Theory of Stochastic Processes
Vol.13 (29), no.4, 2007, pp.210–218

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STATIONARY PROCESSES IN FUNCTIONAL SPACES $L_q(\mathbb{R})$

The paper is devoted to the problem of establishing the conditions on the stochastic process to belong it to the functional space $L_q(\mathbb{R})$ with probability one. The corresponding results were obtained for the strictly Orlicz, stationary in wide sense processes.

1. Introduction

This paper deals with the problem of establishing the condition on the stochastic process from the space of random variables $L_p(\Omega)$ under which it belongs to the functional space $L_q(\mathbb{R})$ with probability one. This two spaces are actually Orlicz spaces generated by the functions $V(x) = |x|^p$, $p \geq 1$ and $U(x) = |x|^q$, $q \geq 1$. The general theory on Orlicz functions and spaces is contained in [1]. Random variables and stochastic processes in Orlicz spaces were investigated in book [2]. For the stochastic process defined in compact set the corresponding conditions of belonging were established for two cases: for $1 \leq p < q$ in [3] and for $1 \leq q \leq p$ in [4]. The stochastic processes with noncompact parametric set were examined in [5,6] for $1 \leq p < q$. In this paper we have completed that results with treatment the case when $1 \leq q \leq p$. The results obtained were applied to strictly Orlicz, stationary in wide sense stochastic processes in both this cases.

2. Some facts from Orlicz space theory.

Definition 2.1 [1] A function $U = U(x)$, $x \in R$ is called an Orlicz S-function if it is continuous, even, convex and $U(0) = 0$, $U(x) > 0$ as $x \neq 0$.

Example 2.2 The functions $U(x) = A|x|^\alpha$, $A > 0$, $\alpha \geq 1$ and $V(x) = \exp\{B|x|^\beta\} - 1$, $B > 0$, $\beta \geq 1$ are the S-functions.

2000 *Mathematics Subject Classifications.* Primary 60G17, secondary 60G07
Key words and phrases. Stationary process, Orlicz space, noncompact parametric set, strictly Orlicz random variables.

Let $\{\mathbf{T}, \mathcal{B}, \mu\}$ be a measurable space.

Definition 2.3 [2] The space of measurable functions $f = \{f(t), t \in \mathbf{T}\}$ such that for all f there exists a constant $r_f > 0$ for which

$$\int_{\mathbf{T}} U\left(\frac{f(t)}{r_f}\right) d\mu(t) < \infty$$

is called the Orlicz space $L_U(\mathbf{T}, \mu)$ generated by the S-function U.

Remark 2.4 [2] The space $L_U(\mathbf{T}, \mu)$ is a Banach space with respect to the Luxemburg norm

$$\|f(t)\|_{L_U(\mathbf{T})} = \inf\Bigg\{r > 0 : \int_{\mathbf{T}} U\left(\frac{f(t)}{r}\right) d\mu(t) \leq 1\Bigg\}.$$

Definition 2.5 [2] Let $\{\Omega, \mathcal{B}, P\}$ be a standard probability space. Then $L_U(\Omega, P)$ is the Orlicz space of random variables generated by the S-function U. The Luxemburg norm in this space is determined as

$$\|\xi\|_{L_U(\Omega)} = \inf\left\{r > 0 : \mathrm{E}U\left(\frac{\xi}{r}\right) \leq 1\right\}.$$

Definition 2.6 [2] A stochastic process $X = \{X(t), t \in \mathbf{T}\}$ belongs to the Orlicz space $L_U(\Omega, P)$ if for all $t \in \mathbf{T}$ random variables $X(t)$ belong to the space $L_U(\Omega)$.

Definition 2.7 [2] A stochastic process $X = \{X(t), t \in \mathbf{T}\}$ belongs to the Orlicz space $L_U(\mathbf{T}, \mu)$ with probability one if the trajectories of the process X are measurable and $X(t) \in L_U(\mathbf{T}, \mu)$ with probability one.

