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DRIFT-KINETIC EQUATIONS IN MAGNETIZED CURRENT-CARRYING PLASMAS

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Kinetic models of magnetized current-carrying plasma have been developed to study the influence of magnetic drift effects on the wave-particle interactions in tokamaks and cylindrical plasma columns. The drift-kinetic equations are derived for the perturbed distribution functions of trapped and untrapped particles in a two-dimensional axisymmetric toroidal plasma, taking into account their bounce oscillations and the finite orbit-widths of their banana trajectories.

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INTRODUCTION

To study the influence of finite Larmor radius effects on the resonant wave-particle interactions in magnetized plasmas one should use the kinetic dielectric tensor accounting for the particle drifts from the magnetic surfaces under their moving along the magnetic field lines. Corresponding kinetic wave theory [1, 2] should be based on the solution of the linearized Vlasov equations or the drift-kinetic equations [3 - 6] for the perturbed distribution functions of ions and electrons.

Usually, the response of a collisionless plasma to global electromagnetic perturbations of an axisymmetric toroidal equilibrium is described by the perturbed distribution functions of charged particles expressed in terms of their linearized guiding center Littlejohn Lagrangian [4, 5], adopting a variational formulation for the guiding center motion and drift effects.

In this work, the drift-kinetic equation is derived directly from the Vlasov equation using the Fourier expansion of the perturbed distribution functions of plasma particles over the polar angle (gyration angle) in velocity space for low-frequency wave processes in axisymmetric tokamaks with circular magnetic surfaces and large aspect ratio, up to first-order corrections in the magnetization parameters.

1. PLASMA MODEL

To describe the stationary magnetic field $\mathbf{H}_0$ in an axisymmetric tokamak with a circular cross-section, we use the system of quasi-toroidal coordinates (r,θ,φ), associated with cylindrical (ρ,φ,z) as follows:

$$\rho = R_0 + r\cos\theta, \quad \varphi=\varphi, \quad z=-r\sin\theta, \qquad (1)$$

where R_0 is the large radius of the torus, Fig. 1, determined by the radius of the magnetic axis; r is the radius of a magnetic surface (magnetic surface equation: $r = const$, $0 \le r \le a$, a is the small plasma radius); θ is the poloidal angle, measured by small azimuth in the cross-section of the torus, $0 \le \theta \le 2\pi$; φ is the toroidal angle measured along the major azimuth in the horizontal section of the torus, $0 \le \varphi \le 2\pi$.

In an axisymmetric 2D tokamak, the plasma configuration is homogeneous in φ. As a result, the equilibrium field $\mathbf{H}_0$ and other steady-state plasma-field parameters do not depend on φ. Moreover, for this reason, the single-mode harmonic approximation is valid for the perturbations, $\sim \exp(in\varphi)$, where integer n is the toroidal mode number. In contrast to the usual notation, the angle θ is measured from the outer side of the torus, shorting the formulas related to trapped particles. The components of the equilibrium magnetic field $\mathbf{H}_0 = \{H_{0r}, H_{0\theta}, H_{0\varphi}\}$ for the considered plasma model are determined from the conditions of the absence of magnetic charges $\nabla\cdot\mathbf{H}_0 = 0$, and have the form:

$$H_{0r}=0, \quad H_{0\theta} = \frac{\bar{H}_{0\theta}(r)}{1+\varepsilon\cos\theta}, \quad H_{0\varphi} = \frac{\bar{H}_{0\varphi}(r)}{1+\varepsilon\cos\theta}, \qquad (2)$$

where $\varepsilon = r/R_0$ is the inverse aspect ratio of a considered toroidal magnetic surface $r = const$; and $\bar{H}_{0\theta}(r)$, $\bar{H}_{0\varphi}(r)$ are the amplitude values of poloidal and toroidal magnetic fields there (at $\theta=\pi/2$).

