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RISK PROCESS WITH STOCHASTIC PREMIUMS

The Cramér-Lundberg model with stochastic premiums which is natural generalization of classical dynamic risk model is considered. Using martingale technique the Lundberg inequality for ruin probability is proved and characteristic equations for Lundberg coefficients are presented for certain classes of stochastic premiums and claims. The simple diffusion and de Vylder approximations for the ruin probability are introduced and investigated similarly to classical Cramér-Lundberg set-up. The weak and strong invariance principles for risk processes with stochastic premiums are discussed. Certain variants of the strong invariance principle for risk process are proved under various assumptions on claim size distributions. Obtained results are used for investigation the rate of growth of the risk process and its increments. Various modifications of the LIL and Erdös-Renyi-type SSLN are proved both for the cases of small and large claims.

0. Introduction. Definition of the model

Suppose that the following objects are defined on the same probability space (Ω, F, P):

– two independent Poisson processes $N(t)$ and $N_1(t)$ with intensities $\lambda > 0$ and $\lambda_1 > 0$ ($EN(t) = \lambda t$, $N(0) = 0$, $EN_1(t) = \lambda_1 t$, $N_1(0) = 0$);

– two sequences of i.i.d.r.v. $(x_i : i \geq 1)$ and $(y_i : i \geq 1)$ independent of the Poisson processes and of each other with d.f. $F(x)$ and $G(x)$, respectively, $F(0) = 0$, $G(0) = 0$.

Within the Cramér-Lundberg risk model with stochastic premiums the risk process $U(t)$, $t \geq 0$, is defined as

$$U(t) = u + \sum_{i=1}^{N_1(t)} y_i - \sum_{i=1}^{N(t)} x_i, \qquad (1)$$

where: $u \geq 0$ is an initial capital; $N(t)$ - the number of claims in the time interval $[0, t]$; positive i.i.d.r.v. $(x_i : i \geq 1)$ are claim sizes; $N_1(t)$ is interpreted as a number of polices bought during $[0, t]$; $(y_i : i \geq 1)$ stand for sizes of premiums paid for corresponding polices.

So, in (1) not only total claim amount process $S(t) = \sum\limits_{i=1}^{N(t)} x_i$ is a compound Poisson process, but also the total premium amount process $\Pi(t) = \sum\limits_{i=1}^{N_1(t)} y_i$ is a process of such type unlike the classical Cramér-Lundberg risk model

$$U(t) = u + ct - \sum_{i=1}^{N(t)} x_i$$

with non-random linear continuous premium income function $\Pi^*(t) = ct$, $c > 0$.

Due to properties of compound Poisson process $Q(t) = \Pi(t) - S(t)$ is again a compound Poisson process with intensity $\lambda^* = \lambda + \lambda_1$ and d.f. of the jumps $G^*(x) = \frac{\lambda_1}{\lambda^*}G(x) + \frac{\lambda}{\lambda^*}F^*(x)$, where $F^*(x)$ is a d.f. of the random variable $-x_1$. In the other words $Q(t)$ admits the representation

$$Q(t) = \sum_{i=1}^{N^*(t)} \xi_i, \tag{2}$$

where $N^*(t)$ is homogeneous Poisson process with intensity $\lambda^* = \lambda + \lambda_1$ and i.i.d.r.v. ξ_i have d.f. $G^*(x)$.

Thus, moment generating function

$$E\{\exp(rQ(t))\} = \exp\left(\lambda^*\left(\frac{\lambda_1}{\lambda^*}M_y(r) + \frac{\lambda}{\lambda^*}M_x(-r) - 1\right)\right),$$

if moment generating function $M_x(r)$ for claim size x_1 and $M_y(r)$ – for premium y_1 exist.

Denote by

$$\tau = \inf(t > 0 : U(t) < 0)$$

the *ruin time*, by

$$\psi(u, T) = P(\inf_{0 < t \leq T} U(t) < 0) = P(\tau \leq T | U(0) = u)$$

the *probability of ruin in a finite time* (in time interval (0,T]) and by

$$\psi(u) = P(\inf_{t > 0} U(t) < 0) = \lim_{T \to \infty} \psi(u, T)$$

the *probability of ultimate ruin* (the ruin probability in infinite time).

We will also use notations $\varphi(u) = 1 - \psi(u)$ and $\varphi(u, T) = 1 - \psi(u, T)$ for non-ruin probabilities in infinite and finite time intervals, respectively.

It is obvious, that $EQ(t) = t(\lambda_1 E y_1 - \lambda E x_1)$. In forthcoming we shall suppose that $EQ(t) > 0$ for $t > 0$, i.e. that

$$\lambda_1 E y_1 > \lambda E x_1. \tag{3}$$

This condition is analog of *net profit condition* in classical Cramér-Lundberg model and, due to the SLLN, provides $\psi(u) < 1$.

Model (1) was studied by A. Boykov [6], L.Gilina [18], D. Gusak [20] and in authors previous work [2]. V.Korolev, V.Bening, S.Shorgin [23] present an interesting example of using (1) for modelling the speculative activity of money exchange point and optimization of its profit.

In [6] Boykov proved that non-ruin probability $\varphi(u)$ satisfy equation

$$(\lambda + \lambda_1)\varphi(u) =$$

$$\lambda \int_0^u \varphi(u - x)dF(x) + \lambda_1 \int_0^\infty \varphi(u + \nu)dG(\nu), \tag{4}$$

and find its solution if:

(a) $P(x_i = 1) = P(y_i = 1) = 1$, in this case

$$\varphi(u) = \varphi([u]) = 1 - \left(\frac{\lambda}{\lambda_1}\right)^{[u]+1}, \quad [u] - \text{ entire of } x;$$

(b) premiums and claims have exponential distributions, i.e. $G(x) = 1 - e^{-bt}$, $F(x) = 1 - e^{-at}$, $a, b > 0$. In this situation

$$\psi(u) = \frac{(a + b)\lambda}{(\lambda + \lambda_1)a} \exp\left(-\frac{\lambda_1 a - \lambda b}{\lambda + \lambda_1}u\right). \tag{5}$$

Note that in this case condition (3) is equivalent to $\lambda_1/b > \lambda/a$.

