

INCLUDING THE OWN FIELDS OF QUANTUM EMITTERS IN DESCRIBING GENERATION REGIMES

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The paper discusses three different modes of electromagnetic field generation by an ensemble of quantum emitters placed at the radiation wavelength in the one-dimensional case. The excitation of the resonator field is considered, which, as a rule, is determined by the geometry of the system, with and without taking into account the eigenfields of the emitters. The superradiance mode of the same ensemble of emitters is also analyzed. Since the main indicator of generation is the level of energy output from the system, the position of the maximum of this indicator determines the operating point of the device. Taking into account the intrinsic field of the emitters enhances the generation intensity and significantly changes the position of the operating point.

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INTRODUCTION

Let us consider the excitation of electromagnetic waves in limited systems, which impose the choice of wavelengths of the modes of the wave packet and form the spatial structure of the field in the resonators. The dominant frequency in the radiation spectrum will be the frequency of the system closest to the natural frequency of the emitter. Let the resonator be filled with a medium representing a two-level system of dipoles. To describe such a system, we use the semiclassical model of interaction between the field and particles (see, for example, [1 - 3]). In this case, the medium is represented quantum mechanically, and the field in the classical representation. As is known, the radiation intensity of a quantum emitter is determined by the amplitude of the electromagnetic field, and the phase of the radiation corresponds to the phase of the field in its volume. If the wave functions of the emitters do not overlap, then their interaction is possible only due to the electromagnetic field (see, for example, [1 - 3]).

When describing the resonator excitation process, in addition to the resonator field, one should take into account the own field of the emitters, which at the initial moment is spontaneous and only the synchronization of the emitters leads to a greater coherence of the total radiation. This phenomenon in the absence of a resonator is called collective spontaneous radiation, or superradiance. To accelerate and stimulate the process of synchronization of emitters in the superradiance regime, a sufficiently large initiating field is used [4], but, of course, it is much smaller than the achievable field in the developed generation regime. In the resonator, the role of the initiating field is taken over by the resonator field. A similar problem for a system of oscillators in the classical representation was considered in [5].

The purpose of this paper is to compare the fields generated by open systems in the cases of 1) excitation of only the resonator field, 2) excitation of only the self-radiation field of active emitter elements without the presence of a resonator, that is, the superradiance field, and 3) joint excitation of the resonator field, taking into account the intrinsic fields of emitters. Let us show that, on the one hand, it is necessary to take into account the intrinsic field of the emitters, because it qualitatively

affects all the characteristics of the mode of oscillation generation. On the other hand, a comparison of the achievable amplitudes of the resonator field and the superradiance field for one system of emitters showed that they are of the same order [6]. This is an additional argument for the need to take into account the eigenfields of the emitters in the resonator.

1. DESCRIPTION OF THE PROCESSES OF GENERATION BY THE SYSTEM OF QUANTUM EMITTERS

Consider a one-dimensional model for perturbations of the electric field E , polarization P and population inversion μ slowly changing with time, describing the excitation of electromagnetic oscillations in a two-level active medium, the equations of which can be represented as ([1, 6, 7])

$$\frac{\partial^2 E}{\partial t^2} + \delta \frac{\partial E}{\partial t} - c^2 \frac{\partial^2 E}{\partial z^2} = -4\pi \frac{\partial^2 P}{\partial t^2}, \quad (1)$$

$$\frac{\partial^2 P}{\partial t^2} + \gamma_{ab} \frac{\partial P}{\partial t} + \omega^2 \cdot P = -\frac{2\omega |d_{ab}|^2}{\hbar} \mu E, \quad (2)$$

$$\frac{\partial \mu}{\partial t} + \gamma_1 (\mu - \mu^0) = \frac{2}{\hbar \omega} \langle E \frac{\partial P}{\partial t} \rangle, \quad (3)$$

where the transition frequency between the levels ω corresponds to the field frequency, we neglect the inversion relaxation due to external causes, δ is the field absorption decrement in the medium, d_{ab} is the matrix element of the dipole moment (more precisely, its projection on the direction of the electric field), $\mu = n \cdot (\rho_a - \rho_b)$ is the population difference per unit volume, ρ_a and ρ_b are the relative populations levels in the absence of a field, γ_{ab} is the width of the spectral line, $\gamma_1 = \gamma_a = \gamma_b$ is the reciprocal of the lifetime of the levels, n is the density of the dipoles of the active medium. Here, the line width is inversely proportional to the lifetime of the states, which is due to relaxation processes. The fields are presented in the form $E = \text{Re}[E(t) \cdot e^{-i\omega t}]$ and $P = \text{Re}[P(t) \cdot e^{-i\omega t}]$, where $E(t), P(t)$ – slowly changing amplitudes. Wherein $\langle E^2 \rangle = |E(t)|^2 / 2$.

