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Mathematical model of arc temperature and conductivity at metallic and gaseous arc phases

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The new criterion of arc stability and instability is introduced, which enables one to find arc duration dependently on given circuit parameters and properties of contact material. The mathematical model of phase transformations inside electrodes during arcing is elaborated which describes dynamics of arc erosion in metallic and gaseous arc phases. Increasing of arc duration and erosion with inductance occurs on account of enlarging of gaseous arc phase, while variation of metallic phase is relatively small.

***Keywords**: mathematical model, arc erosion, arc phases.*

Introduction

Investigation of dynamical arc phenomena in opening electrical contacts is very important for performance build-up of circuit breakers by means of decrease of arc duration and erosion. Mayr's and Cassie's models [1] and their generalization [2] based on the power balance method are not applicable to describe arc temperature field at the initial arc stage just after arc ignition. Elenbaas-Heller equation gives information about radial distribution of the arc temperature however it is correct for stationary arcs only [3]. Arc dynamics should be described by transient heat equation taking into account nonlinear arc characteristic. It is the first intent of this paper. The second one is to device a method for calculation of arc erosion in dynamics.

Mathematical model of arc temperature and conductivity at metallic arc phase

Equation for the temperature

The arc temperature $\theta(r,t)$ in opening contacts just after ignition is less than the threshold value required for gas ionization, θ_{ig}, however it is sufficient to ionize metallic vapours in the contact gap, which takes place at the temperature θ_{im}:

$$\theta_{im} < \theta < \theta_{ig}.$$

This initial stage, called metallic arc phase, has very short duration and occurs in a small contact gap. Therefore the arc takes the form of a disk, which thickness is much less than radius, and the axial temperature component can be neglected in comparison with radial component. In this case the heat equation for the arc should be written in the form

$$C\frac{\partial\theta}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial\theta}{\partial r}\right) + \sigma E^2 - W_r, \qquad (1)$$

where C and λ are thermal capacity and density, λ and σ are heat and electrical conductivities of the arc plasma, E is electrical field and W_r is power loss due to arc radiation and heat conduction from arc column to electrodes. The initial temperature distribution along radius

$$\theta(r,0) = f(r) \tag{2}$$

can be found from the solution of the heat equation for metallic vapours at the pre-arcing stage [4, 5]. We can approximate the function $f(r) = \theta_o J_o(\mu_1 r / r_A)$ by parabola

$$f(r) = \theta_0 (1 - \frac{r^2}{r_A{}^2})$$

or by the Bessel function

$$f(r) = \theta_o J_o(\mu_1 r / r_A) , \tag{3}$$

where $\mu_1 = 2,405$ is the first root of the Bessel function and θ_0 is the temperature maximum at the centre of arc disc.

The temperature on the interface $r = r_A$ between ambient air and arc plasma should be equal to threshold of metal ionization

$$\theta(r_A, t) = \theta_{mi}. \tag{4}$$

It should be noted that thermal and electrical plasma conductivities, λ and σ, depend essentially on the temperature and this dependence can not be averaged. In contrast the arc radiation W_r can be neglected for metallic arc phase, which temperature is relatively low : $\theta_{im} < \theta_{ig} \approx 5000$ °C (fig. 1).

Equation for electrical conductivity σ

To solve the heat equation (1) we use the Kirchhoff's substitution

$$S(\theta) = \int_{\theta_{mi}}^{\theta} \lambda(\theta) d\theta. \tag{5}$$

Then the equation (1) transforms to

$$\frac{C}{\lambda}\frac{\partial S}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial S}{\partial r}) + \sigma E^2 - W_r . \tag{6}$$