In [6] it also proved that non-ruin probability $\varphi(u, t)$ on finite interval satisfy equation

$$\frac{\partial \varphi(u, t)}{\partial t} + (\lambda + \lambda_1)\varphi(u, t) =$$

$$\lambda \int_0^u \varphi(u - x, t)dF(x) + \lambda_1 \int_0^\infty \varphi(u + \nu, t)dG(\nu), \tag{6}$$

In the case of exponential premiums and claims equation (6) can be reduced to

$$\int_0^\infty (1 - \varphi(u, t))e^{-ts}dt = A(s)e^{\alpha(s)u},$$

where

$$\alpha(s) = \frac{\lambda b - \lambda_1 a + s(b-a)}{2(\lambda + \lambda_1 + s)} - \frac{\sqrt{[\lambda b - \lambda_1 a + s(b-a)]^2 + 4ab(\lambda + \lambda_1 + s)s}}{2(\lambda + \lambda_1 + s)},$$

$$A(s) = \frac{\lambda}{s(s + \lambda + \lambda_1 - \lambda_1 b(b - \alpha(s))^{-1})}.$$

Since one can obtain the explicit solution of equations (5), (6) and derive the exact formulas for ruin/non-ruin probabilities only in a few exceptional cases, the problem of finding practically useful estimates or approximations for $\psi(u)$ becomes rather important.

We start with Lundberg's exponential inequality (Section 1) and prove it using standard martingale technique. Also we discussed two examples of finding adjustment (Lundberg) coefficient for exponential premiums and claims with gamma or mixture of exponential distributions.

In Section 2 we extend rather "simple" and "practically useful" de Vylder approach to approximation of ruin probabilities for the models with stochastic premiums.

Section 3 is devoted to diffusion approximation of ruin probabilities in finite/infinite intervals. Unlike ad hoc de Vylder approximation, the diffusion approximation has a solid theoretical base in functional limit theorems (weak invariance principle) for $Q(t)$ and risk process $U(t)$.

In Section 4 we deal with other type of limit theorems, so called *strong invariance principle* (SIP), which in certain sense is a bridge between weak and a.s. convergence. Using rather general results due to SIP for random sums, we proved SIP for risk processes with stochastic premiums. Special cases, i.e. claims with finite second moment and large claims attracted to α-stable law, are studied separately. In the cases of classical and Sparre Andersen risk models SIP can be used to find approximations of ruin probabilities on finite interval, see M.Csörgő, L. Horváth (1993). But in our work we shall use SIP for other purpose: investigation of the rate of growth of risk process and its increments.

Thus, in Section 5 we proved various modifications of the law of iterated logarithm (LIL) and Erdös-Renyi-Révész-Csörgő-type strong law of large numbers (SLLN) for risk processes. Cases of small claims, large claims with finite variance and large claims attracted to asymmetric stable law are discussed.

1. Martingale approach. Lundberg's inequality

Similar to the case of classical risk process, martingale approach leads to a simple, but useful exponential upper bound for ultimate ruin probability for model (1).

Consider the equation, which we traditionally will call *"characteristic"*

$$\lambda_1(Ee^{-Ry_1} - 1) + \lambda(Ee^{Rx_1} - 1) = 0 \qquad (7)$$

which is equivalent to

$$\lambda_1 M_y(-R) + \lambda M_x(R) = \lambda_1 + \lambda.$$

if corresponding moment generating functions for y_1 and x_1 exist in some neighborhood of zero.

Definition *Lundberg coefficient(Lundberg exponent, adjustment coefficient) for model (1) is a positive solution of equation (7), if such solution exists.*

It is easy to check that the process $\exp\{-R(\Pi(t) - S(t))\}$ is a martingale (relative to natural filtration), and ruin time τ is a stopping time, see [6]. Thus, standard considerations as in J. Grandell [19] give the possibility to prove following statement.

Lemma 1. *If $R > 0$ is a solution of (7), then*

$$\psi(u) = \frac{e^{-Ru}}{E\{e^{-RU(\tau)}|\tau < \infty\}}.$$

More useful for applications is following upper bound.

Corollary 1(Lundberg's inequality). Suppose that adjustment coefficient R exists, then
$$\psi(u) < e^{-Ru}.$$

In the case of exponentially distributed premiums and claim sizes,i.e. $G(x) = 1 - \exp(-bx)$, $F(x) = 1 - \exp(-ax)$, one can solve equation (7) and obtain the explicit formula for Lundberg coefficient

$$R = \frac{(\lambda_1 a - \lambda b)u}{\lambda + \lambda_1}.$$

We shall present another two examples when (7) has rather simple form.

Example 1. Assume that in model (1) claim sizes are distributed according to mixture of exponential distributions with d.f. $F(x) = p_1 F_1(x) + p_2 F_2(x)$,

$F_1(x) = 1 - e^{-a_1 x}$, $F_2(x) = 1 - e^{-a_2 x}$, $p_1 + p_2 = 1$, while premiums are exponentially distributed with $G(x) = 1 - e^{-bx}$. Then characteristic equation

$$\lambda_1 \left(\int_0^\infty e^{-Ry} dG(y) - 1 \right) + \lambda \left(\int_0^\infty e^{Rx} dF(x) - 1 \right) = 0$$

implies

$$\lambda_1 \left(\int_0^\infty be^{-(R+b)y} dy - 1 \right) +$$

$$+ \lambda \left(\int_0^\infty p_1 a_1 e^{-(a_1-R)x} dx + \int_0^\infty p_2 a_2 e^{-(a_2-R)x} dx - 1 \right) = 0 \qquad (8)$$

where $R < \min(a_1, a_2)$. Equation (8) is transformed to

$$\frac{\lambda_1 R}{R+b} - \lambda \left(\frac{p_1 a_2 + p_2 a_1 - R}{(a_1 - R)(a_2 - R)} \right) = 0$$

Thus, Lundberg coefficient R is the positive solution of the equation

$$R^2(\lambda_1 + \lambda) - R(\lambda_1 a_1 + \lambda_1 a_2 + \lambda b - \lambda p_2 a_1 - \lambda p_1 a_2) + \lambda_1 a_1 a_2 - \lambda b p_2 a_1 - \lambda b p_1 a_2 = 0,$$

where

$$p_1 + p_2 = 1, \quad a_1 > 0, \quad a_2 > 0, \quad \lambda > 0, \quad \lambda_1 > 0, \quad \frac{\lambda_1}{b} > \lambda \left(\frac{p_1}{a_1} + \frac{p_2}{a_2} \right).$$

Here the last inequality provides (3). Solution must satisfy additional condition $0 < R < \min(a_1, a_2)$.