1.1. SUPERRADIATION

Quantum emitters are located along the Oz axis, the field emitted by them is directed along the Ox . Their total number $M = b \cdot n$ is divided into S particles, n is the density, b is the length of the system, M/S is the number of emitters in one modeling particle, z_i ($i = 1, \dots, S$) are the coordinates of the particles distributed evenly along the length of the system.

The polarization of one molecule [1] is $p = (d_{ba}\rho_{ab} + d_{ab}\rho_{ba})$. Equations (1)-(3) use the polarization density (polarization per unit volume) $P = np = n(d_{ba}\rho_{ab} + d_{ab}\rho_{ba})$.

The field equation for a polarized molecule (the molecule at the point z_0) has the form

$$\frac{\partial^2 E_x}{\partial t^2} - c^2 \frac{\partial^2 E_x}{\partial z^2} = 4\pi\omega^2 p(t) \cdot e^{-i\omega t} \cdot \delta(z - z_0). \quad (4)$$

From equation (4) for a slowly varying amplitude of the field emitted by one molecule, one obtains

$$E(z, t) = i \cdot 2\pi \cdot p \cdot \omega \cdot c^{-1} \cdot e^{ik|z-z_0|}. \quad (5)$$

Then, the amplitude of the field emitted by the system (field of superradiance) is equal to

$$E_{\text{sup}}(z, t) = \frac{i \cdot 2\pi \cdot \omega}{\delta} \cdot \frac{1}{S} \sum_{s=1}^S P_s(t) \cdot e^{ik|z-z_s|}. \quad (6)$$

Here $\delta = c/b$, that is, is actually the ratio of the energy flow from the system to its total energy.

1.2. WAVEGUIDE/RESONATOR FIELD

Equation (1) for one polarized molecule at the point z_0 takes the form

$$\frac{\partial^2 E}{\partial t^2} + \delta \frac{\partial E}{\partial t} - c^2 \frac{\partial^2 E}{\partial z^2} = 4\pi\omega^2 p e^{-i\omega t} \delta(z - z_0). \quad (7)$$

Assuming that the waveguide field has the form

$$E = E_+(t) \cdot e^{-i\omega t + ikz} + E_-(t) \cdot e^{-i\omega t - ikz}, \quad (8)$$

and, consequently, for slowly varying field amplitudes we have

$$\frac{\partial E_{\pm}(t)}{\partial t} + \delta E_{\pm}(t) = i \cdot 2\pi \cdot \omega \cdot p \cdot \delta(z - z_0) e^{\mp ikz}. \quad (9)$$

For a system of particles, we obtain the equations for the amplitudes of the resonator field

$$\frac{\partial E_{\pm}(t)}{\partial t} + \delta E_{\pm}(t) = i \cdot 2\pi \cdot \omega \cdot \frac{1}{S} \sum_s P_s \cdot e^{\mp ikz_s} \quad (10)$$

and, in fact, the field itself

$$E_{\text{res}}(z, t) = E_+(t) \cdot e^{ikz} + E_-(t) \cdot e^{-ikz}. \quad (11)$$

1.3. EQUATION FOR AMPLITUDE POLARIZATIONS AND INVERSIONS

From equation (2) for a slowly varying polarization amplitude, one can easily obtain

$$\frac{\partial P(z, t)}{\partial t} + \gamma_{ab} P(z, t) = -i \frac{|d_{ab}|^2}{\hbar} \mu(z, t) \cdot E(z, t). \quad (12)$$

For a slowly varying inversion, from equation (3) we obtain

$$\frac{\partial \mu}{\partial t} + \gamma_1 (\mu - \mu^0) = \frac{i}{2\hbar} (EP^* - E^*P). \quad (13)$$