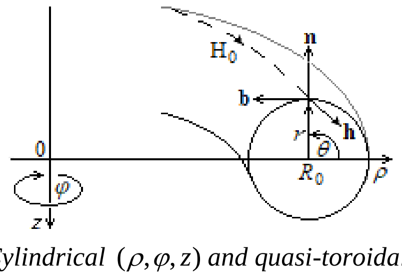

Fig. 1. Cylindrical (ρ,φ,z) and quasi-toroidal (r,θ,φ) coordinates, describing an axisymmetric tokamak with circular magnetic surfaces

In this case, the unit vector along the $\mathbf{H}_0$ field does not depend on the poloidal angle and has projections:

$$\mathbf{h} = \frac{\mathbf{H}_0}{|\mathbf{H}_0|} = \{0, h_\theta, h_\varphi\} = \left\{0, \frac{\bar{H}_{0\theta}}{\sqrt{\bar{H}_{0\varphi}^2+\bar{H}_{0\theta}^2}}, \frac{\bar{H}_{0\varphi}}{\sqrt{\bar{H}_{0\varphi}^2+\bar{H}_{0\theta}^2}}\right\}. \qquad (3)$$

When describing the plasma particle distribution functions in velocity space $F_\alpha(t,\mathbf{r},\mathbf{v})$, it is convenient to use the orthogonal normal, binormal, and parallel velocity components:

$$\mathbf{v}=\upsilon_1\mathbf{n}+\upsilon_2\mathbf{b}+\upsilon_3\mathbf{h}=\upsilon_\perp\cos\sigma\,\mathbf{n}+\upsilon_\perp\sin\sigma\,\mathbf{b}+\upsilon_\parallel\mathbf{h}, \quad (4)$$

where **n** is the unit vector normal (radial) to a magnetic surface, $\mathbf{b}=\mathbf{h}\times\mathbf{n}$ is the binormal to **n** and **h**, Fig. 1; $\upsilon_\parallel$ and $\upsilon_\perp$ are the parallel and perpendicular velocity components, respectively; σ is the gyration angle (or the polar angle in velocity space).

Using the small perturbation method, the plasma particle distribution functions can be found as

$$F_\alpha(t,\mathbf{r},\mathbf{v})=\bar{F}_\alpha(\mathbf{r},\mathbf{v})+f_\alpha(t,\mathbf{r},\mathbf{v}), \quad (5)$$

where $\bar{F}_\alpha(\mathbf{r},\mathbf{v})$ and $f_\alpha(t,\mathbf{r},\mathbf{v})$ are the steady-state and the perturbed distribution functions of ions and/or electrons (α=*i*,*e*), respectively, under the condition: $f_\alpha(t,\mathbf{r},\mathbf{v})<<\bar{F}_\alpha(\mathbf{r},\mathbf{v})$. The steady-state functions $\bar{F}_\alpha(\mathbf{r},\mathbf{v})$ must take into account the presence of a stationary equilibrium current in tokamaks, $\mathbf{j}_0=j_{0\theta}\mathbf{e}_\theta+j_{0\varphi}\mathbf{e}_\varphi$, diamagnetic currents and be self-consistent with the confining magnetic field, $\mathbf{H}_0=H_{0\theta}\mathbf{e}_\theta+H_{0\varphi}\mathbf{e}_\varphi$. In the general case, $\bar{F}_\alpha(\mathbf{r},\mathbf{v})$ can be defined in the velocity space $(\upsilon_\parallel,\upsilon_\perp,\sigma)$ by the Fourier expansion

$$\bar{F}_\alpha(\mathbf{r},\mathbf{v})=\bar{F}_\alpha(r,\theta,\varphi,\upsilon_\perp,\sigma,\upsilon_\parallel)=\sum_{\ell=-\infty}^{+\infty}F_{\ell\alpha}(r,\theta,\varphi,\upsilon_\perp,\upsilon_\parallel)\,e^{-i\ell\sigma}. \quad (6)$$

The harmonics $F_{0\alpha}(r,\theta,\varphi,\upsilon_\perp,\upsilon_\parallel)$ allow us to define the main contribution of plasma particles to the field-aligned stationary current (parallel to $\mathbf{H}_0$),

$$j_{0\parallel}=\mathbf{j}_0\cdot\mathbf{h}=2\pi\sum_\alpha^{e,i}q_\alpha\int_0^\infty\upsilon_\perp d\upsilon_\perp\int_{-\infty}^\infty\upsilon_\parallel F_{0\alpha}(\upsilon_\perp,\upsilon_\parallel)d\upsilon_\parallel. \quad (7)$$

Whereas the harmonics $F_{\pm1\alpha}(r,\theta,\varphi,\upsilon_\perp,\upsilon_\parallel)$ are necessary to describe the diamagnetic currents, connected with the Larmor radius gyration of plasma particles along the helical magnetic field lines and the gradients of their density and temperature.