Example 2. Now suppose that premiums have exponential distribution with parameter $b > 0$ and claims have $\Gamma(\alpha, \beta)$-distribution with density $\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$.

In this case Lundberg coefficient R is a solution of the system

$$\begin{cases} \lambda_1 + \lambda = \frac{\lambda_1 b}{b+R} + \frac{\lambda \beta^\alpha}{(\beta-R)^\alpha} \\ 0 < R < \beta \\ \frac{\lambda_1}{b} > \frac{\lambda \alpha}{\beta} \\ \lambda > 0, \lambda_1 > 0, b > 0, \alpha > 0, \beta > 0. \end{cases}$$

Martingale approach and Lundberg inequality are applicable in the case of risk process with stochastic premiums perturbed by a Wiener process

$$U_1(t) = u + \sum_{i=1}^{N_1(t)} y_i - \sum_{i=1}^{N(t)} x_i + \varsigma W(t),$$

where $W(t)$ is a standard Wiener process, $\varsigma > 0$.

For such model Lundberg coefficient r is the positive solution of the equation

$$E \exp\{-r(\Pi(t) - S(t) + \varsigma W(t))\} = 1$$

which leads to equation

$$\lambda_1(E e^{-ry_1} - 1) + \lambda(E e^{rx_1} - 1) + \varsigma^2 r^2/2 = 0 \qquad (9)$$

if corresponding moment generating functions for x_1 and y_1 exist in some neibourhood of zero.

Notice that the process $\exp\{-r(\Pi(t) - S(t) + \varsigma W(t))\}$ is a martingale (relative to natural filtration), and ruin time τ^* for perturbed risk model is a stopping time. Thus, we have variant of Lundberg inequality:

Corollary 2. Suppose that adjustment coefficient r exists, then ruin probability $\psi^*(u)$ for perturbed risk model satisfies

$$\psi^*(u) < e^{-ru}.$$

The proof is standard:

$$1 = E \exp\{-r(\Pi(t) - S(t) + \varsigma W(t))\} =$$

$$= E \exp\{-r(\Pi(t \wedge \tau^*) - S(t \wedge \tau^*) + \varsigma W(t \wedge \tau^*))\} \geq$$

$$\geq E \exp\{-r(\Pi(t \wedge \tau^*) - S(t \wedge \tau^*)) + \frac{(r\varsigma)^2}{2}(t \wedge \tau^*))\}\chi_{\{\tau^* \leq t\}} \geq$$

$$\geq \exp\{ru\}P(\tau^* \leq t) \Rightarrow$$

$$\Rightarrow P(\tau^* \leq t) = 1 - \varphi^*(u, t) \leq \exp\{-ru\}.$$

So,

$$\psi^*(u) \leq \exp\{-ru\}.$$

2. DE VILDER APPROXIMATION OF RUIN PROBABILITY

Since the problem of derivation the exact formulas for ruin probabilities for model (1) or even solution of characteristic equation (7) is more complicated than for classical risk model, the methods which give rather "simple" and practically applicable approximations for $\psi(u)$ or $\psi(u, T)$ become very

actual. One of approaches to solution such a problem, similar to classical case, is based on a simple idea to replace the origin risk process $U(t)$ with an approximating risk process $\tilde{U}(t)$ for which calculations of ruin probabilities, i.e. distributions of infimum/supremum are easier. Some approximations are chosen based on functional limit theorems (invariance principles for random sums and risk processes), other – are ad hog procedures based on considerations of simplicity for applications. De Vylder approximation belongs to the second group. Below we demonstrate its application to a model with stochastic premiums.

We propose to replace the original risk process

$$U(t) = u + Q(t) = u + \sum_{i=1}^{N_1(t)} x_i - \sum_{i=1}^{N(t)} y_i$$

with the risk process

$$\tilde{U}(t) = u + \tilde{Q}(t) = u + \sum_{i=1}^{\tilde{N}_1(t)} \tilde{x}_i - \sum_{i=1}^{\tilde{N}(t)} \tilde{y}_i,$$

where premiums and claims are exponentially distributed, i.e. $\{\tilde{x}_i\}$ and $\{\tilde{y}_i\}$ are independent sequences of i.i.d.r.v. exponentially distributed with parameters α and β, relatively; $\tilde{N}_1(t)$ and $\tilde{N}(t)$ - mutually independent Poisson processes with intensities $\tilde{\lambda}_1 > 0$ and $\tilde{\lambda} > 0$, respectively. Parameters $\alpha, \beta, \tilde{\lambda}_1$ and $\tilde{\lambda}$ are chosen in such a way that first four moments of $\tilde{Q}(t)$ and $Q(t)$ are equal. More precise we shall demand that

$$EQ(t) = E\tilde{Q}(t), \quad E(Q(t) - EQ(t))^k = E(\tilde{Q}(t) - \tilde{Q}(t))^k, \quad k = 2, 3, 4.$$