2. MODELS OF GENERATION BY THE QUANTUM EMITTER SYSTEM

Next, we will compare the fields generated by open systems in the cases of 1) excitation of only the resonator field, 2) excitation of only the eigenradiation field of active emitting elements, that is, the superradiance field, and 3) joint excitation of the resonator field, taking into account the eigenfields of active emitting elements. Let us show that, in addition to the resonator field, it is necessary to take into account the self-radiation field of oscillators, because it qualitatively affects all characteristics of the oscillation generation mode. Let us rewrite equations (6), (10)-(13) in dimensionless form using the following notation:

$$E_0 = \sqrt{2\pi\omega\hbar}\mu_0, \quad P_0 = |d_{ab}| \cdot \mu_0, \quad \gamma_0 = \frac{|d_{12}| \cdot E_0}{\hbar},$$

$$\Theta = \delta / \gamma_0, \quad \mathbf{E} = E / E_0, \quad \mathbf{P} = P / P_0, \quad \hat{\mathbf{P}} = i\mathbf{P},$$

$$Z = kz / 2\pi, \quad \tau = \gamma_0 t.$$

Assuming additionally that $\gamma_{ab} / \gamma_0 \ll 1$ and $\gamma_1 / \gamma_0 \ll 1$, we obtain the following system of equations.

$$\frac{\partial \hat{\mathbf{P}}}{\partial \tau} = \mathbf{M} \cdot \mathbf{E}, \quad (14)$$

$$\frac{\partial \mathbf{M}}{\partial \tau} = -\frac{1}{2} (\mathbf{E}\hat{\mathbf{P}}^* + \mathbf{E}^*\hat{\mathbf{P}}), \quad (15)$$

$$E_{\text{sup}}(Z, \tau) = \frac{1}{\Theta} \cdot \frac{1}{S} \sum_{s=1}^S \hat{\mathbf{P}}_s(\tau) \cdot e^{i2\pi Z - Z_s}, \quad (16)$$

$$E_{\text{res}}(Z, \tau) = E_+(\tau) \cdot e^{i2\pi Z} + E_-(\tau) \cdot e^{-i2\pi Z}, \quad (17)$$

$$\frac{\partial E_{\pm}}{\partial \tau} + \Theta E_{\pm} = \frac{1}{S} \sum_s \hat{\mathbf{P}}_s \cdot e^{\mp i2\pi Z_s}. \quad (18)$$

As already noted, the system is divided into S large particles, so equations (14), (15) are solved for each particle, in formulas (16), (17) the fields are also calculated for each particle.

In all three cases, equations (14), (15) are solved, but the form of the fields is different.

In the first case (excitation of only the resonator field) $\mathbf{E} = E_{\text{res}}(Z, \tau)$, equations (18) are solved, the field is found by formula (17).

In the second case (excitation of only the self-radiation field of active emitting elements), the field is taken in the form

$$\mathbf{E} = E_{\text{sup}}(Z, \tau) + E_{\text{ext}}(Z), \quad (19)$$

$$E_{\text{ext}}(Z) = E_{0+} \cdot e^{i2\pi Z} + E_{0-} \cdot e^{-i2\pi Z}, \quad (20)$$

where $E_{\text{ext}}(Z)$ is the external, process-initiating field.

In the third case (joint excitation of the resonator field, taking into account the eigenfields of the oscillators) – $\mathbf{E} = E_{\text{sup}}(Z, \tau) + E_{\text{res}}(Z, \tau)$.

3. NUMERICAL SIMULATION RESULTS

A comparison was made of the three above models for describing generation. For all three models, calculations were carried out under the same initial conditions and different values of the loss factor Θ , which defined

as the ratio of the energy flow from the system to the total energy in its volume.

Initial conditions: the length of the system is equal to one wavelength ($Z_j \in (0;1)$); number of particles $S=800$; initial inversion $M_i(0)=1$; the initial polarization has an amplitude of 0.1 and random phases φ ($P_i(0)=0.1 \cdot \exp(i\varphi)$); $E_{0\pm}=0.01$ are the initial values (15) for the amplitudes of the resonator field, as well as constants for the external field (16).