Condition M. [3] It is said that condition M is satisfied for S-function V in a set G if for all measurable stochastic processes $Z = \{Z(t), t \in G\}$ such that $Z(t) \in L_V(\Omega)$ and $\|Z(t)\|_{L_V(\Omega)} = 1$, random variables $\|Z(t)\|_{L_V(G)}$ also belong to $L_V(\Omega)$ and there exists an absolute constant a_V such that

$$\big\|\|Z(t)\|_{L_V(G)}\big\|_{L_V(\Omega)} \leq a_V.$$

Example 2.8 [3] The functions $U(x) = A|x|^\alpha$, where $A > 0$, $\alpha \geq 1$ and $V(x) = \exp\{B|x|^\beta\} - 1$, where $B > 0$, $\beta \geq 1$ satisfy the condition M in every compact set G.

Consider the measurable parametric space $(\mathbf{T}, \mathcal{B}(\mathbf{T}), \mu)$ with $\sigma-$ finite measure μ. That is, there exists such partition $\{T_l \in \mathcal{B}(\mathbf{T})\}_{l\geq 1}$ of space $\mathbf{T}$

that $\mathbf{T} = \bigcup_{l=1}^{\infty} T_l$ and $\mu(T_l) < \infty$. Furthermore, let for all $l^{'}, l^{''}$ the following condition holds : $T_{l'} \bigcap T_{l''} = \varnothing$ as $l^{'} \neq l^{''}$.

Assume that U and V are such Orlicz $S-$ functions that $W = W(x) = V^{-1}(U(x))$, $x \in \mathbb{R}$ is convex function too. In this case the following theorem takes place.

Theorem 2.9 [5] *Assume that the stochastic process $X = \{X(t), t \in \mathbf{T}\}$ ($\mu(\mathbf{T}) = \infty$), which belongs to the Orlicz space of random variables $L_V(\Omega)$, is separable, measurable and*

$$\forall B \in \mathcal{B}(T) \qquad \sup_{\substack{\rho(t,s) \leq h \\ t,s \in B}} \|X(t) - X(s)\|_{L_V(\Omega)} \leq \sigma_B(h)$$

where $\sigma_B = \sigma_B(h)$, $h > 0$ is the continuous, monotone nondecreasing function and $\sigma_B(h) \to 0$ as $h \to 0$.

Then, if the condition M is satisfied for the Orlicz S-function V in every set T_l from the partition $\mathbf{T}$ and there exists such sequence $\{0 < \delta_l < 1\}_{l \geq 1}$ that

$$\sum_{l \geq 1} \delta_l < \infty \qquad \text{and} \qquad \sum_{l \geq 1} \left(V \left(\frac{\delta_l}{\left\| \|X(t)\|_{L_U(T_l)} \right\|_{L_V(\Omega)}} \right) \right)^{-1} < \infty,$$

then the stochastic process $X = \{X(t), t \in \mathbf{T}\}$ belongs to the space $L_U(\mathbf{T})$ with probability one.

3. Conditions for stochastic process belonging to the functional space $L_q(\mathbb{R})$.

Consider stochastic processes from the Orlicz space of random variables $L_p(\Omega)$. This space is generated by the S-function $V(x) = |x|^p$, $p \geq 1$. The functional space $L_q(\mathbb{R})$ is generated by the Orlicz S-function $U(x) = |x|^q$, $q \geq 1$. In this section we will present sufficient conditions for belonging of the stochastic processes defined on set of all real numbers $\mathbb{R}$ to functional space $L_q(\mathbb{R})$ with probability one in two cases, according to the type of subordination between p and q : $1 \leq q \leq p$ and $1 \leq p < q$.

Consider measurable parametric space $(\mathbb{R}, \mathcal{B}(\mathbb{R}), \mu)$, where $\mathcal{B}(\mathbb{R})$ is a σ-algebra of Borelian subsets on $\mathbb{R}$, μ is the Lebegue measure on $\mathbb{R}$.