Denote by $m_k = Ey_1^k$, $\mu_k = Ex_1^k$, $k \geq 1$. Keeping in mind that

$$M_1(t) = EQ(t) = (\lambda_1 m_1 - \lambda \mu_1)t,$$

$$M_k(t) = E(Q(t) - EQ(t))^k = (\lambda_1 m_k - \lambda \mu_k)t, \quad k = 2, 3, 4.$$

we get following system for determining parameters $\alpha, \beta, \tilde{\lambda}_1$ and $\tilde{\lambda}$

$$\lambda_1 m_1 - \lambda \mu_1 = \frac{\tilde{\lambda}_1}{\alpha} - \frac{\tilde{\lambda}}{\beta}, \quad \lambda_1 m_2 + \lambda \mu_2 = \frac{2\tilde{\lambda}_1}{\alpha^2} + \frac{2\tilde{\lambda}}{\beta^2}$$

$$\lambda_1 m_3 - \lambda \mu_3 = \frac{6\tilde{\lambda}_1}{\alpha^3} - \frac{6\tilde{\lambda}}{\beta^3}, \quad \lambda_1 m_4 + \lambda \mu_4 = \frac{24\tilde{\lambda}_1}{\alpha^4} + \frac{24\tilde{\lambda}}{\beta^4}. \tag{10}$$

Finally, let us input obtained values of parameters in the formula (5), i.e. formula for ruin probability in the case of exponential claims and premiums. Thus, we obtain *de Vylderm approximation of the ultimate ruin probability*

$$\psi(u) \approx \psi_{DV}(u) = \frac{\alpha + \beta}{\alpha} \frac{\tilde{\lambda}_1}{\tilde{\lambda}_1 + \tilde{\lambda}} \exp\left\{ \frac{\tilde{\lambda}\beta - \tilde{\lambda}_1\alpha}{\tilde{\lambda}_1 + \tilde{\lambda}} u \right\}, \tag{11}$$

De Vylder approximation can be applied in the case when theoretical distributions of $\{x_i\}$ and $\{y_i\}$ are unknown, but there are experience data about premiums $(\hat{y}_1, ..., \hat{y}_{n_1})$ and claim sizes $(\hat{x}_1, ..., \hat{x}_{n_1})$ in time interval $[0, t]$. Then, using sample moments instead of theoretical ones, put

$$\hat{\lambda}_1 = \frac{n_1}{t}, \quad \hat{\lambda} = \frac{n}{t},$$

$$\hat{m}_k = \sum_{i=1}^{n} \hat{y}_i/n \quad \hat{\mu}_k = \sum_{i=1}^{n} \hat{x}_i/n$$

in the mentioned above system (10) and calculate corresponding parameters $\alpha, \beta, \tilde{\lambda}_1, \tilde{\lambda}$. Formula (11) again provides de Vylder approximation $\psi_{DV}(u)$.

3. DIFFUSION APPROXIMATION

On the first glance diffusion approximation is similar to de Vylder approach, it can be considered as if origin risk process $U(t)$ is replaced by a Wiener process with a drift in such a way that first two moments coincide. Then well known expressions for distribution of infimum of the Wiener process on the finite/infinite interval provide approximations for ruin probabilities.

Denote by $W_{\tilde{a},\tilde{\sigma}^2}(t)$ the Wiener process with the mean $\tilde{a}t$, variance - $\tilde{\sigma}^2 t$. Random process $W_{\tilde{a},\tilde{\sigma}^2}(t)$ is stochastically equivalent to $\tilde{a}t + \tilde{\sigma}W(t)$, where $W(t)$ is a standard Wiener process.

According to diffusion approximation risk process $U(t) = u + Q(t) = u + \Pi(T) - S(t)$ is replaced by process $u + W_{\tilde{a},\tilde{\sigma}^2}(t)$ such that $EQ(t) = \tilde{a}t$, $VarQ(t) = \tilde{\sigma}^2 t$. Thus we have

$$\tilde{a} = \lambda_1 m_1 - \lambda\mu_1, \quad \tilde{\sigma}^2 = \lambda_1 m_2 + \lambda\mu_2. \tag{12}$$

Therefore *diffusion approximation for ultimate ruin probability* for model (1) is given by

$$\psi(u) \approx \psi_D(u) = P\{\inf_{t>0} W_{\tilde{a},\tilde{\sigma}^2}(t) < -u\} = \exp(-2u\tilde{a}/\tilde{\sigma}^2) \tag{13}$$

and *diffusion approximation for ruin probability on finite interval* is determined by expression

$$\psi(u, T) \approx \psi_D(u, T) = P\{\inf_{0<t\leq T} W_{\tilde{a},\tilde{\sigma}^2}(t) < -u\} =$$

$$= 1 - \Phi(\frac{\tilde{a}T + u}{\tilde{\sigma}\sqrt{T}}) + \exp(-2u\tilde{a}/\tilde{\sigma}^2)\Phi(\frac{\tilde{a}T - u}{\tilde{\sigma}\sqrt{T}}), \tag{14}$$

where $\Phi(.)$ is d.f. of standard normal distribution, values of $\tilde{a}, \tilde{\sigma}$ are presented in (12).

Unlike De Vylder approach diffusion approximation has a solid theoretical base in functional limit theorems (weak invariance principles) for randomly stopped sums. A number of general results can be find, for example, in P. Billingsley [1968, p.17], A. Gut [1989 ,ch.5], P. Embrechts et al. [1997, p.2.5], W. Whitt [2002], D. Silvestrov [2004], V. Korolev et al. [2007 , ch.7], applications to diffusion approximation for classical model - in J.Grandell [1991]. Weak convergence of risk processes to α-stable Lévy process was studied in [15, 16].

Following theorem from P.Embrechts et al. [1997, p.2.5] will serve as an auxiliary result for further conclusions.

Theorem A1 (Invariance principle for randomly stopped sums).
Let $\{\xi_i\}$ be a sequence of i.i.d.r.v., $E\xi_1 = M_1$, $Var\xi_1 = \sigma_\xi^2 < \infty$, partial sum process $S(n) = \sum_{i=1}^n \xi_i$, $S(0) = 0$. Assume that the renewal counting process $N(t) = \sup\{n \geq 1 : \sum_{i=1}^n \eta_i \leq t\}$, $t \geq 0$ and $\{\xi_i\}$ are independent, $E\eta_1 = 1/\lambda$, $Var\eta_1 = \sigma_\eta^2 < \infty$. Then as $n \to \infty$

$$\left((\sigma_\xi^2 + (M_1\lambda\sigma_\eta)^2)\lambda n\right)^{-1/2}\left(S(N(nt)) - \lambda M_1 nt\right) \Rightarrow W(t), \qquad (15)$$

where $W(t)$, $t \geq 0$ is a standard Wiener process, $\Rightarrow$ means weak convergence in Skorokhod space $D[0, \infty)$ equipped with J_1-metric.

