2/3)\sigma_Y[1 - \ln(r/r_e)^3]; \ \sigma_t = \sigma_r - \sigma_Y.$$

The pressure $p = -\sigma_r - (2/3)\sigma_Y = -2\sigma_Y \ln(r/r_e)$ determines the distribution of the elastic strain ε of volume compression and the density ρ of a material:

$$\varepsilon = p/K = 2(\sigma_Y/K)\ln(r/r_e),$$

$$\rho = \rho_0(1+\varepsilon) = \rho_0[1 - 2(\sigma_Y/K)\ln(r/r_e)].$$

The equation of mass

$$(4/3)\rho_0 \pi r_e^3 (1 - \varepsilon_{tY})^3 = \int_{r_c}^{r_e} 4\pi\rho r^2 p dr$$

after the Taylor expansion of its left part, the substitution of $\varepsilon_{tY} = (1+\nu)\varepsilon_Y$ and of the equation for density, takes on the form:

$$(4/3)\rho_0 \pi r_e^3 [1 - (1+\nu)\varepsilon_Y] = 4\rho_0 \pi \int_{r_c}^{r_e} [1 - 2(\sigma_Y/K)\ln(r/r_e)] r^2 dr =$$

$$= (4/3)\rho_0 \pi \{(r_e^3 - r_c^3)[1 + (2/3)(\sigma_Y/K)] - 2(\sigma_Y/K) r_c^3 \ln(r_e/r_c)\}.$$

From this equation, the relation

$$(1/\alpha^3)[1 + 2(1-2\nu)\varepsilon_Y(1 + \ln\alpha^3)] = 3\varepsilon_Y(1-\nu)$$

is derived, which allows for the nonlinear dependence of the relative radius of an elastoplastic interface on the strain ε_Y at yield stress and for the influence of compressibility (displayed as the effect of the strain ε_Y and the Poisson ratio ν).

At a real value of $\alpha > 5$ (valid for major metals), the latter relation with an error not exceeding 2% can be approximated by

$$1/\alpha^3 = 3\varepsilon_Y(1-\nu); \ T_3 = \sigma_{rc} = (2/3)\sigma_Y\{1 - \ln[3(1-\nu)\varepsilon_Y]\},$$

and if $\nu = 0.5$, it corresponds to the calculations for a rigid plastic medium without the account of the volume compressibility. This result coincides with the equation derived for the final stage of the spherical cavity expansion [8].

An increase in the strength of a material (increase in the strain ε_Y) as well as the account of compressibility ($\nu < 0.5$) leads to a smaller calculated elastoplastic interface (decrease in the relative radius α) at the formation of a spherical cavity and a decrease in the pressure T_3 at an expanding cavity surface. A decrease in α and T_3 in comparison with α' and T_3' for a rigid-plastic medium is presented below:

$$\alpha' / \alpha = [2(1-\nu)]^{1/3}, \quad T_3' / T_3 = \{1 - \ln[(3/2)\varepsilon_Y]\} / \{1 - \ln[3(1-\nu)\varepsilon_Y]\}.$$

For a steel with the yield stress $\sigma_Y = 1.2$ GPa, Young's modulus $E = 210$ GPa ($\varepsilon_Y = 0.0057$), and the Poisson ratio $\nu = 0.3$, these values are

$$\alpha' / \alpha = 1.12; \quad T_3' / T_3 = 1.06.$$

1.2.2. *Strain Distribution near the Spherical Cavity in an Ideally Plastic Medium*. The radial displacement of a material in the expansion of a spherical cavity is accompanied with intense plastic flow. The logarithmic plastic strain of the material in the tangential direction as the result of its radial displacement u from the initial position with the radius r_0 is determined by the summation of the true strain increments $d\varepsilon_t = du / (r_0 + u)$.