Let partition the set $\mathbb{R}$ into half-open intervals $[x_{l-1}, x_l)$ with properties

$$\forall l \geq 1 \quad \Delta x_l = |x_l - x_{l-1}| < \infty; \quad \bigcup_{l \in \mathbb{Z}} [x_{l-1}, x_l) = \mathbb{R}; \tag{1}$$

$$[x_{l'-1}, x_{l'})\bigcap[x_{l''-1}, x_{l''}) = \varnothing \text{ , as } l' \neq l''.$$

The partition determined by the sequence

$$\left\{ x_l = sign(l) \sum_{i=1}^{|l|} \frac{1}{i^\gamma}, 0 < \gamma < 1 \right\}_{l\in\mathbb{Z}} \tag{2}$$

has property (1) and for all $l \geq 1$

$$x_l = |x_{-l}| = \sum_{i=1}^{l} \frac{1}{i^\gamma} > (l+1)^{1-\gamma} - 1 \sim l^{1-\gamma} \text{ as } l \to \infty$$

Really, since for all $0 < \gamma < 1$, $u \geq 1$ function $1/u^\gamma$ is monotonely decreasing. Then for all $i \geq 1$

$$\frac{1}{i^\gamma} = \int_i^{i+1} \frac{1}{i^\gamma} du \geq \int_i^{i+1} \frac{1}{u^\gamma} du = \left.\frac{u^{1-\gamma}}{1-\gamma}\right|_i^{i+1} = \frac{1}{1-\gamma}\left((i+1)^{1-\gamma} - i^{1-\gamma}\right),$$

and thus

$$\begin{aligned} x_l &= |x_{-l}| = \sum_{i=1}^{l} \frac{1}{i^\gamma} \geq \frac{1}{1-\gamma} \sum_{i=1}^{l} \left((i+1)^{1-\gamma} - i^{1-\gamma}\right) = \\ &= \frac{1}{1-\gamma}\left((l+1)^{1-\gamma} - 1\right) > (l+1)^{1-\gamma} - 1 \sim l^{1-\gamma} \text{ as } l \to \infty. \end{aligned}$$

Theorem 3.1 *Let $1 \leq q \leq p$. Suppose that the measurable stochastic process $X = \{X(t), t \in \mathbb{R}\}$ belongs to the space of random variables $L_p(\Omega)$ and for all $-\infty < a < b < +\infty$ such that $b - a \leq 1$*

$$\sup_{a \leq t < b} (E|X(t)|^p)^{1/p} \leq \frac{A}{(\max\{|a|, |b|\})^\tau},$$

where $A > 0$, $\tau > \frac{1}{1-\gamma} + \frac{1}{p}$ and $0 < \gamma < 1$. Then trajectories of the process X belong to the functional space $L_q(\mathbb{R})$ with probability one.

Proof. Let partition the set $\mathbb{R}$ using sequence $\{x_l\}_{l\in\mathbb{Z}}$ (2). Then

$$\Delta x_l = |x_l - x_{l-1}| = \begin{cases} \frac{1}{l^\gamma}, & l \geq 1; \\ \frac{1}{(-l+1)^\gamma}, & l \leq 0. \end{cases}$$

Notice, that for all $l \in \mathbb{Z}$ $x_l = -x_{-l}$ and $\Delta x_l = \Delta x_{-l+1}$.