Reminding that Poisson process is renewal counting process with $\{\eta_i\}$ exponentially distributed with parameter $\lambda > 0$, $E\eta_1 = 1/\lambda$, $Var\eta_1 = 1/\lambda^2$ we have

Corollary 2 (Functional CLT for Poisson sums). *Let $\{\xi_i\}$ and $S(n)$ be as in Theorem A1, $N(t)$ - Poisson process with intensity $\lambda > 0$, $S(N(t))$ - compound Poisson process. Then as $n \to \infty$*

$$\left(M_2\lambda n\right)^{-1/2}\left(S(N(nt)) - \lambda M_1 nt\right) \Rightarrow W(t), \qquad (16)$$

in Skorokhod J_1-metric, $M_2 = E\xi_1^2 = \sigma_\xi^2 + M_1^2$.

Corollary 3 (Functional CLT for risk process with stochastic premiums). *Let $Q(t) = \Pi(t) - S(t)$ be a risk process (1) with $Ex_1^k = \mu_k$, $Ey_1^k = m_k$, $k = 1, 2$, λ_1 and λ be parameters of corresponding Poisson processes $N_1(t)$ and $N(t)$, then*

$$\left(\tilde{\sigma}^2 n\right)^{-1/2}\left(Q(nt) - (\lambda_1 m_1 - \lambda\mu_1)nt\right) \Rightarrow W(t), \qquad (17)$$

where $W(t)$, $t \geq 0$ is a standard Wiener process, $\tilde{\sigma}^2 = \lambda_1 m_2 + \lambda \mu_2$.

Proof obviously follows from Corollary 2 due to the fact that $Q(t)$ is a compound Poisson process, see (2), with intensity $\lambda^* = \lambda + \lambda_1$, whose jumps have mean $\frac{\tilde{a}}{\lambda^*} = \frac{\lambda_1}{\lambda^*} m_1 - \frac{\lambda}{\lambda^*} \mu_1$ and second moment $\frac{\tilde{\sigma}^2}{\lambda^*} = \frac{\lambda_1}{\lambda^*} m_2 + \frac{\lambda}{\lambda^*} \mu_2$.

Corollary 3 once more underline the reasonability of proposed diffusion approximation for model (1). Detail discussion of diffusion approximation for classical Cramér-Lundberg model is presented by J.Grandell [19]; his conclusions in whole can be extended on model (1) with stochastic premiums.

Note that in the case, when claims are so heavy-tailed that $Ex_1^2 = \infty$, more precise $\{x_i\}$ belong to the domain of attraction of α-stable law, $0 < \alpha < 2$, appropriate approximating process is α-stable Lévy process.

4. Strong invariance principle for risk process with stochastic premiums

The other type of limit theorems is *strong invariance principle* (SIP), which occurs to be a bridge between weak and a.s. convergence.

Strong invariance principle (almost sure approximation) is a class of limit theorems that provide sufficient (or necessary and sufficient) conditions for the possibility to construct the i.i.d.r.v. $\{\xi_i, i \geq 1\}$ and Lévy process $\{Y(t), t \geq 0\}$ on the same probability space in such a way that a.s.

$$\left| \sum_{i=1}^{[t]} \xi_i - mt - Y(t) \right| = o(r(t)), \tag{20}$$

where $[a]$ is entire of $a > 0$, $m = E\xi_1$, $r(t)$ - non-random function - *approximation error (error term)*, depending on additional assumptions posed on $\{X_i, i \geq 1\}$. Concrete assumptions on $\{\xi_i, i \geq 1\}$ clear up the type of $Y(t)$ and the form of $r(.)$. Since we deal with i.i.d.r.v. it is natural to consider Wiener process $W(t)$ or α-stable Lévy process $Y_\alpha(t)$, $t \geq 0$, as an approximation process $Y(t)$ in (20).

While usual invariance principle deals with convergence of distributions of functionals of S_n, SIP tells how "small" can be difference between sample pathes of S_n and limiting process $Y(t)$.

It is obvious that using (20) with appropriate error term one can easily (almost without the proof) transfer the results about the asymptotic behavior of Lévy process $Y(t)$ or its increments on the rate of growth of partial sums and corresponding increments.

In forthcoming we will use the concept of SIP in a wider sense, and say that a random process $\xi(t)$ admits the a.s. approximation by the random

process $\eta(t)$ if $\xi(t)$ (or stochastically equivalent process $\{\xi'(t), t \geq 0\}$) can be constructed on the rich enough probability space together with $\eta(t), t \geq 0$, in such a way that a.s.

$$|\xi(t) - \eta(t)| = o(r_1(t)) \vee O(r_1(t)), \tag{21}$$

where $r_1(.)$ is again a non-random function.

The investigations of SIP for partial sum process was originated by pioneer works of A.Skorokhod (1961) (Skorokhod embedding scheme) and W.Strassen (1965, 1967) in the middle of 60-th; for detail bibliography see M.Csörgő, P.Révész (1981) and more recent M.Csörgő, L. Horváth (1993), M.Alex and J. Steinebach (1994), also survey by N.Zinchenko (2000).

Summarizing all known results for i.i.d.r.v. with finite variance we have as in M.Csörgő, L. Horváth (1993)

Theorem A2. *I.i.d.r.v.* $\{\xi_i, i \geq 1\}$ *with* $E\xi_1 = M_1$ *can be defined on the same probability space together with standard Wiener process* $\{W(t), t \geq 0\}$ *in such a way that a.s.*

$$\sup_{0 \leq t \leq T} |S(t) - M_1 t - W(t)| = o(r(T)), \tag{22}$$

where
- $r(T) = T^{1/p}$ iff $E|\xi_1|^p < \infty, p > 2$;
- $r(T) = (T \ln \ln T)^{1/2}$ iff $E|\xi_1|^2 < \infty$;
- right hand side of (22) is $O(\ln T)$ iff $E\exp(uX) < \infty$ for $u \in (0, u_0)$.