Further, we consider the simplest pressureless plasma model of a 2D tokamak, where the equilibrium current must be force-free, i.e., the current density must be parallel to the magnetic field: $\mathbf{j}_0\parallel\mathbf{H}_0$ or $\mathbf{j}_0\times\mathbf{H}_0=0$. Describing such plasma models, it is assumed that the steady-state current is created by electrons having the velocity $\upsilon_{0e}=\upsilon_0$, whereas $\upsilon_{0i}=0$ for heavy ions. In this case, according to Ampere's law,

$$(\nabla\times\mathbf{H}_0)\cdot\mathbf{h}=\frac{4\pi}{c}j_{0\parallel}=-\frac{4\pi}{c}n_{0e}e\upsilon_0, \quad (8)$$

where

$$\upsilon_0=-\frac{\kappa_2H_0c}{4\pi n_{0e}e}. \quad (9)$$

Here n_{0e} is the electron density, *e* is the elementary charge, and the magnetic field parameter κ_2 is equal to

$$\kappa_2=\frac{h_\varphi^2}{r}\frac{d}{dr}\left(\frac{rR_0h_\theta}{h_\varphi(R_0+r\cos\theta)}\right), \quad (10)$$

h_θ and h_φ are the poloidal and toroidal projections of unit vector **h** along a helical $\mathbf{H}_0$-field line, Eq. (3).

The steady-state distribution functions for such 2D current-carrying toroidal plasma can be done by the following harmonics, satisfying the Vlasov equation:

$$F_{0\alpha}=\frac{n_{0\alpha}}{\pi^{3/2}\upsilon_{T\alpha}^3}\exp\left(-\frac{\upsilon_\perp^2+(\upsilon_\parallel-\upsilon_{0\alpha})^2}{\upsilon_{T\alpha}^2}\right)\approx F_{M\alpha}\left(1+2\frac{\upsilon_{0\alpha}\upsilon_\parallel}{\upsilon_{T\alpha}^2}\right), \quad (11)$$

$$F_{1\alpha}-F_{-1\alpha}=i\frac{\upsilon_\perp}{\Omega_\alpha}\frac{\partial F_{0\alpha}}{\partial r}-2i\frac{h_\theta^2\upsilon_\parallel\upsilon_\perp\upsilon_{0\alpha}}{r\Omega_\alpha\upsilon_{T\alpha}^2}F_{0\alpha}-2i\frac{h_\varphi^2\upsilon_\parallel\upsilon_\perp\upsilon_{0\alpha}}{R_0\Omega_{0\alpha}\upsilon_{T\alpha}^2}\cos\theta F_{0\alpha},$$

$$F_{1\alpha}+F_{-1\alpha}=-\frac{\upsilon_\perp}{r\Omega_\alpha}\frac{\partial F_{0\alpha}}{\partial\theta}-2\frac{h_\varphi\upsilon_\parallel\upsilon_\perp\upsilon_{0\alpha}}{R_0\Omega_{0\alpha}\upsilon_{T\alpha}^2}\sin\theta\,F_{0\alpha}, \quad (12)$$

where

$$\Omega_\alpha=\frac{\tilde{q}_\alpha}{M_\alpha c}\frac{\sqrt{\bar{H}_{0\varphi}^2+\bar{H}_{0\theta}^2}}{1+\varepsilon\cos\theta}=\frac{\Omega_{0\alpha}}{1+\varepsilon\cos\theta},\quad \upsilon_{T\alpha}^2=\frac{2T_{0\alpha}}{M_\alpha} \quad (13)$$

are the cyclotron (Larmor) frequency and the squared thermal velocity of species α=*i*,*e* plasma particles with the mass M_α, charge $\tilde{q}_\alpha$, and temperature $T_{0\alpha}$. Further, we assume that the drift-current velocity is much less than the thermal velocity, $\upsilon_{0\alpha}<<\upsilon_{T\alpha}$.