The tangential strain, allowing for the condition of incompressibility ($r_0^3 = (r_0 + u)^3 - r_c^3 = r^3 - r_c^3$), is calculated from the equation:

$$\varepsilon_t = \int du / (r_0 + u) = \ln[(1 + u / r_0) = -(1/3) \ln(1 - r_c^3 / r^3).$$

The plastic strain intensity

$$\varepsilon_i = 2\varepsilon_t = -(2/3)\ln(1 - r_c^3 / r^3),$$

increases from a very small plastic strain value at the elastoplastic interface to an unlimited one near the surface of a cavity. A real metal exhibits strain hardening and structural changes, and an increase in the expansion rate causes the adiabatic heating of the material essential at large strains. These effects can considerably change the strength of the material and its resistance to penetration. However, their evaluation requires additional studies. Thus, the simplified elastoplastic model, if applied to large strains near the cavity surface, should be used with certain caution.

The elastic volume compressibility of a material in the plastic region reduces the extent of plastic strain corresponding to the volume changes. This increases a strain gradient in the plastic region.

1.2.3. *Surface Pressure for an Expanding Cavity in the Plastic Medium with Strain Hardening*. The effect of strain hardening is examined in many papers, however, analytical solutions are limited in number. The analysis of linear strain hardening [8] is not applicable to the description of flow at large strains. Really, an increase in strain should induce the asymptotic growth of resistance to the utmost value σ_*, which cannot exceed the theoretical strength of a material, neglecting softening (as the result of structural changes, damage, and heating). The stress-strain relation (for the logarithmic plastic strain $\varepsilon_i = 2\varepsilon_t = \ln(r/r_0)^2$ in the form

$$\sigma_i = \sigma_* - (\sigma_* - \sigma_Y)\exp(-A\varepsilon_i)$$

corresponds to $\sigma_i = \sigma_Y$ at small plastic strains $\varepsilon_i = 0$ (at the initial stage of strain hardening) and to the utmost stress $\sigma_i \approx \sigma_*$ at large strains. The relation for the strain hardening module

$$M = d\sigma_i / d\varepsilon_i = M_0 \exp(-A\varepsilon_i), \quad M_0 = A(\sigma_* - \sigma_Y)$$

permits to represent the stress-strain relation for spherical symmetry in the form:

$$\sigma_i = \sigma_* - (M_0/A)\exp(-A\varepsilon_i) = \sigma_* - (M_0/A)\exp[A\ln(r_0/r)^2] =$$

$$= \sigma_* - (M_0/A)(r_0/r)^{2A}.$$

For this relation of strain hardening, the differential equation of equilibrium has the form:

$$d\sigma_r / dr = 2\sigma_i / r = 2\{\sigma_*/r - (M_0/A)[1-(r_c/r)^3]^{2A/3}/r\}$$

(if the equations of incompressibility $r^3 - r_c^3 = r_0^3$ or $(r_0/r)^3 = 1-(r_c/r)^3$ are used). The radial stress in a spherical layer is determined by integration of the above equation.

For $A = 3/2$ (example used only for the evaluation of a strengthening effect), from the simplified differential equation

$$d\sigma_r / dr = 2[\sigma_Y + (\sigma_* - \sigma_Y)(r_c/r)^3]/r,$$

we get the stress distribution:

$$\sigma_r = 2[\sigma_Y \ln r - (\sigma_* - \sigma_Y)(r_c/r)^3/3] + C.$$

At the elastoplastic interface,

$$\sigma_r = -(2/3)\sigma_Y = 2[\sigma_Y \ln r_e + (\sigma_* - \sigma_Y)(r_c/r_e)^3/3] + C,$$

hence

$$\sigma_r = -(2/3)\{\sigma_Y[1 - \ln(r/r_e)^3] + (\sigma_* - \sigma_Y)[(r_c/r)^3 - (r_c/r_e)^3]\}.$$

The maximum radial stress at the cavity surface, corresponding to $(r_c/r) = 1$,

$$\sigma_{r\max} = -(2/3)\{\sigma_Y[1 - \ln(r_c/r_e)^3] + (\sigma_* - \sigma_Y)[1 - (r_c/r_e)^3]\}$$

is essentially higher than for an ideal plastic medium, and the behavior of a material with intense strain hardening is mainly dependent on the utmost stress σ_*.