Now consider the case $E\xi_i^2 = \infty$. More precise assume that $\{\xi_i, i \geq 1\}$ are in domain of normal attraction of $G_{\alpha,\beta}$ (notation $\{\xi_i\} \in DNA(G_{\alpha,\beta})$). This means weak convergence

$$S_n^* = n^{-1/\alpha}(S(n) - a_n) \Rightarrow G_{\alpha,\beta},$$

where

$$a_n = \begin{cases} nE\xi_1 & if \quad 1 < \alpha < 2, \\ 0 & if \quad 0 < \alpha < 1, \\ (2/\pi)\beta \ln n & if \quad \alpha = 1. \end{cases}$$

It is well known that $\{\xi_i\} \in DNA(G_{\alpha,\beta})$ iff for large x the tails of its d.f.$F(x)$ satisfy

$$1 - F(x) = c_1 x^{-\alpha} + d_1(x)x^{-\alpha}, \quad F(-x) = c_2 x^{-\alpha} + d_2(x)x^{-\alpha},$$

where $c_1 > 0$, $c_2 > 0$, $d_1(x) \to 0$, $d_2(x) \to 0$ as $x \to \infty$. Thus, for $X \in DNA(G_{\alpha,\beta})$, $E|X|^p < \infty, \forall p < \alpha$, but $E|X|^p = \infty$ for any $p > \alpha$.

It occurs that the fact $\{\xi_i\} \in DNA(G_{\alpha,\beta})$ is not enough to obtain "good" error term in (20), thus, certain additional assumptions are needed. We formulate them in terms of ch.f. (see Zinchenko [31, 33, 34], Berkes et al. [4,5], Mijnheer [26]).

Assumption (C) : *there are $a_1 > 0, a_2 > 0$ and $l > \alpha$ such that for $|u| < a_1$*

$$|f(u) - g_{\alpha,\beta}(u)| < a_2 |u|^l$$

where $f(u) = e^{-iuM_1}\varphi(u)$ is a ch.f. of $(\xi_1 - E\xi_1)$ if $1 < \alpha < 2$ and $f(u) = \varphi(u)$, i.e. ch.f. of ξ_1 if $0 < \alpha \leq 1$.

Assumption **(C)** not only provides the weak convergence $S_n \Rightarrow G_{\alpha,\beta}$, but also determines the rate of convergence.

As in [33] we have

Theorem A3. *Let $M_1 = E\xi_1$ for $1 < \alpha < 2$ and $M_1 = 0$ for $0 < \alpha \leq 1$. Under assumption (**C**) it is possible to define α-stable process $Y_{\alpha,\beta}(t), t \geq 0$, such that a.s.*

$$\sup_{0 \leq t \leq T} |S(t) - M_1 t - Y_{\alpha,\beta}(t)| = o(T^{1/\alpha - \rho_1}), \qquad (23)$$

for all $0 < \rho_1 < \varrho_0)$, $\varrho_0 = (l - \alpha)/80\alpha$.

In our work we shall focus on SIP for randomly stopped sums

$$D(t) = S(N(t)) = \sum_{i=1}^{N(t)} \xi_i, \qquad (24)$$

where $N(t)$ is a renewal (counting) process

$$N(t) = \inf\{x > 0 : Z(x) > t\}$$

associated with the sums of i.i.d.r.v. $Z(n) = \sum_{i=1}^{n} \eta_i$, $0 < E\eta_1 < 1/\lambda < \infty$, $\{Z_i, i \geq 1\}$ are independent of $\{\xi_i, i \geq 1\}$.

A number of results concerning SIP for $N(t)$, $D(t)$ and wider classes of inverse processes and superposition of the processes can be find in M.Csörgő, L. Horváth [8] for the case of a.s. approximation with Wiener process, particularly, for $\{\xi_i\}$ and $\{\eta_i\}$ with finite variance. Case of heavy-tailed summands attracted to α-stable law was studied by N.Zinchenko [35] , see also N.Zinchenko and M.Safonova [36].

We will need following results:

Theorem A4 [8]. *Denote by* $Var\xi_1 = \sigma^2$, $Var\eta_1 = \tau^2$, $\nu^2 = \lambda\sigma^2 + \lambda^3 M_1^2 \tau^2$.

(i) Suppose that $E|\xi_1|^p < \infty$, $E|\eta_1|^p < \infty$, $p > 2$, then $\{\xi_i\}$ and $N(t)$ can be constructed on the same probability space together with Wiener process $\{W(t), t \geq 0\}$ in such a way that a.s.

$$\sup_{0 \leq t \leq T} |D(t) - \lambda M_1 t - \nu W(t)| = o(T^{1/p}), \qquad (25)$$

(ii) If $E\exp(u\xi_1) < \infty$ and $E\exp(u\eta_1) < \infty$ for all $u \in (0, u_o)$ then right side of (25) is $O(\ln T)$.

Corollary (SIP for Poisson sums, summands with finite variance).
Let $N(t)$ be Poisson process with intensity $\lambda > 0$ ($\{\eta_i\}$ are exponentially distributed with parameter λ) then under condition $E|\xi_1|^p < \infty$, $p > 2$, there is a Wiener process $\{W(t), t \geq 0\}$ such that a.s.

$$\sup_{0 \leq t \leq T} \left| D(t) - \lambda M_1 t - \sqrt{\lambda(\tau^2 + M_1^2)}W(t) \right| = o(T^{1/p}), \qquad (26)$$

and for $\{\xi_i\}$ with light tails, i.e. with finite moment generating function $E\exp(u\xi_1) < \infty$, $u \in (0, u_o)$, right side of (26) is $O(\ln T)$.