2. DRIFT-KINETIC EQUATION

The linearized Vlasov equation [7, 8] for the perturbed plasma particle distribution functions $f_\alpha(t,r,\theta,\varphi,\upsilon_\parallel,\upsilon_\perp,\sigma)$ in the considered tokamak plasma model can be rewritten in the explicit form as

$$\frac{\partial f}{\partial t}-\Omega\frac{\partial f}{\partial\sigma}-\frac{h_\varphi\upsilon_\parallel}{2}\left[h_\varphi\frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\varphi}\right)+\frac{3h_\theta R_0}{r(R_0+r\cos\theta)}\right]\frac{\partial f}{\partial\sigma}-$$
$$\frac{h_\theta\upsilon_\perp\sin\theta}{2(R_0+r\cos\theta)}\hat{V}f+\frac{h_\theta\upsilon_\parallel}{r}\frac{\partial f}{\partial\theta}+\frac{h_\varphi\upsilon_\parallel}{R_0+r\cos\theta}\frac{\partial f}{\partial\varphi}+ \quad (14)$$
$$\sin\sigma\left[\frac{h_\varphi\upsilon_\perp}{r}\frac{\partial f}{\partial\theta}-\frac{h_\theta\upsilon_\perp}{R_0+r\cos\theta}\frac{\partial f}{\partial\varphi}+\frac{h_\varphi\upsilon_\parallel\sin\theta}{R_0+r\cos\theta}\hat{V}f-\right.$$
$$\left.\left(\frac{h_\theta^2\upsilon_\parallel^2}{r\upsilon_\perp}+\frac{h_\varphi^2\upsilon_\perp}{r}+\frac{h_\varphi^2\upsilon_\parallel^2\cos\theta}{\upsilon_\perp(R_0+r\cos\theta)}+\frac{h_\theta^2\upsilon_\perp\cos\theta}{R_0+r\cos\theta}\right)\frac{\partial f}{\partial\sigma}\right]+$$
$$\cos\sigma\left[\upsilon_\perp\frac{\partial f}{\partial r}-\frac{h_\theta^2\upsilon_\parallel}{r}\hat{V}f-\frac{h_\varphi^2\upsilon_\parallel\cos\theta}{R_0+r\cos\theta}\hat{V}f-\frac{h_\varphi\upsilon_\parallel^2\sin\theta}{\upsilon_\perp(R_0+r\cos\theta)}\frac{\partial f}{\partial\sigma}\right]+$$
$$\frac{\sin2\sigma}{2}\left\{h_\varphi\upsilon_\perp\left[h_\varphi\frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\varphi}\right)-\frac{h_\theta R_0}{r(R_0+r\cos\theta)}\right]\hat{V}f+\frac{h_\theta\upsilon_\parallel\sin\theta}{R_0+r\cos\theta}\frac{\partial f}{\partial\sigma}\right\}+$$
$$\frac{\cos2\sigma}{2}\left\{\frac{h_\theta\upsilon_\perp\sin\theta}{R_0+r\cos\theta}\hat{V}f-h_\varphi\upsilon_\parallel\left[h_\varphi\frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\varphi}\right)-\frac{h_\theta R_0}{r(R_0+r\cos\theta)}\right]\frac{\partial f}{\partial\sigma}\right\}=$$
$$=-\frac{e}{M}\left\{E_3\frac{\partial\bar{F}}{\partial\upsilon_\parallel}+\cos\sigma\left[E_1\frac{\partial\bar{F}}{\partial\upsilon_\perp}+\frac{H_2}{c}\hat{V}\bar{F}+\frac{1}{\upsilon_\perp}\left(E_2+\frac{\upsilon_\parallel}{c}H_1\right)\frac{\partial\bar{F}}{\partial\sigma}\right]-\right.$$
$$\left.-\frac{H_3}{c}\frac{\partial\bar{F}}{\partial\sigma}+\sin\sigma\left[E_2\frac{\partial\bar{F}}{\partial\upsilon_\perp}-\frac{H_1}{c}\hat{V}\bar{F}-\frac{1}{\upsilon_\perp}\left(E_1-\frac{\upsilon_\parallel}{c}H_2\right)\frac{\partial\bar{F}}{\partial\sigma}\right]\right\}.$$