2. **Pressure onto the Surface of an Expanding Cylindrical Cavity**.

2.1. **Rigid-Plastic Material**. For the elementary volume of the cylindrical layer of a material in the inelastic region (at $r_c < r < r_e$), using the equation of static equilibrium and the condition of plastic flow for the case of cylindrical symmetry $(\sigma_r - \sigma_t) = 2\tau_Y$, we get

$$d\sigma_r / dr = -(\sigma_r - \sigma_t)/r = -2\tau_Y / r \tag{7}$$

similar to Eq. (1). Integrating this equation (with the account that $\sigma_{re} = -\tau_Y$ at the cylindrical elastoplastic interface, $r = r_e$) for the case of the infinite external boundary of the elastic region, we obtain the distribution of radial stresses in the form

$$\sigma_r = -\tau_Y[1 - \ln(r/r_e)^2]. \tag{8}$$

The stress and plastic strain components in cylindrical coordinates (Fig. 2) for a material incompressible at plastic flow ($\nu = 0.5$)

$$\sigma_1 = \sigma_r;\ \sigma_2 = (\sigma_r + \sigma_3)/2;\ \sigma_3 = \sigma_t = \sigma_r - 2\tau_Y;\ \tau_{12} = \tau_{23} = \tau_{31} = 0;$$

$$\varepsilon_1 = \varepsilon_r;\ \varepsilon_2 = 0;\ \varepsilon_3 = \varepsilon_t = -\varepsilon_r;\ \varepsilon_{12} = \varepsilon_{23} = \varepsilon_{31} = 0$$

correspond to the stress and plastic strain intensities in the plastic region:

$$\begin{aligned} \sigma_i^2 &= [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + \\ &\quad + 3(\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2)]/2 = 3\tau_Y^2; \\ \varepsilon_i^2 &= (2/9)[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2 + \\ &\quad + 3(\varepsilon_{12}^2 + \varepsilon_{23}^2 + \varepsilon_{31}^2)] = (4/3)\varepsilon_r^2. \end{aligned} \tag{9}$$

Assuming that the stress intensity σ_i at plastic flow in a 1D stress state $(\sigma_i = \sigma_Y)$ and at cylindrical symmetry are equal, we obtain from Eq. (9): $\tau_Y = \sigma_Y / \sqrt{3}$.

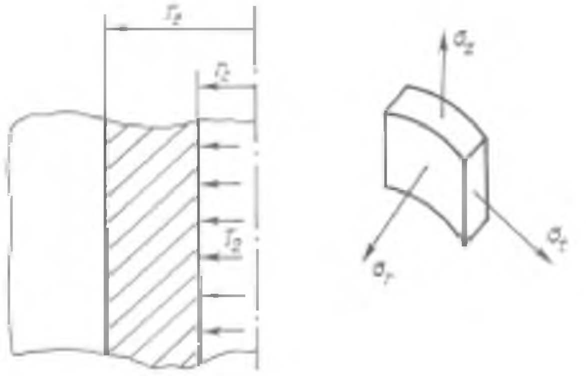

Fig. 2. Stress state near a cylindrical cavity at its expansion.

The radius of the elastoplastic interface r_e is calculated from the equality of the cylindrical cavity volume with the radius r_c and the change of the cavity volume in an elastic material with the radius r_e under the surface pressure $p_e = \tau_Y$, which initiates plastic flow:

$$(r_c / r_e)^2 = 2\varepsilon_{te} = \sqrt{3}\, \varepsilon_Y. \tag{10}$$

The pressure onto the surface of a cylindrical cavity, formed in the plane layer of a material by its radial displacement, is derived using Eqs. (8), (9), and (10):

$$T_2 = -\sigma_{rc} = (\sigma_Y / 2)[1 - \ln(\sqrt{3}\, \varepsilon_Y)]. \tag{11}$$

At $\varepsilon_Y = 0.002$, this pressure

$$T_2 = (\sigma_Y / (\sqrt{3})[1 - \ln 0.0035] = 3.85 \sigma_Y$$

does not depend on the cavity radius (due to the constant ratio between the radii of the elastoplastic interface and the cavity). The specific work spent for an increase in the cavity volume

$$a_2 = dA_2 / dW = 2\pi R dR T_2 / 2R dR = T_2 \tag{12}$$

is constant for the expansion of a cavity and is 15% less than the value for the expansion of a spherical cavity (given in the previous section).