Theorem 4.1 [4] and properties of the sequence (2) imply that for every $l \geq 1$

$$\begin{aligned} &\left\| \|X(t)\|_{L_q[x_{l-1}, x_l)} \right\|_{L_p(\Omega)} \leq \frac{(1 + \Delta x_l)\Delta x_l^{1/q}}{x_l^\tau} \leq \frac{\left(1 + \frac{1}{l^\gamma}\right)\frac{1}{l^{\gamma/q}}}{\left((l+1)^{1-\gamma} - 1\right)^\tau} \leq \\ &\leq \frac{2}{l^{\gamma/q}\left((l+1)^{1-\gamma} - 1\right)^\tau} \sim \frac{2}{l^{\gamma/q + \tau(1-\gamma)}} =: B_l^+, \text{ as } l \to +\infty, \end{aligned}$$

and for $l \leq 0$,

$$\left\| \|X(t)\|_{L_q[x_{l-1},x_l)} \right\|_{L_p(\Omega)} \leq \frac{(1+\Delta x_l)\Delta x_l^{1/q}}{|x_{l-1}|^{\tau}} =$$
$$= \frac{(1+\Delta x_l)\Delta x_l^{1/q}}{x_{-l+1}^{\tau}} \leq \frac{\left(1+\frac{1}{(1-l)^{\gamma}}\right)\frac{1}{(1-l)^{\gamma/q}}}{((2-l)^{1-\gamma}-1)^{\tau}} \leq$$
$$\leq \frac{2}{(1-l)^{\gamma/q}\left((2-l)^{1-\gamma}-1\right)^{\tau}} \sim \frac{2}{(1-l)^{\gamma/q+\tau(1-\gamma)}} =: B_l^-, \text{ as } l \to -\infty.$$

It is obvious that for all $l \geq 1$ $B_l^+ = B_{-l+1}^-$. Consider the following sequence

$$\delta_l = \begin{cases} \frac{1}{l^{\varepsilon}}, & l \geq 1; \\ \frac{1}{(-l+1)^{\varepsilon}}, & l \leq 0, \end{cases} \qquad \varepsilon > 1.$$

Then

$$\sum_{l=-\infty}^{+\infty} \delta_l = \sum_{l=-\infty}^{0} \frac{1}{(-l+1)^{\varepsilon}} + \sum_{l=1}^{+\infty} \frac{1}{l^{\varepsilon}} = 2\sum_{l=1}^{+\infty} \frac{1}{l^{\varepsilon}} < \infty,$$

and

$$\sum_{l=-\infty}^{+\infty} \left(\frac{B_l}{\delta_l}\right)^p = \sum_{l=-\infty}^{0} \left(\frac{B_l^-}{(-l+1)^{\varepsilon}}\right)^p + \sum_{l=1}^{+\infty} \left(\frac{B_l^+}{l^{\varepsilon}}\right)^p =$$
$$= 2\sum_{l=1}^{+\infty} \left(\frac{B_l^+}{l^{\varepsilon}}\right)^p \sim \sum_{l \geq 1} \frac{1}{l^{\gamma+\tau(1-\gamma)p-\varepsilon p}}.$$

The last series converges as $\gamma + \tau(1-\gamma)p - \varepsilon p > 1$, that is

$$1 < \varepsilon < (1-\gamma)\left(\tau - \frac{1}{p}\right). \tag{3}$$

According to the conditions of the theorem

$$(1-\gamma)\left(\tau - \frac{1}{p}\right) > (1-\gamma)\left(\frac{1}{1-\gamma} + \frac{1}{p} - \frac{1}{p}\right) = 1.$$

and such ε exists. So, the statement of the theorem follows from the Theorem 2.9. □

Theorem 3.2 [6] *Let $1 \leq p < q$. We consider separable measurable stochastic process $X = \{X(t), t \in \mathbb{R}\}$, which belongs to the space of random variables $L_p(\Omega)$. Suppose that for some $-\infty < a < b < +\infty$, $b - a \leq 1$:*

i) $\sup_{a \leq t < b} (E|X(t)|^p)^{1/p} \leq \frac{A}{(\max\{|a|,|b|\})^{\tau}}$,
where $A > 0$, $\tau > \frac{1}{1-\gamma}\left(1 + \frac{1}{p} - \frac{\gamma}{q}\right)$, $0 < \gamma < 1$*;*

ii) $\sup_{\substack{|t-s|\le h \\ t,s\in[a,b)}} (E|X(t)-X(s)|^p)^{1/p} \le C_{a,b}h^{\alpha}$, *where* $h>0$, $\alpha > \frac{1}{p}-\frac{1}{q}$;

iii) $\exists\, 0<c<\infty : C_{a,b} \le \frac{c}{(b-a)^{\alpha}} \sup_{a\le t<b} (E|X(t)|^p)^{1/p}$.