Here the differential operator

$$\hat{V}f=\upsilon_\perp\frac{\partial f}{\partial\upsilon_\parallel}-\upsilon_\parallel\frac{\partial f}{\partial\upsilon_\perp} \quad (15)$$

has been used in velocity space to shorten the kinetic equations; the index α=*i*,*e* of particle species is omitted in Eqs. (14), (15). The connection between the projections of vector values $\mathbf{A}=\{\mathbf{E},\mathbf{H},\mathbf{v},\mathbf{j}\}$ in the quasi-toroidal coordinates (A_r,A_θ,A_φ) and their projections ($A_1\equiv A_n$, $A_2\equiv A_b$, $A_3\equiv A_h$) on the orts of an orthogonal trihedron generated by a magnetic field $\mathbf{H}_0$, i.e. into the unit vectors **n**, **b**, **h** (see Fig. 1), is given by the formulas

$$A_1 = \mathbf{A}\cdot\mathbf{n} = A_r\,, \qquad A_2 = \mathbf{A}\cdot\mathbf{b} = h_\varphi A_\theta - h_\theta A_\varphi\,,$$
$$A_3 = \mathbf{A}\cdot\mathbf{h} = h_\varphi A_\varphi + h_\theta A_\theta\,. \qquad (16)$$

The linearized Vlasov equation, Eq. (14), is suitable for studying a wide class of electrodynamic problems in 2D tokamaks with circular magnetic surfaces, provided that the particle Larmor radius $\upsilon_{T\alpha}/\Omega_\alpha$ is the smallest among all the characteristic dimensions-scales of the problem in the direction perpendicular to the equilibrium magnetic field, using the periodicity of distribution functions on the polar angle σ in velocity space. In this case, as for other axisymmetric plasma models in the cylindrical current-carrying plasmas [9 - 11] or elongated tokamaks [12, 13], the perturbed distribution functions can be expanded into the Fourier series in σ:
$f_\alpha(t,r,\theta,\varphi,v_\parallel,v_\perp,\sigma) = \sum_\ell^{\pm\infty} f_{\ell,\alpha}(r,\theta,v_\parallel,v_\perp)\exp(-i\omega t + in\varphi - i\ell\sigma)$.

As a result, Eq. (14) can be reduced to the set of coupled equations for harmonics $f_\ell(r,\theta,v_\parallel,v_\perp)$, $f_{l\pm1}(r,\theta,v_\parallel,v_\perp)$ and $f_{\ell\pm2}(r,\theta,v_\parallel,v_\perp)$:

$$\begin{aligned}
&-i\omega f_\ell + i\ell\Omega f_\ell + i\ell\frac{h_\varphi\upsilon_\parallel}{2}\left[h_\varphi\frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\varphi}\right) + \frac{3h_\theta R_0}{r(R_0 + r\cos\theta)}\right]f_\ell - \\
&-\frac{h_\theta\upsilon_\perp\sin\theta}{2(R_0 + r\cos\theta)}\hat{V}f_\ell + \frac{h_\theta\upsilon_\parallel}{r}\frac{\partial f_\ell}{\partial\theta} + \frac{ih_\varphi\upsilon_\parallel n f_\ell}{R_0 + r\cos\theta} - \\
&-i\frac{h_\varphi\upsilon_\perp}{2r}\frac{\partial\left(f_{\ell+1} - f_{\ell-1}\right)}{\partial\theta} - \frac{h_\theta\upsilon_\perp n\left(f_{\ell+1} - f_{\ell-1}\right)}{2(R_0 + r\cos\theta)} + \\
&+\frac{1}{2}\left(\frac{h_\theta^2\upsilon_\parallel^2}{r\upsilon_\perp} + \frac{h_\varphi^2\upsilon_\perp}{r} + \frac{h_\varphi^2\upsilon_\parallel^2\cos\theta}{\upsilon_\perp(R_0 + r\cos\theta)} + \frac{h_\theta^2\upsilon_\perp\cos\theta}{R_0 + r\cos\theta}\right)\times \\
&\times\left[(\ell+1)f_{\ell+1} - (\ell-1)f_{\ell-1}\right] - \frac{ih_\varphi\upsilon_\parallel\sin\theta}{2(R_0 + r\cos\theta)}\hat{V}(f_{\ell+1} - f_{\ell-1}) + \\
&+\frac{\upsilon_\perp}{2}\frac{\partial\left(f_{\ell+1} + f_{\ell-1}\right)}{\partial r} - \frac{h_\theta^2\upsilon_\parallel}{2r}\hat{V}\left(f_{\ell+1} + f_{\ell-1}\right) + \qquad (17)\\
&+\frac{h_\varphi^2\upsilon_\parallel\cos\theta}{2(R_0 + r\cos\theta)}\hat{V}\left(f_{\ell+1} + f_{\ell-1}\right) + \frac{ih_\varphi\upsilon_\parallel^2\sin\theta}{2\upsilon_\perp(R_0 + r\cos\theta)}\times \\
&\times\left[(\ell+1)f_{\ell+1} + (\ell-1)f_{\ell-1}\right] + \frac{h_\theta\upsilon_\perp\sin\theta}{4(R_0 + r\cos\theta)}\hat{V}(f_{\ell+2} + f_{\ell-2}) + \\
&+i\frac{h_\varphi\upsilon_\parallel}{4}\left[h_\varphi\frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\varphi}\right) - \frac{h_\theta R_0}{r(R_0 + r\cos\theta)}\right]\left[(\ell+2)f_{\ell+2} + (\ell-2)f_{\ell-2}\right] - \\
&-i\frac{h_\varphi\upsilon_\perp}{4}\left[h_\varphi\frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\varphi}\right) - \frac{h_\theta R_0}{r(R_0 + r\cos\theta)}\right]\hat{V}(f_{\ell+2} - f_{\ell-2}) - \\
&-\frac{h_\theta\upsilon_\parallel\sin\theta}{4(R_0 + r\cos\theta)}\left[(\ell+2)f_{\ell+2} - (\ell-2)f_{\ell-2}\right] = \\
&= -\frac{e}{M}\left\{E_3\frac{\partial F_\ell}{\partial\upsilon_\parallel} + \frac{E_1}{2}\frac{\partial\left(F_{\ell+1} + F_{\ell-1}\right)}{\partial\upsilon_\perp} + \frac{H_2}{2c}\hat{V}\left(F_{\ell+1} + F_{\ell-1}\right) - \right.\\
&\quad -\frac{i}{2\upsilon_\perp}\left(E_2 + \frac{\upsilon_\parallel}{c}H_1\right)\left[(\ell+1)F_{\ell+1} + (\ell-1)F_{\ell-1}\right] - \\
&\quad +i\ell\frac{H_3}{c}F_\ell - i\frac{E_2}{2}\frac{\partial\left(F_{\ell+1} - F_{\ell-1}\right)}{\partial\upsilon_\perp} + i\frac{H_1}{2c}\hat{V}\left(F_{\ell+1} - F_{\ell-1}\right) + \\
&\quad \left. +\frac{1}{2\upsilon_\perp}\left(E_1 - \frac{\upsilon_\parallel}{c}H_2\right)\left[(\ell+1)F_{\ell+1} - (\ell-1)F_{\ell-1}\right]\right\}.
\end{aligned}$$

As is well known, by the plasma particle distribution functions one can estimate, in the scope of kinetic wave theory, the perturbations of particle densities and current density components involved in Maxwell's equations for the perturbed electromagnetic fields $(\mathbf{E},\mathbf{H})$ in a considered plasma model. However, we have no exact solution for Eq. (17) in the general case. It is necessary to apply the approximation methods using the small parameters (e.g., the smallness of the Larmor radius of plasma particles or magnetization parameters) and the restrictions on the wave frequencies ω.