Solving the equation (5) with respect to θ, get

$$\theta = \theta_{mi} + g(S) . \tag{7}$$

Since the function $\sigma = \sigma(\theta)$ is given (fig. 1), can write this function in term of σ using (6), i. e. $\sigma = \sigma(S)$. Linearization of this function gives the expression (fig. 2)

$$\sigma = bS \; ; \; b = \tan\varphi = \frac{\sigma_g}{S_{gi}} , \tag{8}$$

where σ_g is given electrical conductivity at the transition from metallic arc phase to gaseous arc phase, when $\theta = \theta_{gi}$, and

$$S_{gi} = \int_{\theta_{mi}}^{\theta_{gi}} \lambda(\theta) d\theta . \tag{9}$$

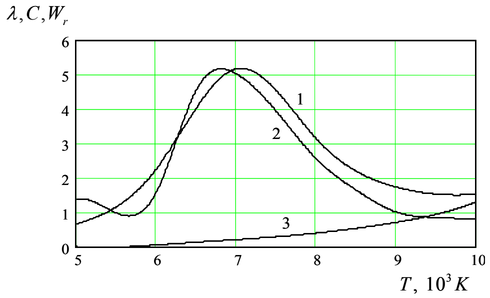


Fig. 1. Temperature dependence of λ, σ and W_r: 1 — λ, $Wm^{-1} K^{-1}$; 2 — C, $\cdot 10^2$ $Jm^{-3} K^{-1}$; 3 — W_r, $\cdot 10^{11}$ Wm^{-3} [6].

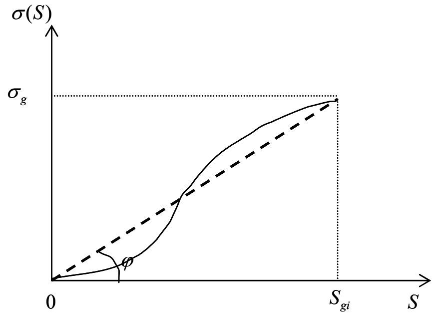


Fig. 2. Linear approximation of $\sigma(S)^{gi}$.

Substituting (8) in (7) and using notation

$$r = \frac{x}{E\sqrt{b}}; \; \tau = \frac{C}{\lambda E^2 b}, \tag{10}$$

can write the equation with respect to σ

$$\tau \frac{\partial \sigma}{\partial t} = \frac{\partial^2 \sigma}{\partial x^2} + \frac{1}{x}\frac{\partial \sigma}{\partial x} + \sigma . \tag{11}$$

It should be noted that τ can be considered as constant because the thermal diffusivity $a^2 = C/\lambda$ is approximately constant (fig. 1). The domain for this equation is $0 < x < x_0$, where $x_0 = r_A E\sqrt{b}$.

The boundary conditions (2)—(4) transform to the type

$$\sigma(x,0) = F(x) \tag{12}$$

with

$$F(x) = b \int_{\theta_{mi}}^{f(x/E\sqrt{b})} \lambda(\theta) d\theta , \; \sigma(x_0, t) = 0 . \tag{13}$$

The solution of the problem (11)—(13) can be found in the form of Fourier-Bessel series

$$\sigma(x,t) = \sum_{n=1}^{\infty} C_n \exp[-(k_n^{\,2} - 1)t/\tau] J_0(k_0 x) \; , \tag{14}$$

where

$$C_n = \frac{2}{x_0^{\,2} J_1^{\,2}(\mu_n)} \int_0^{x_0} F(x) J_0(k_n x) x dx \, , \; k_n = \mu_n / x_0 ,$$

and μ_n are roots of the Bessel function: $J_0(\mu_n) = 0, \quad n = 1,2,3,...$.
For approximation (3)

$$\sigma(x,0) = \sigma_0 J_0(\mu_1 x / x_0)$$

and the solution (14) takes the simple form

$$\sigma(x,t) = \sigma_0 \exp[-(\frac{\mu_1^{\,2}}{x_0^{\,2}} - 1)t/\tau] J_0(\mu_1 x / x_0) ,$$

Taking into account (10), we get finally the expression for arc electrical conductivity in the form

$$\sigma(r,t) = \sigma_0 \exp[-(\mu_1^2 - E^2 b r_A^2)\frac{a^2 t}{r_A^2}] J_0(\mu_1 r / r_A) . \tag{15}$$

The arc temperature can be found now from the expressions (5) and (8).
Let us introduce the criterion of arcing

$$\xi = E^2 b r_A^2 - \mu_1^2 . \tag{16}$$

We should distinguish three cases (fig. 3):
1) $\xi > 0$. Rise of arc conductivity, power and temperature due to Joule heating.
2) $\xi = 0$. Maximum value of arc conductivity and power.
3) $\xi < 0$. Arc conductivity, power and temperature decrease, thus the arc should extinguish.