In this case trajectories of the stochastic process X belong to the functional space $L_q(\mathbb{R})$ with probability one.

Remark 3.3 Conditions in the Theorem 3.1 and Theorem 3.2 on stochastic process will be weaker if γ is closer to 0.

4. Stationary strictly Orlicz stochastic processes.

In this section we consider special kind of stochastic processes, namely stationary strictly Orlicz processes. Let us recall some definitions first.

Definition 4.1 [7] Let $V=V(x)$, $x\in\mathbb{R}$ be Orlicz S-function, such that, there exist $x_0>0$ and $k>0$ that for all $x>x_0$, $x^2\le V(kx)$. Family Δ of random variables ξ_i ($E\xi_i=0$) from space $L_V(\Omega)$ is called strictly Orlicz if there exists constant C_Δ, such that for every finite set of random variables $\xi_i\in\Delta$, $i\in I$ and for all $\lambda_i\in\mathbb{R}$ the following inequality holds true

$$\left\|\sum_{i\in I}\lambda_i\xi_i\right\|_{L_V(\Omega)} \le C_\Delta\left(E\left(\sum_{i\in I}\lambda_i\xi_i\right)^2\right)^{1/2}.$$

C_Δ is determining constant for the family of strictly Orlicz random variables Δ.

Properties of the Orlicz S-function V imply that there exists constant $B>0$ such that

$$\left(E\left(\sum_{i\in I}\lambda_i\xi_i\right)^2\right)^{1/2} \le B\left\|\sum_{i\in I}\lambda_i\xi_i\right\|_{L_V(\Omega)},$$

i.e. norms $\|\cdot\|_{L_V(\Omega)}$ and $(E(\cdot)^2)^{1/2}$ are equivalent for family Δ.

Example 4.2 Family of Gaussian centered random variables is strictly Orlicz in the exponential Orlicz space $Exp_2(\Omega)$(See book [2]).

Definition 4.3 Stochastic process $X=\{X(t), t\in\mathbf{T}\}$ belonging to space $L_V(\Omega)$ is called strictly Orlicz if the family of random variables $\{X(t)\}_{t\in\mathbf{T}}$ is strictly Orlicz.

Definition 4.4 Stochastic process $X=\{X(t), t\in\mathbb{R}\}$ is called stationary in wide sense if $EX(t)=m=const$ and its correlation function

$R(t,s) = r(t-s)$, $t,s \in \mathbb{R}$.

In the space $L_p(\Omega)$, $p \geq 2$ the family of the strictly Orlicz random variables can be easily determined. For more details see [7].

Theorem 4.5 *Let* $1 \leq q \leq p$, $Y = \{Y(t), t \in \mathbb{R}\}$ *be centered, measurable, stationary in wide sense, strictly Orlicz in* $L_p(\Omega)$ *stochastic process with determining constant* $C_{\Delta p}$ *and correlation function* $R(t,s) = r(t-s)$, $t,s \in \mathbb{R}$.