Figs. 1-3 show for three modes the dependence on the level of energy loss Θ of the maximum intensity of the field in the volume of the emitter $|E_{\max}|^2$ (see Fig.

1), the rate of energy output $\Theta|E_{\max}|^2$ (see Fig. 2) and the inverse time (increment) of the process

$$\gamma = \left(\frac{1}{|E|^2} \frac{d|E|^2}{d\tau} \right)_{\max}$$

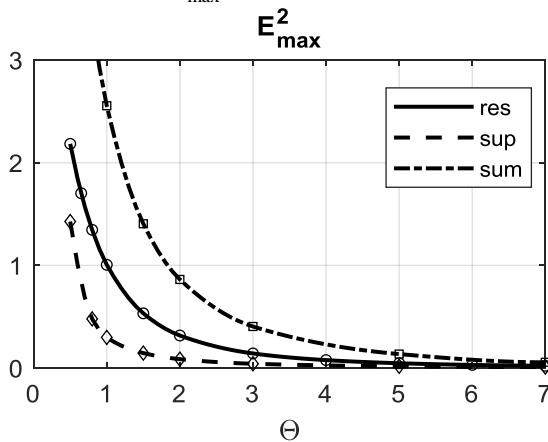


Fig. 1. Dependence on Θ of the maximum field intensity $\propto |E|^2$ for the cases: solid line (res) – generation of the resonator field without taking into account the radiation field of oscillators; dotted line – superradiance mode (sup); dash-dotted line – generation of the resonator field together with the radiation field of oscillators

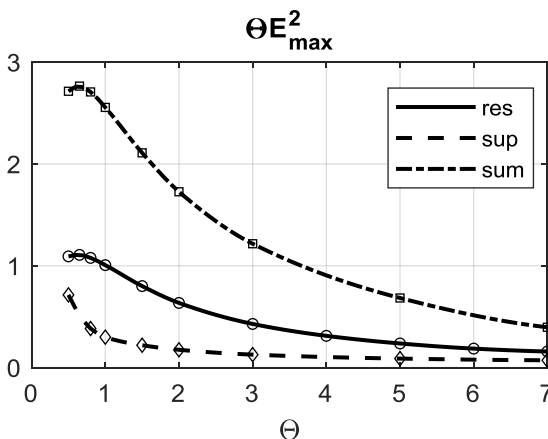


Fig. 2. Dependence on Θ of the rate of energy output for the cases: solid line (res) – generation of the resonator field without taking into account the radiation field of oscillators; dotted line – superradiance mode (sup); dash-dotted line – generation of the resonator field together with the radiation field of oscillators

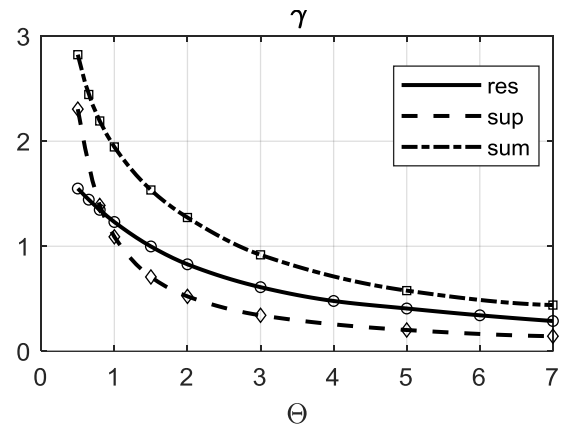


Fig. 3. Dependence on Θ of the reciprocal time (increment) of the process: $\propto |E|^{-2} \cdot (d|E|^2/dt)$ solid line (res) – generation of the resonator field without taking into account the radiation field of oscillators; dotted line – superradiance mode (sup); dash-dotted line – generation of the resonator field together with the radiation field of oscillators

The solid line marks the resonator mode without taking into account the influence of oscillator fields on the generation process, the superradiance mode is represented by dotted lines in the graphs, and the dash-dotted line in the graphs corresponds to the case of cavity excitation with allowance for the influence of oscillator fields.

It is important to note that taking into account the eigenfields of the emitters when the resonator field is excited leads to a higher field intensity and a higher rate of energy output and reduces the characteristic time of the process development. The maximum rate of energy output for this case occurs at $\Theta=0.7$.