The use of the Tresca criterion of plastic flow results in a lower value of pressure necessary for the expansion of the cavity. For this criterion, the maximum shear stress $\tau_Y = \sigma_Y / 2$ corresponds to the maximum strain in shear $\gamma_Y = (3/2)\varepsilon_Y$, and instead of Eq. (11), we should take

$$T_2 = -\sigma_{rc} = (\sigma_Y / 2)\{1 - \ln[(3/2)\varepsilon_Y]\}. \tag{13}$$

At $\varepsilon_Y = 0.002$, the calculated pressure for the cavity expansion is approximately 25% less than T_3 in Eq. (5) for a spherical cavity:

$$T_2 = (\sigma_Y / 2)[1 - \ln 0.003] = 3.4\sigma_Y.$$

2.2. **Elastoplastic Material**.

2.2.1. *Evaluation of the Radius of a Cylindrical Elastoplastic Interface*. The radial pressure at the elastoplastic interface $(r = r_e)$ under the static loading of the surface of a cylindrical cavity is determined from a known solution of the elastic problem for an unlimited medium. The elastic volume compressibility of a material in the plastic region affects the relative radius of the elastoplastic interface $\alpha = r_e / r_c$ at the formation of a cavity induced by the radial displacement of the material. The approximate evaluation of this effect proceeds from the equation of mass equality within the cylindrical elastoplastic interface with the final radius r_e and that of the initial radius r_{e0}:

$$r_{e0} = r_e(1 - \varepsilon_{tY}) = r_e[1 - \sqrt{(2/3)}\,\varepsilon_Y]$$

($\varepsilon_{tY} = u / r_e$ is the tangential strain as the result of the radial displacement u of a material at the elastoplastic interface).

According to the Mises criterion, for the cylindrical symmetry of plastic flow with the stresses at the elastoplastic interface $\sigma_r = -\tau_Y$; $\sigma_t = \tau_Y$; $\sigma_z = 0$ and the tangential strain $\varepsilon_{tY} = (\sigma_t - \nu\sigma_r) / E = \tau_Y (1 + \nu) / E$, we get:

$$\tau_Y = \sigma_Y / \sqrt{3} = E\varepsilon_Y / \sqrt{3}; \;\; \varepsilon_{tY} = (\varepsilon_Y / \sqrt{3})(1 + \nu).$$

The distribution of the pressure p along the radius in the cylindrical plastic region (accounting for $\sigma_r = -\tau_Y[1 - 2\ln(r / r_e)]; \sigma_t = \sigma_r + 2\tau_Y$; $\sigma_z = \sigma_r + \tau_Y$)

$$p = -2\tau_Y \ln(r / r_e)$$

determines the distribution of volume compression strains and the density of a material in the plastic region:

$$\varepsilon = p / K = -(2\tau_Y / K)\ln(r / r_e),$$

$$\rho = \rho_0(1 + \varepsilon) = \rho_0[1 - 2(\tau_Y / K)\ln(r / r_e)].$$

The equation of conservation of mass for the layer of unit thickness

$$\rho_0 \pi r_e^2 (1 - \varepsilon_{tY})^2 = \int_{r_c}^{r_e} \pi \rho r dr$$

after the Taylor expansion of its left part and the substitution of relations for tangential strain and density, takes on the form:

$$\rho_0 \pi r_e^2 [1 - 2\varepsilon_{tY}] = 2\rho_0 \pi \int_{r_c}^{r_e} [1 - 2(\tau_Y / K)\ln(r / r_e)] r dr =$$

$$= \rho_0 \pi \{(r_e^2 - r_c^2)[1 + (\tau_Y / K)] + 2(\tau_Y / K) r_c^2 \ln(r_c / r_e)\}.$$