Interaction between arc and contact surface

At the first stage of contact opening $\xi > 0$ and then changes the sign. To find the critical point $\xi = 0$ we need to know the dynamics of arc radius r_A, which expands during arcing. Then using formula

$$E = \frac{I}{\pi r_A^2 \sigma} \tag{17}$$

and the expression (16) we can find the critical time $t = t_{cr}$ at which $\xi = 0$. For this purpose we consider the region D_A occupied by arc interacting with contact surface (fig. 4). This interaction results into phase transformations of contact material and formation of three zones:
1. The zone of evaporated material:

$$D_b : 0 \le r \le r_b(t),\ \ 0 \le z \le \sigma_b(r,t) .$$

2. The zone of melted material:

$$D_m : \sigma_b(r, t) \le z \le \sigma_m(r, t),\ \text{ if }\ 0 \le r \le r_b(t),\ 0 \le z \le \sigma_m(r, t),\ \text{ if }\ r_b(t) \le r \le r_m(t).$$

3. The solid zone:

$$D_s : \sigma_m(r,t) \le z \le \infty,\ \text{ if }\ 0 \le r \le r_m(t),\ \ 0 \le z \le \infty\ \text{ if }\ r_m(t) \le r \le \infty .$$

The contact temperature $T_C(r,z,t)$ can be presented as the sum

$$T_C(r,z,t) = T_J(r,z,t) + T_S(r,z,t) , \tag{18}$$

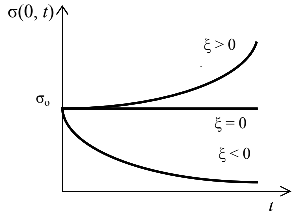
where $T_J(r,z,t)$ and $T_S(r,z,t)$ are the temperature components due to volumetric Joule heating and due to surface arc flux heating respectively. The expression for calculating of the first component is given above. It can be shown that the Joule component $T_J(r,z,t)$ is important at the pre-arcing stage only, and it can be neglected after arc ignition. The expression for the second component can be found similarly in the form

$$T_S(r,z,t) = \int_0^t dt_1 \int_0^\infty [P_c(r_1,t_1) - P_b(r_1,t_1) - P_m(r_1,t_1)] G(r,r_1,z,t-t_1) r_1 dr_1 .$$

(19)

Here $P_c(r,t)$ is the total heat flux (power per unit area) entering the contact surface during arcing, $P_b(r,t)$ and $P_m(r,t)$ are portions of this flux consumed for

Fig. 3. Evolution of arc conductivity.



evaporation and melting of contact material, which can be found by the expressions

$$P_b(r,t) = L_b \gamma \frac{\partial \sigma_b(r,t)}{\partial t}\,; \; P_m(r,t) = L_m \gamma \frac{\partial \sigma_m(r,t)}{\partial t}\,; \quad (20)$$

where L_b and L_m are specific heat for evaporation and melting, γ is density of contact material.

It reasonable to assume that the isothermal surfaces $z = \sigma_b(r,t)$ and $z = \sigma_\mathrm{m}(r,t)$ are ellipsoids of revolution that can be found from the equations

$$\frac{r^2}{r_b(t)^2} + \frac{z^2}{z_b(t)^2} = 1; \qquad \frac{r^2}{r_m(t)^2} + \frac{z^2}{z_m(t)^2} = 1\,;$$

in other words

$$\sigma_b(r,t) = z_b(t)\sqrt{1 - r^2 / r_b(t)^2}\,; \;\; \sigma_\mathrm{m}(r,t) = z_m(t)\sqrt{1 - r^2 / r_m(t)^2}\;. \qquad (21)$$

The functions $r_b(t)$, $z_b(t)$, $r_m(t)$ and $z_m(t)$ should be found from the equations

$$T_\mathrm{C}(r_b(t),0,t) = T_b\,; \;\; T_\mathrm{C}(0,z_b(t),t) = T_b\,; \;\; T_\mathrm{C}(r_m(t),0,t) = T_m\,;$$
$$T_\mathrm{C}(0,z_m(t),t) = T_m\,, \qquad (22)$$

where T_m is the melting temperature of the contact material.