As a next step assume that $\{\xi_i\}$ are attracted to α-stable law $G_{\alpha,\beta}$ with $1 < \alpha < 2$, $|\beta| \leq 1$ (condition $\alpha > 1$ needed to have a finite mean). SIP for randomly stopped sums in this case was studied in [35,36].

Theorem A5. *Let $\{\xi_i, i \geq 1\}$ satisfy (C) with $1 < \alpha < 2$, $\beta \in [-1, 1]$, $E\eta_1^2 < \infty$. Then $\{\xi_i, i \geq 1\}$, $\{\eta_i, i \geq 1\}$, $N(t)$ can be defined together with α-stable process $Y_\alpha(t) = Y_{\alpha,\beta}(t)$, $t \geq 0$ so that a.s.*

$$|S(N(t)) - M_1\lambda t - Y_{\alpha,\beta}(\lambda t)| = o(t^{1/\alpha - \varrho_2}), \quad \rho_2 \in (0, \rho_0^*), \qquad (27)$$

for some $\varrho_0^* = \varrho_0^*(\alpha, l) > 0$.

Corollary 4 (SIP for Poisson sums with summands attracted to α-stable law). *Theorem A5 holds if $N(t)$ is a Poisson process with intensity $\lambda > 0$.*

We will use such general results to investigate the possibility of a.s. approximation of risk process $Q(t)$ with stochastic premiums.

Corollary 5 (SIP for risk process with stochastic premiums, finite variance case).
(I) If in model (1) both premiums $\{y_i\}$ and claims $\{x_i\}$ have moments of order $p > 2$, then there is a Wiener process $\{W_{\tilde{a}, \tilde{\sigma}^2}(t), t \geq 0\}$ with

$\tilde{a} = \lambda_1 m_1 - \lambda \mu_1$, $\tilde{\sigma}^2 = \lambda_1 m_2 + \lambda \mu_2$ *such that a.s.*

$$\sup_{0 \le t \le T} |Q(t) - W_{\tilde{a},\tilde{\sigma}^2}(t)| = o(T^{1/p}). \qquad (26)$$

(II) If premiums $\{y_i\}$ and claims $\{x_i\}$ are light-tailed with finite moment generating function in some positive neighborhood of zero, then a.s.

$$\sup_{0 \le t \le T} |Q(t) - W_{\tilde{a},\tilde{\sigma}^2}(t)| = O(\log T), \qquad (27)$$

Proof immediately follows from Corollary 4 since $Q(t)$ is a compound Poisson process (see (2)) with intensity $\lambda^* = \lambda + \lambda_1$, whose jumps have mean $\frac{\tilde{a}}{\lambda^*} = \frac{\lambda_1}{\lambda^*} m_1 - \frac{\lambda}{\lambda^*} \mu_1$, and second moment $\frac{\tilde{\sigma}^2}{\lambda^*} = \frac{\lambda_1}{\lambda^*} m_2 + \frac{\lambda}{\lambda^*} \mu_2$.

The other way to prove (26) or (27) is a.s. approximation of $\Pi(t)$ and $S(t)$ separately by $(\lambda_1 m_2)^{1/2} W_1(t)$ and $(\lambda \mu_2)^{1/2} W_2(t)$ with corresponding error terms, where $W_1(t)$ and $W_2(t)$ are independent standard Wiener processes.

Remark. In model (1) it is natural to suppose that premiums have distributions with light tails or tails which are lighter than for claim sizes. Therefore moment conditions, which determine the error term in SIP, are in fact conditions on claim sizes.

Now consider the case when claims are so large that they have infinity variance.

Corollary 6 (SIP for risk process with stochastic premiums and large claims attracted to α-stable law). *Suppose that claim sizes $\{x_i\}$ satisfy (C) with $1 < \alpha < 2$, $\beta \in [-1, 1]$, premiums $\{y_i\}$ are i.i.d.r.v. with finite moments of order $p > 2$, then a.s.*

$$|Q(t) - (\lambda_1 m_1 - \lambda \mu_1)t - \lambda^{1/\alpha} Y_{\alpha,-\beta}(t)| = o(t^{1/\alpha - \varrho_2}), \quad \rho_2 \in (0, \rho_0^*), \qquad (28)$$

for some $\varrho_0^* = \varrho_0^*(\alpha, l) > 0$.

Proof can be derived with the help of a.s. approximation of $\Pi(t)$ by $(\lambda_1 m_2)^{1/2} W_1(t)$ with the error term $o(t^{1/p})$ (Theorem A4) and $-S(t) = -\sum_{i=1}^{n} x_i$ – by independent of $W_1(t)$ α-stable process $\lambda^{1/\alpha} Y_{\alpha,-\beta}(t)$ with error term $o(t^{1/\alpha - \varrho_2})$, $1 < \alpha < 2$ (Theorem A5) and application of LIL for standard Wiener process. Note that $Y_{\alpha,-\beta}(t) = -Y_{\alpha,\beta}(t)$.

The other way of the proof – application of Theorem A5 to process $Q(t)$ in form (2) taking in consideration the form of intensity and scale parameter.

5. The rate of growth of risk process with stochastic premiums and magnitude of its increments

Now we shall apply the results of previous section about SIP to investigation the rate of growth of risk process $Q(t)$ as $t \to \infty$ and its increments $Q(t + a_t) - Q(t)$ on intervals whose length a_t grows but not faster than t. Such question about the order of magnitude of increments for classical risk process and renewal model was asked in Embrechts et al. [10], partial answers on this question can be found in [7-9], [11-14], [24,25].

We will follow the approach developed in Zinchenko and Safonova [36] for investigation the rate of growth of random sums. The key moments are application of SIP for $Q(t)$ with appropriate error term and known results about asymptotic behavior of the Wiener and stable processes and their increments, namely, various modifications of the LIL and Erdös-Rényi-Csörgő-Révész type SLLN [7,8,11].