From the above equation, we have

$$1/\alpha^2 = (\varepsilon_Y / \sqrt{3})(5 - 4\nu) / \{1 + (\varepsilon_Y / \sqrt{3}) 3(1 - 2\nu)[1 + \ln \alpha^2]\}.$$

At $\alpha > 5$ (valid for major metals), the latter equation is equal to

$$1/\alpha^2 = (\varepsilon_Y / \sqrt{3})(5 - 4\nu),$$

with sufficient accuracy, and it coincides with Eq. (10) for a rigid-plastic medium, if $\nu = 0.5$. An increase in the strength of a material (increase in the strain ε_Y) as well as the account of the volume compressibility leads to a decease in the radius of an interface and in the pressure onto the cavity surface necessary for its expansion. A decrease in α and T_2 for an elastoplastic medium in comparison with α' and T_2' for a rigid-plastic one can be calculated from the relations:

$$\alpha' / \alpha = [(5 - 4\nu)/3]^{1/2}; \quad T_2' / T_2 = [1 - \ln(\sqrt{3}\,\varepsilon_Y)] / \{1 - \ln[\varepsilon_Y (5 - 4\nu)/\sqrt{3}]\}.$$

For a steel with $\sigma_Y = 1.2$ GPa, $E = 210$ GPa ($\varepsilon_Y = 0.0057$), and $\nu = 0.3$, these values are

$$\alpha' / \alpha = 1.12; \quad T_2' / T_2 = 1.04.$$

Thus, the effect of compressibility on the radius of an elastoplastic interface is essential, which does not contradict earlier findings [13]. The results of this analysis are close to the approximate solution for the dynamic expansion of a cylindrical cavity, based on the similarity solution for plastic flow [11].

2.2.2. *Strain Distribution near the Surface of an Expanding Cylindrical Cavity.* The radial displacement of a material at the formation of a cavity leads to intense plastic flow. The logarithmic strain in the tangential direction, caused by the radial displacement u of a material from the initial radius r_0, is determined by summation of the true strain increments $d\varepsilon_t = du / (r_0 + u)$. The tangential component of plastic strain (for zero compressibility, i.e., for $r_0^2 = (r_0 + u)^2 - - r_0^2 = r^2 - r_0^2$) can be calculated by the equation:

$$\varepsilon_t = \int du / (r_0 + u) = \ln[(r_0 + u)/r_0] = -(1/2)\ln(1 - r_0^2 / r^2).$$

The strain intensity $\varepsilon_i = \varepsilon_t \sqrt{4/3} = \ln[(r_0 + u)/r_0] = -(1/\sqrt{3})\ln(1 - r_0^2/r^2)$ is low near the elastoplastic interface. The calculated strains increase to an unlimited value near the cavity surface. Intense strains near the cavity surface give rise to strain hardening and an increase in temperature with an increase in a penetration velocity.

The effects of strain hardening and compressibility are similar to such effects for the expansion of a spherical cavity.

3. **Specific Work Spent for the Cavity Formation at Rod Penetration**. The static penetration of a rigid rod in a plastic medium results in a cylindrical cavity with a diameter approximately equal to the diameter of the rod with the cross section S. An increase in the penetration depth (at steady-state penetration) in an infinite plastic medium, accompanied with the radial displacement of a material (neglecting the friction at the contact surface of the bodies), is proportional to the strength of the material at plastic flow and is independent of the shape of the rod head.

An increase in the work $\delta A = SP_a \delta L$ spent for the deformation of the medium to increase the penetration depth by δL under the action of the mean pressure P_a onto the contact surface should be equal to the total work spent for the formation of a residual cylindrical cavity as the result of axisymmetric plastic flow in the plane layer of the thickness δL (Fig. 3). The total specific work of the cavity formation in such a plane layer includes the terms T_{02} and P_{02} associated with the radial and axial displacements of a material in the plastic region.