Consider function $c(t) = \frac{A}{(|t|+1)^\tau}$, $t \in \mathbb{R}$, $A = C_{\Delta p}\sqrt{r(0)}$, $\tau > \frac{1}{1-\gamma} + \frac{1}{p}$, $0 < \gamma < 1$. *Then stochastic process* $X(t) = \{c(t)Y(t), t \in \mathbb{R}\}$ *belongs to the functional space* $L_q(\mathbb{R})$ *with probability one.*

Proof. The statement of the theorem follows from the Theorem 3.1. Indeed, for all $-\infty < a < b < +\infty$, $b-a \leq 1$ (Note: 0 must not be inside the interval (a,b))

$$\sup_{a \leq t < b} \|X(t)\|_{L_p(\Omega)} = \sup_{a \leq t < b} (E|X(t)|^p)^{1/p} = \tag{4}$$

$$= \sup_{a \leq t < b} (E|c(t)Y(t)|^p)^{1/p} \leq C_{\Delta p} \sup_{a \leq t < b} |c(t)| \left(E|Y(t)|^2\right)^{1/2} =$$

$$= C_{\Delta p}\sqrt{r(0)} \sup_{a \leq t < b} |c(t)| = \frac{C_{\Delta p}\sqrt{r(0)}}{(\min\{|a|,|b|\}+1)^\tau} \leq \frac{A}{(\max\{|a|,|b|\})^\tau},$$

since $b - a = \max\{|a|,|b|\} - \min\{|a|,|b|\} \leq 1$. $\square$

Theorem 4.6 *Let* $2 \leq p \leq q$ *and* $Y = \{Y(t), t \in \mathbb{R}\}$ *be centered, measurable, separable, stationary in wide sense, strictly Orlicz in* $L_p(\Omega)$ *stochastic process with determining constant* $C_{\Delta p}$ *and correlation function* $R(t,s) = r(t-s)$, $t,s \in \mathbb{R}$ $(r(0) \neq 0)$, *such that for all* $0 < h \leq 1$ $\quad r(0) - r(h) \leq C_0 h^{2\alpha}$, $C_0 > 0$, $1/p - 1/q < \alpha \leq 1$.

Consider function $c(t) = \frac{A}{(|t|+1)^\tau}$, $t \in \mathbb{R}$, $\tau > \frac{1}{1-\gamma}\left(1 + \frac{1}{p} - \frac{\gamma}{q}\right)$, $0<\gamma<1$, $A = C_{\Delta p}\sqrt{r(0)}$. *Then stochastic process* $X(t) = \{c(t)Y(t), t \in \mathbb{R}\}$ *belongs to the functional Orlicz space* $L_q(\mathbb{R})$ *with probability one.*

Proof. For stochastic process $X(t) = \{c(t)Y(t), t \in \mathbb{R}\}$ the inequality (4) takes place, i.e the first condition of Theorem 3.2 holds.

Now, let's check the other two conditions of the Theorem 3.2.

For $-\infty < a < b < +\infty$, $b - a \leq 1$

$$\sup_{\substack{|t-s| \leq h \\ t,s \in [a,b)}} \|X(t) - X(s)\|_{L_p(\Omega)} = \sup_{\substack{|t-s| \leq h \\ t,s \in [a,b)}} (E|X(t) - X(s)|^p)^{1/p} \leq$$

$$\leq C_{\Delta p} \sup_{\substack{|t-s| \leq h \\ t,s \in [a,b)}} \left(E|X(t) - X(s)|^2\right)^{1/2} =$$

$$= C_{\Delta p} \sup_{\substack{|t-s|\le h\\ t,s\in[a,b)}} \left(E|c(t)Y(t)-c(s)Y(s)|^2\right)^{1/2} =$$

$$= C_{\Delta p} \sup_{\substack{|t-s|\le h\\ t,s\in[a,b)}} \left(E|c(t)Y(t)-c(s)Y(t)+c(s)Y(t)-c(s)Y(s)|^2\right)^{1/2} \le$$

$$\le C_{\Delta p} \sup_{\substack{|t-s|\le h\\ t,s\in[a,b)}} \left[|c(t)-c(s)|\left(EY^2(t)\right)^{1/2} + c(s)\left(E(Y(t)-Y(s))^2\right)^{1/2}\right] =$$

$$= C_{\Delta p} \sup_{\substack{|t-s|\le h\\ t,s\in[a,b)}} \left[|c(t)-c(s)|\sqrt{r(0)} + c(s)\sqrt{2(r(0)-r(t-s))}\right] =$$

$$= C_{\Delta p}\Big[(c(\min\{|a|,|b|\}) - c(\min\{|a|,|b|\}+h))\sqrt{r(0)} +$$
$$+ c(\min\{|a|,|b|\})\sqrt{2(r(0)-r(h))}\Big].$$