If the heat fluxes $P_c(r,t)$ $P_b(r,t)$, $P_m(r,t)$ obeys the normal Gauss's radial distribution

$$P_c(r,t) = P_c(t)\exp(-\frac{r^2}{r_A(t)^2})\,; \;\; P_b(r,t) = P_b(t)\exp(-\frac{r^2}{r_A(t)^2})\,;$$
$$P_m(r,t) = P_m(t)\exp(-\frac{r^2}{r_A(t)^2})\,, \qquad (23)$$

then the integral with respect to *r* in the formula (19) can be calculated and the expression for the contact temperature becomes more simple form

$$T_S(r,z,t) = \frac{a}{\lambda\sqrt{\pi}} \int_0^t \frac{[P_c(t_1) - P_b(t_1) - P_m(t_1)] r_A(t_1)^2}{[r_A(t_1)^2 + 4a^2(t-t_1)]\sqrt{t-t_1}} \exp[-\frac{z^2}{4a^2(t-t_1)} - \frac{r^2}{r_A(t_1)^2 + 4a^2(t-t_1)}] d\tau. \quad (24)$$

The heat flux $P_c(t)$ should be calculated taking into account positive components due to arc radiation, electron (or ion) bombardment of anode (cathode) contact surface, inverse electrons from the arc column, and negative components due to power losses for evaporation, radiation, electron emission cooling and heat conduction inside the contact body. The expressions for all these components can be found in the paper [7]. However the model in considered case can be simplified because the information about current, voltage and displacement is available from experiment. Therefore it is more convenient to calculate power generated by arc W_A directly from the measured values of arc voltage $U_A(t)$, arc current $I_A(t)$ and then arc heat flux entering contact is

$$P_c(t) = \frac{I_A(t)\cdot U_A(t)}{2\pi r_A{}^2(t)} = \frac{P_A(t)}{2\pi r_A{}^2(t)}\,. \qquad (25)$$

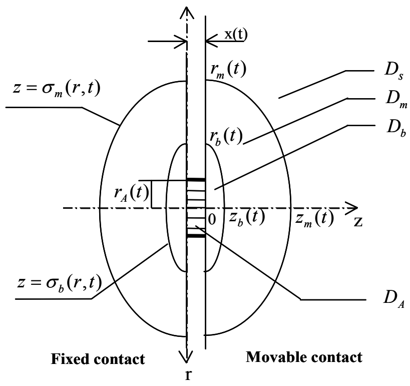


Fig. 4. The arc and contacts geometry: arc region D_A, evaporated D_b, melted D_m and solid D_s zone.

This expression is the final equation, which enables in the aggregate with other cited above equations to calculate dynamics of contact melting, evaporation, arc radius $r_A(t)$ and arc power $P_A(t)$.

Fig. 5 and fig. 6 depict dynamics of arc power and temperature for AgCdO contacts calculated using above considered model at the conditions: supplied voltage $U_0 = 14$ V, current $I_0 = 20$ A, inductance $L = 47{,}5$ mH, opening velocity $V = 0{,}2$ m/s [2]. One can see that critical time in this case is $t_{cr} = 10$ ms, however the maximum of arc temperature occurs a little bit later, at $t = 15$ ms due to thermal inertia.

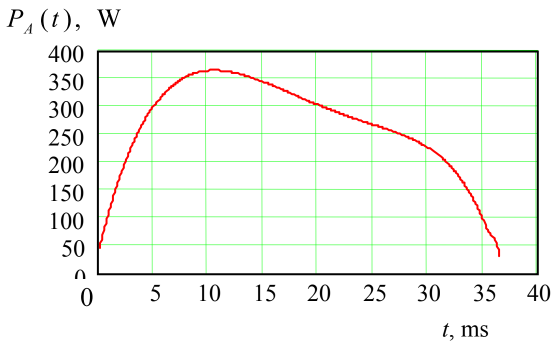


Fig. 5. Dynamics of arc power $P_A(t)$.