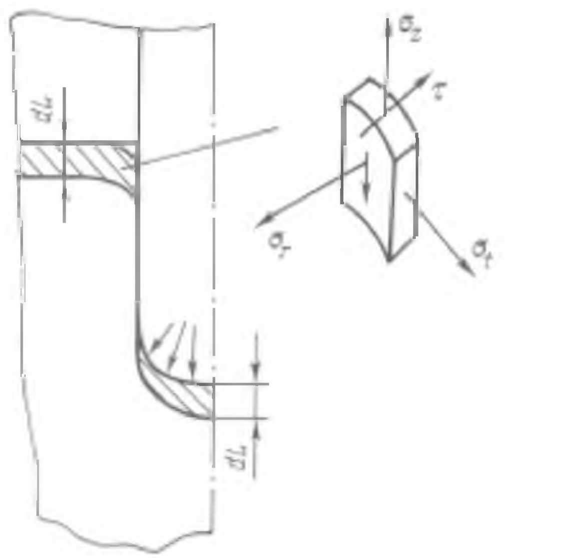

Fig. 3. Scheme of stresses in the material at long-rod penetration.

The specific works T_3 spent for the formation of a spherical cavity (caused by the radial displacement of a material) and P_a spent for the formation of a cylindrical cavity at the rod penetration (as the result of the axisymmetric radial and axial displacements of a material, taking $P_a = T_{02} + P_{02}$) can be assumed equal. From Eq. (5), it follows:

$$T_3 = P_a = T_{02} + P_{02} = (2/3)\sigma_Y\{1 - \ln[(3/2)\varepsilon_Y]\}. \qquad (14)$$

Hence, the specific work of shear deformation at the cavity formation, as it follows from (13) and (14), is equal to

$$P_2 = (1/6)\sigma_Y\{1 - \ln[(3/2)\varepsilon_Y]\} = (1/4)P_a, \qquad (15)$$

which is 25% of the total specific work.

4. **Stress-Strain Kinetics for the Material near the Contact Surface at Rod Penetration**. The change of the stress-strain state in a volume within the plastic region is caused by the axial and radial displacements of a material as the result of its flow at the penetration of a rod. Such displacements are associated with the stress-strain state if the cylindrical coordinates are used. The radial and axial displacements and the tensor components of a plastic strain rate for the material in the elementary ring depend on its initial distance from the axis of the rod.

The pressure onto the rod head is determined by the stress-strain state of the material along the zero line of flow adjacent to the contact surface. Along the section of this line (on the axis of symmetry), from the elastoplastic interface to the point of branching, the plastic deformation of a material is accompanied with an increase in the axial compression strain ε_z and the tangential tensile strains $\varepsilon_r = \varepsilon_t$. The stress-strain state of the material is similar to that of the expanding spherical cavity.

The stress and plastic strain intensities from Eq. (3)

$$\sigma_i = \sigma_Y = 2\tau_Y; \;\; \varepsilon_i = \varepsilon_Y = 2\varepsilon_t$$

determine the specific work (per unit volume of a material) of plastic deformation:

$$a_v = \sigma_i \varepsilon_i = \sigma_{ij}\varepsilon_{ij}. \qquad (16)$$

With further motion along the zero line of flow away from the point of branching over the contact surface of interacting bodies (or over the contact surface with a stagnant zone), the axial and radial displacements of a material result in the development of 3D strains $\varepsilon_r, \varepsilon_t, \varepsilon_{rt}$ (strain components ε_r and ε_t are not equal, the plastic strain in shear ε_{rt} is nonzero). It should be noted that the expansion of a cavity in the radial direction and plastic shear in the axial direction activate the two groups of main shear planes (*rt* and *rz*) with the critical values of shear stresses which are assumed to be equal:

$$\tau_{rt} = (\sigma_r - \sigma_t)/2 = \tau_Y; \;\; \tau_{rz} = \tau_Y.$$

The stress state in this case is characterized by the tensor components:

$$\sigma_r; \;\; \sigma_t = \sigma_r - 2\tau_Y; \;\; \sigma_z = 0; \;\; \tau_{rz} = \tau_Y,$$

which correspond to the stress intensity

$$\sigma_i^2 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 +$$

$$+ 6(\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2)]/2 = \sigma_Y^2 = 6\tau_Y^2$$

and the shear stress $\tau_Y^2 = \sigma_i^2 / 6$.