Theorem 6 (LIL for risk process with stochastic premiums). *If in model (1) both premiums $\{y_i\}$ and claims $\{x_i\}$ have moments of order $p > 2$, then*

$$\limsup_{t \to \infty} \frac{|Q(t) - \tilde{a}t|}{\sqrt{2t \ln \ln t}} = \tilde{\sigma}, \ \text{where } \tilde{a} = \lambda_1 m_1 - \lambda \mu_1, \quad \tilde{\sigma}^2 = \lambda_1 m_2 + \lambda \mu_2. \ (29)$$

Proof follows from classical LIL for standard Wiener process and SIP for $Q(T)$ with error term $o(t^{1/p})$ (Corollary 5).

Notice that Theorem 6 covers not only the case of small claims, but also the case of large claims with finite variance.

Next result deals with the case of large claims with infinite variance. More precise, we shall consider the case when r.v. $\{x_i, i \geq 1\}$ are attracted to an asymmetric stable law $G_{\alpha,1}$, but premiums have $Ey_1^p < \infty$.

Theorem 7. *Let $\{x_i, i \geq 1\}$ satisfy* **(C)** *with $1 < \alpha < 2$, $\beta = 1$ and $Ey_1^p < \infty$, $p > 2$. Then a.s.*

$$\limsup_{t \to \infty} \frac{Q(t) - (\lambda_1 m_1 - \lambda \mu_1)t}{(\lambda t)^{1/\alpha}(B^{-1} \ln \ln t)^{1/\theta}} = 1, \tag{30}$$

where

$$B = B(\alpha) = (\alpha - 1)\alpha^{-\theta}|\cos(\pi\alpha/2)|^{1/(\alpha-1)}, \quad \theta = \alpha/(\alpha - 1). \tag{31}$$

Proof. Condition **(C)** provides normal attraction of $\{x_i\}$ to a stable law with $1 < \alpha < 2$, $\beta = 1$ and, therefore, $Q(t) - (\lambda_1 m_1 - \lambda \mu_1)t$ can be a.s. approximated by the stable Lévy process $\lambda^{1/\alpha} Y_{\alpha,-1}(t)$ via Corollary 6. Stable process $Y_{\alpha,-1}(t)$ has only negative jumps and obeys the following modification of the LIL [17, 32]

$$\limsup_{t \to \infty} \frac{Y_{\alpha,-1}(t)}{t^{1/\alpha}(B^{-1} \ln \ln t)^{1/\theta}} = 1. \tag{32}$$

Combining (32) and order of error term in SIP (28), we obtain (30).

Investigation of the asymptotic of increments of $Q(t)$ we shall also carry out step by step.

We shall start with the light-tailed case.

Theorem 8. *Let claims* $\{x_i, i \geq 1\}$ *and premiums* $\{y_i, i \geq 1\}$ *be indepen-dent sequences of i.i.d.r.v. with* $Ey_1 = m_1$, $Ey_1^2 = m_2$, $Ex_1 = \mu_1$, $Ex_1^2 = \mu_2$, *and finite moment generating functions*

$$E \exp(rx_1) < \infty, \quad E \exp(ry_1) < \infty \text{ as } |r| < r_0, \quad r_0 > 0. \tag{33}$$

Assume that function a_T, $T \geq 0$, *satisfies following conditions:*
(i) $0 < a_T < T$,
(ii) T/a_T *does not decrease in* T.
Also let

$$a_T / \ln T \to \infty \text{ as } T \to \infty. \tag{34}$$

Then a.s.

$$\limsup_{T \to \infty} \frac{|Q(T + a_T) - Q(T) - a_T(\lambda_1 m_1 - \lambda \mu_1)|}{\gamma(T)} = \tilde{\sigma}, \tag{35}$$

where

$$\gamma(T) = \{2a_T(\ln \ln T + \ln T/a_T)\}^{1/2}, \quad \tilde{\sigma}^2 = \lambda_1 m_2 + \lambda \mu_2.$$

Theorem 9. *Let* $\{x_i, i \geq 1\}$, $\{y_i, i \geq 1\}$ *and* a_T *satisfy all conditions of Theorem 8 with following assumption used instead of (34)*

$$Ex_1^p < \infty, \quad Ey_1^p < \infty, \quad p > 2.$$

Then (35) is true if $a_T > c_1 T^{2/p} / \ln T$ *for some* $c_1 > 0$.

On the second step we assume that i.i.d.r.v. $\{x_i, i \geq 1\}$ are attracted to an asymmetric α-stable law. Denote by

$$d_T = a_T^{1/\alpha}\{B^{-1}(\ln\ln T + \ln T/a_T)\}^{1/\theta},$$

where

$$B_1 = B(\alpha) = (1-\alpha)\alpha^{-\theta}\left|\cos(\pi\alpha/2)\right|^{1/(\alpha-1)}, \quad \theta = \alpha/(\alpha-1).$$

Theorem 10. *Suppose that $\{x_i, i \geq 1\}$ satisfy (**C**) with $1 < \alpha < 2$, $\beta = 1$ and $Ey_1^p < \infty$, $p > 2$, $Ex_1 = \mu_1$, $Ey_1 = m_1$. Function a_T is non-decreasing, $0 < a_T < T$, T/a_T is also non-decreasing and provides $d_T^{-1}T^{1/\alpha-\varrho_2} \to 0$ as $T \to \infty$ for certain $\varrho_2 > 0$ determined by error term in SIP. Then a.s.*

$$\limsup_{T\to\infty} \frac{Q(T + a_T) - Q(T) - (\lambda_1 m_1 - \lambda\mu_1)a_T}{d_T} = \lambda^{1/\alpha}. \qquad (36)$$

Proof. Az in theorem 7, $Q(t) - (\lambda_1 m_1 - \lambda\mu_1)t$ can be a.s. approximated by asymmetric stable Lévy process $\lambda^{1/\alpha}Y_{\alpha,-1}(t)$ via Corollary 6. Thus, in the proof we use Erdös-Rényi-Csörgő-Révész type limit theorem for α-stable process $Y_{\alpha,-1}$ (see [32])

$$\limsup_{T\to\infty} d_T^{-1}\left(Y_{\alpha,-1}(T + a_T) - Y_{\alpha,-1}(T)\right) = 1$$

and error term in strong invariance principle for $Q(T)$ from Corollary 6.

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