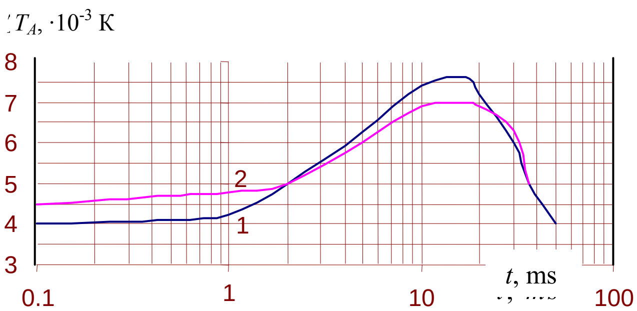


Fig. 6. Arc temperature: 1 — experimental data [2]; 2 — calculation.

Transition from metallic arc phase to gaseous arc phase
Temperature field and erosion

The duration of metallic phase is very short, therefore the arc thickness is still small and above considered model can be applied to describe the transition from metallic to gaseous phase if we replace all parameters of metallic vapours by parameters of gaseous vapours. Dynamics of this transitions is represented in fig. 7. One can see, that at the first stage of arcing, when the contact gap does not exceed 20 µm, anode temperature rises very sharp in comparison with cathode temperature.

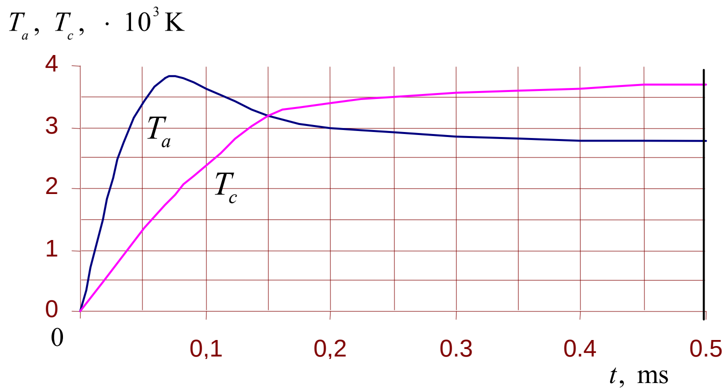


Fig. 7. Dynamics of anode and cathode temperature at the centre of arc root.

It can be explained by the fact that in a short arc, which length is comparable with the length of ionization zone, electron temperature T_e is much greater than ion temperature T_i, therefore kinetic energy of electrons bombarding anode, $\frac{3}{2}\frac{kT_e}{e}j_e$, exceeds significantly kinetic energy of ions entering cathode, $\frac{3}{2}\frac{kT_i}{e}j_i$. Moreover, calculation shows that in this range of contact gap electron component of current density j_e is much greater than ion component j_i, that is an additional reason for anode overheating and material transfer from anode to cathode. However intensive evaporation from anode and increasing of anode arc spot radius, that entails decreasing of current density, cause anode cooling and decreasing of its temperature, while cathode temperature continues to increase. The point of intersection of anode and cathode temperature occurring at $t_{ac} = 0{,}15$ ms corresponds to change the direction of material transfer for inverse and to beginning of compensation arc stage, which continues up to $t_1 = 1{,}8$ ms and transforms then into cathodic stage (fig. 8).

Calculation enables to conclude that cathodic arc stage begins in metallic phase with temperature about 4700 K, that is less than threshold ionisation, however transition to gaseous phase occurs just at $t_1 = 2$ ms. Results of calculated erosion given in fig. 8 are evidence of the fact, that the main portion of erosion in inductive circuits occurs in gaseous phase. Calculated values for

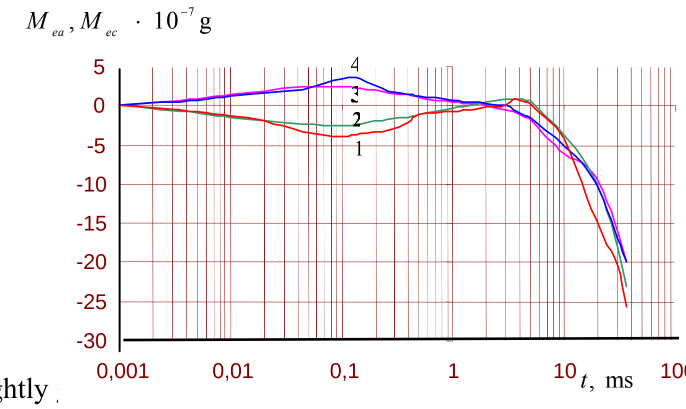


Fig. 8. Anode and cathode mass transfer. From bottom to top: 1, 2 — M_{ec} (calculated and experiment [2]); 3. 4 — M_{ea} (experiment [2] and calculated).

metallic phase are slightly cycling phenomenon, i. e. re-deposition of evaporated material, which is ignored in the mathematical model.