Figs. 4, 5 show the time dependences of the field at the ends of the system and the maximum in the volume at $\Theta=1$ for the waveguide model (see Fig. 4) and the model, which takes into account the own fields of the emitters when the resonator field is excited (see Fig. 5).

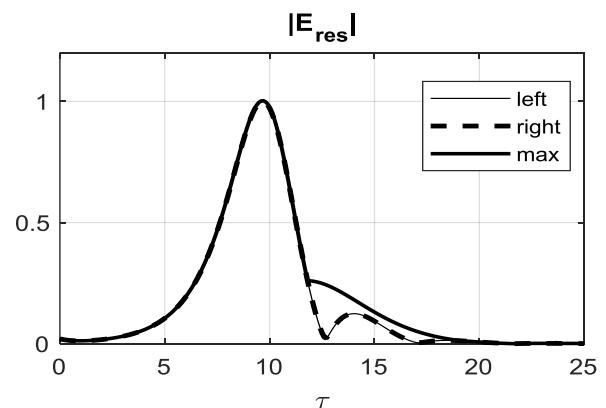


Fig. 4. Time dependence of the fields on the left and right edges $E_{res}(Z=0)$ (left), $E_{res}(Z=1)$ (right) and the maximum in the volume (max) at $\Theta=1$ for the resonator model without taking into account the eigenfields of the emitters

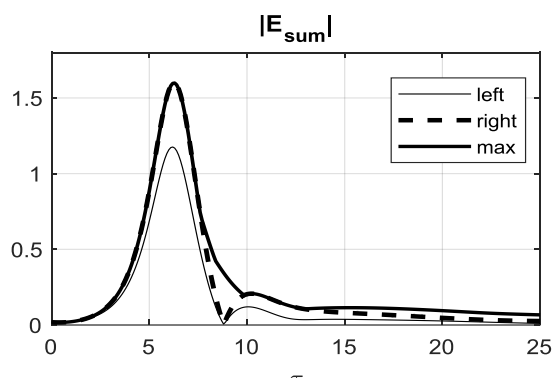


Fig. 5. Time dependence of the fields on the left and right edges $E_{sum}(Z=0)$ (left), $E_{res}(Z=1)$ (right) and the maximum in the volume (max) at $\Theta=1$ for the model of joint excitation of the resonator field, taking into account the eigenfields of the emitters

CONCLUSIONS

The generation of electromagnetic oscillations in a resonator filled with a two-level system of dipoles is considered. The field of the resonator is determined by the geometry of the system, and the emitters exchange energy with this field without affecting its spatial distribution. In addition, the emitters generate their own fields, and even in the absence of a resonator, the intensity of the total fields of the emitters in the superradiance mode is comparable to the generation efficiency in the resonator [5]. Therefore, when describing the excitation of a resonator filled with a two-level system of dipoles, in addition to the resonator field, it is necessary to take into account the sum of the eigenfields of the emitters. It turned out that the joint consideration of the resonator field and the eigenfields of the emitters leads to a change in the characteristics of the generation mode compared to the case when the eigenfield was not taken into account.

УРАХУВАННЯ ВЛАСНИХ ПОЛІВ КВАНТОВИХ ВИПРОМІНЮВАЧІВ ПРИ ОПИСІ РЕЖИМІВ ГЕНЕРАЦІЇ

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Обговорюються три різні режими генерації електромагнітного поля ансамблем квантових випромінювачів, розміщених на довжині хвилі випромінювання в одновимірному випадку. Розглянуто збудження резонаторного поля, яке, як правило, визначається геометрією системи, з урахуванням та без урахування власних полів випромінювачів. Аналізується також режим надвипромінювання такого ж ансамблю випромінювачів. Оскільки основним показником генерації є рівень виведення енергії із системи, то положення максимуму цього показника визначає робочу точку пристрою. Врахування свого поля випромінювачів посилює інтенсивність генерації і значно змінює становище робочої точки.

Since the main indicator of generation is the level of energy output from the system (here it is $\Theta|E_{max}|^2$), the position of the maximum of this indicator determines the operating point of the device. It can be seen that taking into account the self-field of the emitters enhances the intensity and significantly changes the position of the operating point.

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