The plastic strain tensor components in this case

$$\varepsilon_r;\ \varepsilon_t = -\varepsilon_r;\ \varepsilon_z = 0;\ \varepsilon_{rz};\ \varepsilon_{rt} = \varepsilon_{zt} = 0$$

determine the strain intensity (assuming that $\varepsilon_{rz}^2 = \varepsilon_r^2$)

$$\varepsilon_i^2 = (2/9)[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2 +$$

$$+ 6(\varepsilon_{12}^2 + \varepsilon\tau_{23}^2 + \varepsilon_{31}^2)] = (8/3)\varepsilon_r^2$$

and the shear strain $\varepsilon_{rz}^2 = (3/8)\varepsilon_i^2$.

The specific work of plastic deformation in axial shear

$$a_{sh} = \tau_{rz}\varepsilon_{rz} = \sigma_i \varepsilon_i / 4, \qquad (17)$$

which is 25% of the total specific work of plastic deformation, in accordance with the estimation by Eq. (16).

The condition of plastic flow (neglecting strain hardening) is valid for all the points on the the zero line of flow. This line includes the points on the axis, from the elastoplastic boundary to the point of branching (with the axial compression of a material), and the points on the contact surface of interacting bodies (with the radial and axial displacements of a material). Hence, the resistance of a plastic medium to the penetration of a rod can be evaluated by the specific work spent for the formation of a spherical cavity or the work of axial force spent for the formation of a cylindrical cavity. The evaluation of this characteristic by T_3 and P_a (calculated by Eqs. (14) and (15)) is approximate because of the possible pronounced effect of strain hardening and the friction on the contact surface. The account for an error caused by these effects and the real distribution of pressure would increase the reliability of calculations for determining the penetration depth in the materials with high strain hardening.

Further investigations of effects, associated with the possible formation of a stagnant zone at the penetration of rods with a blunt head and studies on the specific features of plastic flow near the point of branching on the contact surface are of importance for the reliable prediction of a penetration depth. Some of these effects were analyzed using the theory of kernel formation [17].

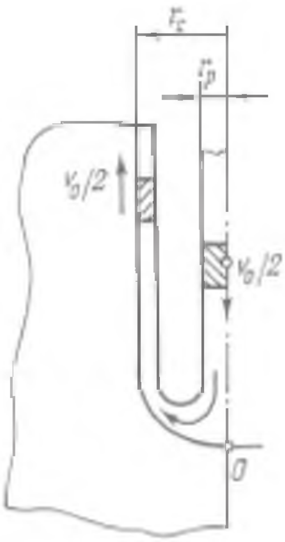

Fig. 4. Scheme of plastic flow at long-rod penetration.

5. **Deformation of a Cylindrical Rod at Penetration**. The plastic flow in a long rod at high-velocity penetration leads to its transformation into a tubular element adjacent to the surface of a cylindrical cavity with the radius r_c in a plastic medium (Fig. 4). The expansion of the tubular element approximately simulates the deformation of an eroded rod induced by its plastic flow near the contact surface. The radial stress σ_r in the wall of this element at its expansion (under pressure onto the surface of the cylindrical cavity with the radius r_i) corresponds to the solution of Eq. (7). Assuming that the pressure at the external surface of the tubular element with the radius r_{cp} equals zero ($\sigma_r = 0$ at $r = r_{cp}$), the distribution of radial stresses is represented by the relation:

$$\sigma_r = 2\tau_Y \ln(r_{cp} / r). \qquad (18)$$

From the condition of the incompressibility of a rod material at plastic flow, the outer and inner tubular radii (r_{cp} and r_i) are related to the initial radius r_p of the rod by

$$r_{cp}^2 - r_i^2 = r_p^2 \quad \text{or} \quad r_i^2 / r_{cp}^2 = 1 - r_p^2 / r_{cp}^2.$$