Influence of inductance on arc duration

Similar calculations were carried out for different values of inductance in the range from 1 to 400 mH. It was found that arc duration increases proportionally inductance and depends on current at relatively small values of inductance (fig. 9). However for inductance greater than 10 mH this dependence becomes negligible. This result correlates with experimental data observed in work [2].

Increasing of arc duration with inductance occurs on account of enlarging of gaseous arc phase, while variation of metallic phase is relatively small. The same conclusion may be proposed for increasing of erosion. However further increasing of inductance up to a few hundred millihenry in the range of low current leads to decrease arc duration and erosion due to arc-to-glow transformation, which is considered below.

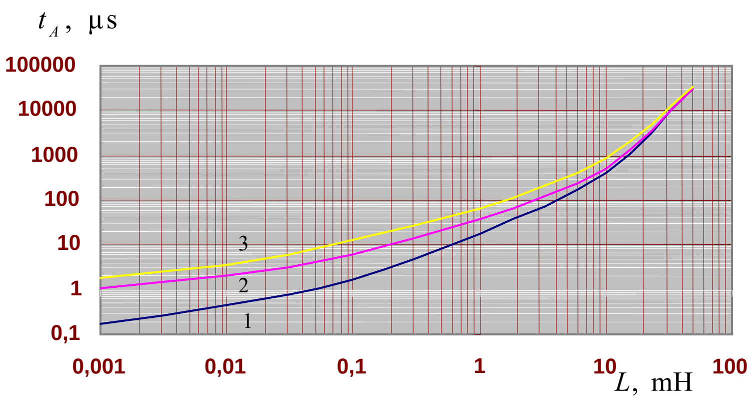


Fig. 9. Arc duration versus inductance and current 1 — I_o = = 0,6 A, 2 — I_o = = 1 A and I_o = = 20 A.

Conclusions

Dynamics of temperature and electrical conductivity can be described and analysed satisfactorily by the model based on a non-linear problem for axisymmetric heat equation.

The new criterion of arc stability and instability is introduced, which enables one to find arc duration dependently on given circuit parameters and properties of contact material.

The mathematical model of phase transformations inside electrodes during arcing is elaborated which describes dynamics of arc erosion in metallic and gaseous arc phases.

Increasing of arc duration and erosion with inductance occurs on account of enlarging of gaseous arc phase, while variation of metallic phase is relatively small.

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Математическая модель температуры и электрической проводимости дуги в металлической и газовой фазах

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Получен новый критерий стабилизации дуги, который позволяет найти зависимость продолжительности металлической фазы дуги от заданных параметров цепи и свойств контактного материала. Предложена математическая модель фазовых переходов внутри электродов в процессе горения дуги, которая описывает динамику дуговой эрозии в металлической и газовой фазах дуги. Установлено, что причиной возрастания продолжительности дуги и эрозии контактов, происходящего с ростом индуктивности цепи, является расширение газообразной фазы дуги относительно ее металлической фазы.

***Ключевые слова**: математическая модель, дуговая эрозия, дуговые фазы.*

Механічна модель температури та електричної провідності дуги в металічній і газовій фазах

С. М. Харін, Ю. Р. Шпади, А. Т. Кулахметова

Отримано новий критерій стабілізації дуги, який дозволяє знайти залежність тривалості металічної фази дуги від заданих параметрів ланцюга і властивостей контактного матеріалу. Запропоновано математичну модель фазових переходів в середині електродів в процесі горіння дуги, яка описує динаміку дугової ерозії в металічній і газовій фазах дуги. Встановлено, що причиною зростання тривалості дуги і ерозії контактів, яке має місце із зростанням індуктивності ланцюга, є розширення газоподібної фази дуги відносно її металічної фази.

***Ключові слова**: математична модель, дугова ерозія, дугові фази.*