As long as function $c(t)$ is continuous then there exists $\theta \in [\min\{|a|,|b|\}, \min\{|a|,|b|\}+h]$ such that

$$\begin{aligned}
&c(\min\{|a|,|b|\}) - c(\min\{|a|,|b|\}+h) = \\
&= \frac{A}{(\min\{|a|,|b|\}+1)^\tau} - \frac{A}{(\min\{|a|,|b|\}+h+1)^\tau} = \\
&= \frac{-\tau A}{(|\theta|+1)^{\tau+1}}(\min\{|a|,|b|\} - \min\{|a|,|b|\} - h) = \\
&= \frac{\tau A}{(|\theta|+1)^{\tau+1}} h \le \frac{\tau A}{(\min\{|a|,|b|\}+1)^{\tau+1}} h
\end{aligned}$$

then

$$\begin{aligned}
&\sup_{\substack{|t-s|\le h\\ t,s\in[a,b)}} \|X(t)-X(s)\|_{L_p(\Omega)} \le \\
&\le C_{\Delta p}\left[\frac{A\tau\sqrt{r(0)}\cdot h}{(\min\{|a|,|b|\}+1)^{\tau+1}} + \frac{A\sqrt{2C_0}\cdot h^\alpha}{(\min\{|a|,|b|\}+1)^\tau}\right] \le \\
&\le \frac{AC_{\Delta p}\sqrt{r(0)}h^\alpha}{(\min\{|a|,|b|\}+1)^\tau}\left[\frac{\tau}{(\min\{|a|,|b|\}+1)} + \sqrt{\frac{2C_0}{r(0)}}\right] \le \\
&\le \frac{AC_{\Delta p}\sqrt{r(0)}\left(\tau + \sqrt{\frac{2C_0}{r(0)}}\right)}{(\min\{|a|,|b|\}+1)^\tau} h^\alpha = C_{a,b}h^\alpha.
\end{aligned}$$

So, condition ii) holds true. Let's check condition iii). Since $p \ge 2$ then there exists constant $\tilde{C}$ such that $\|\cdot\|_{L_2(\Omega)} \le \tilde{C}\|\cdot\|_{L_p(\Omega)}$. So,

$$C_{a,b} \;=\; \frac{AC_{\Delta p}\left(\tau + \sqrt{\frac{2C_0}{r(0)}}\right)}{(\min\{|a|,|b|\}+1)^\tau}\sqrt{r(0)} \le$$

$$\leq \frac{AC_{\Delta p}\left(\tau+\sqrt{\frac{2C_0}{r(0)}}\right)}{(b-a)^{\alpha}} \cdot (b-a)^{\alpha} \cdot \sup_{\substack{a \leq t < b \\ |b-a| \leq 1}} \left(E|X(t)|^2\right)^{1/2} \leq$$

$$\leq \frac{AVC_{\Delta p}\left(\tau+\sqrt{\frac{2C_0}{r(0)}}\right)\tilde{C}}{(b-a)^{\alpha}} \sup_{\substack{a \leq t < b \\ |b-a| \leq 1}} \left(E|X(t)|^p\right)^{1/p}.$$

This concludes that all conditions of Theorem 3.2 hold for stochastic process $X(t) = \{c(t)Y(t), t \in \mathbb{R}\}$. The theorem has been proved. □

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