The pressure T_i at the inner surface of the tubular element, necessary for its radial expansion according to Eq. (18), is calculated as:

$$T_i = (\sigma_Y / 2) \ln(r_{cp} / r_i)^2 = -(\sigma_Y / 2) \ln[1 - (r_p / r_{cp})^2].$$

The work of expansion of the tubular element per its unit length, which is assumed to be equal to the work of plastic deformation of the rod, is calculated by the equation:

$$A_p = \int_0^{r_i} 2\pi r_i T_i dr_i = \pi(\sigma_Y/2) r_p^2 [(r_c/r_p)^2 - 1] \ln [(r_c/r_p)^2 - 1].$$

A relative decrease in the kinetic energy of the rod (for the initial velocity of the rod V and the densities of the materials of the rod and plastic medium ρ) at its transformation into the tubular element (assuming $(r_c/r_p)^2 = 4$) is rather considerable:

$$A_p/(\pi r_p^2 \rho V^2/2) = (\sigma_Y/2)\{[(r_c/r_p)^2 - 1] \ln [(r_c/r_p)^2 - 1]\}/(\rho V^2/2) =$$

$$= 3.4\sigma_Y/(\rho V^2/2). \qquad (19)$$

This reduction of the kinetic energy of a steel rod ($\sigma_Y =$ 1.2 GPa, $\rho =$ $= 7800$ kg/m^3), caused by its plastic deformation at the penetration into the same steel with the initial velocity $V = 2000$ m/s, exceeds 25% as calculated by Eq. (19). Hence, the work of deformation of the rod entering in the equation of energy balance should be accounted for at such velocities.

CONCLUSIONS

The static resistance of a plastic material to the penetration of a rigid rod can be evaluated by the pressure required for the expansion of a spherical cavity as well as by the mean pressure onto the rod head at penetration. This value does not depend on the geometry of the head.

The specific work spent for the cavity formation (per its unit volume) is the same for a spherical or cylindrical cavity formed by the penetration of a rigid or eroded rod; in the latter case, an increment in the cavity depth causes the axisymmetric radial and axial displacements of a medium and the deformation (erosion) of the rod.

For the plastic medium with intense strain hardening, the pressure required for the expansion of the spherical cavity or the resistance of the medium to the rod penetration is controlled by the ultimate stress at large strains.

At the low-rate expansion of a spherical cavity and the penetration of a rod in an elastoplastic medium, the elastic volume compression affects the relative radius of the elastoplastic interface and the surface pressure for the cavity expansion (or the mean pressure at the contact surface for the rod penetration).

The evaluation of the resistance of a plastic medium to the penetration of a rod at low velocities, neglecting the effects of strain hardening, friction on the contact surface, and the specific flow near the point of branching, can be taken only as the first approximation. Therefore, the detailed studies on these effects would be of paramount importance for further progress in this field.

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Р е з ю м е

Розглянуто вплив статичної міцності (границі текучості) матеріалу на основні параметри процесу проникання – питома робота утворення каверни й тиск на поверхню контакту. Отримано аналітичні вирази й кількісні оцінки питомої роботи, що затрачена на утворення сферичної й циліндричної каверн, і тиску на поверхні таких каверн при статичному розширенні останніх для жорсткопластичної та пружнопластичної моделей матеріалу. Показана адекватність оцінок питомої роботи деформування пластини при утворенні сферичної каверни та при утворенні каверни в результаті проникання жорсткого стержня. Оцінено вплив деформаційного зміцнення матеріалу на максимальну радіальну напругу на поверхні каверни. Встановлено, що при низьких швидкостях проникання суттєвий вплив на опір прониканню й розширення каверни має пружна стисливість матеріалу в області непружних деформацій. Проаналізовано кінетику напружено-деформованого стану матеріалу при проникання деформівного стержня.

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