As for the magnetization parameters, all of them, as usual, are inversely proportional to the cyclotron frequency, $\kappa_{X\alpha}\upsilon_{\perp,\parallel}/\Omega_\alpha << 1$, where $\kappa_{X\alpha}$ characterize the spatial scales of the inhomogeneity of particle density $\kappa_{n\alpha} = \partial\ln(n_{0\alpha})/\partial r$, temperature $\kappa_{T\alpha} = \partial\ln(T_{0\alpha})/\partial r$, wave numbers k_r, $k_\theta = m/r$, $k_\varphi = n/R_0$, where m and n are the poloidal and toroidal eigenmode numbers, respectively. It should be noted that, in contrast to the case of a straight equilibrium magnetic field ($\mathbf{H}_0$=$H_0\mathbf{e}_z$, where $\mathbf{k} = k_\perp\mathbf{e}_x + 0\mathbf{e}_y + k_\parallel\mathbf{e}_z$) in a current-carrying plasma confined by a helical magnetic field, the wave vector $\mathbf{k} = k_r\mathbf{n} + k_b\mathbf{b} + k_\parallel\mathbf{h}$ always has three components, where the parallel $k_\parallel$ and binormal k_b projections of **k** are defined as

$$k_\parallel = \frac{h_\theta m}{r} - \frac{h_\varphi n}{R_0} \quad \text{and} \quad k_b = \frac{h_\varphi m}{r} - \frac{h_\theta n}{R_0}. \qquad (18)$$

Moreover, evaluating the main contribution of plasma particles to the perturbed longitudinal (j_3) and transverse (j_1, j_2) current density components, there is enough to find the harmonics $f_{0,\alpha}$ and $f_{\pm1,\alpha}$:

$$\begin{aligned}
&j_1 \equiv j_n = \mathbf{j}\cdot\mathbf{n} = j_{(1)} + j_{(-1)}\,,\\
&j_2 \equiv j_b = \mathbf{j}\cdot\mathbf{b} = i[\,j_{(-1)} - j_{(1)}\,]\,. \qquad (19)\\
&j_3 \equiv j_h = \mathbf{j}\cdot\mathbf{h} = 2\pi\sum_\alpha^{e,i_1,i_2,\ldots}\tilde{q}_\alpha\int_{-\infty}^{\infty}\upsilon_\parallel d\upsilon_\parallel\int_0^\infty f_{0,\alpha}\upsilon_\perp d\upsilon_\perp\,,\\
&j_{(\ell)} = \pi\sum_\alpha^{e,i_1,i_2,\ldots}\tilde{q}_\alpha\int_{-\infty}^{\infty}d\upsilon_\parallel\int_0^\infty f_{\ell,\alpha}\upsilon_\perp^2 d\upsilon_\perp\,, \quad \ell = \pm1\,.
\end{aligned}$$

In our previous papers [9 - 12] we have solved the Vlasov equations for harmonics $f_{0,\alpha}$ and $f_{\pm1,\alpha}$ in the simplest case, i.e., in the zeroth-order over the magnetization parameters, neglecting the drift effects proportional to Larmor radius $\upsilon_{T\alpha}/\Omega_\alpha$. In this case, the harmonics $f_{0,\alpha}$ and $f_{\pm1,\alpha}$ become independent of each other, satisfying first-order differential equations with three partial derivates with respect to θ, $\upsilon_\perp$ and $\upsilon_\parallel$:

$$\begin{aligned}
&-i\omega f_\ell + i\ell\Omega_c f_\ell + \frac{h_\theta\upsilon_\parallel}{r}\left\{\frac{\partial f_\ell}{\partial\theta} + i\left[\frac{nq}{1+\varepsilon\cos\theta} + \frac{\ell h_\varphi}{2}\left(\frac{4+\varepsilon\cos\theta}{1+\varepsilon\cos\theta} - \right.\right.\right.\\
&\left.\left.\left. -\frac{r}{q}\frac{\partial q}{\partial r}\right)f_\ell\right]\right\} - \frac{h_\theta\upsilon_\perp\sin\theta}{2R_0(1+\varepsilon\cos\theta)}\hat{V}f_\ell = Q_\ell\,, \quad \ell = 0,\pm1\,, \quad (20)
\end{aligned}$$

where q is the tokamak safety factor, see Eq. (22), $\varepsilon = r/R_0$, and Q_ℓ terms for the equilibrium (Maxwellian) distribution functions are equal to

$$Q_0 = \frac{\tilde{q}}{T_0} E_3 \upsilon_\| F_M , \quad Q_{\pm 1} = \frac{\tilde{q}}{T_0}\left(E_1 \pm iE_2\right)\upsilon_\perp F_M , \quad (21)$$

$$F_M = \frac{n_0}{(\pi \upsilon_T^2)^{1.5}} \exp\left(-\frac{\upsilon_\|^2 + \upsilon_\perp^2}{\upsilon_T^2}\right), \qquad q = \frac{r h_\varphi}{R_0 h_\theta} . \quad (22)$$

Eqs. (20)-(22) are suitable to study the wave-particle interaction accounting for the Cherenkov, cyclotron, and bounce resonances for both the trapped and untrapped particles in 2D axisymmetric tokamaks.

In this paper, we derive the drift-kinetic equation for $f_{0,\alpha}$ in the first order over the magnetization parameters for the low-frequency perturbations, $\omega << \Omega_i$.

After substituting $\ell = 0$ in Eq. (17), the equation for f_0 can be rewritten in the form

$$-i\omega f_0 + \frac{h_\theta \upsilon_\|}{r}\frac{\partial f_0}{\partial \theta} + \frac{i h_\varphi \upsilon_\| n f_0}{R_0 + r\cos\theta} - \frac{h_\theta \upsilon_\perp \sin\theta}{2(R_0 + r\cos\theta)}\hat{V} f_0 -$$
$$-i\frac{h_\varphi \upsilon_\perp}{2r}\frac{\partial\left(f_1 - f_{-1}\right)}{\partial\theta} - \frac{h_\theta \upsilon_\perp n\left(f_1 - f_{-1}\right)}{2(R_0 + r\cos\theta)} + \quad (23)$$
$$+\frac{1}{2}\left(\frac{h_\theta^2 \upsilon_\|^2}{r\upsilon_\perp} + \frac{h_\varphi^2 \upsilon_\perp}{r} + \frac{h_\varphi^2 \upsilon_\|^2 \cos\theta}{\upsilon_\perp (R_0 + r\cos\theta)} + \frac{h_\theta^2 \upsilon_\perp \cos\theta}{R_0 + r\cos\theta}\right)(f_1 + f_{-1}) -$$
$$-\frac{i h_\varphi \upsilon_\| \sin\theta}{2(R_0 + r\cos\theta)}\hat{V}(f_1 - f_{-1}) + \frac{\upsilon_\perp}{2}\frac{\partial(f_1 + f_{-1})}{\partial r} - \frac{h_\theta^2 \upsilon_\|}{2r}\hat{V}(f_1 + f_{-1}) +$$
$$+\frac{h_\varphi^2 \upsilon_\| \cos\theta}{2(R_0 + r\cos\theta)}\hat{V}(f_1 + f_{-1}) + \frac{i h_\varphi \upsilon_\|^2 \sin\theta}{2\upsilon_\perp (R_0 + r\cos\theta)}(f_1 - f_{-1}) +$$
$$+\frac{h_\theta \upsilon_\perp \sin\theta}{4(R_0 + r\cos\theta)}\hat{V}(f_2 + f_{-2}) + i\frac{h_\varphi \upsilon_\|}{2}\left[h_\varphi \frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\varphi}\right) -\right.$$
$$\left. - \frac{h_\theta R_0}{r(R_0 + r\cos\theta)}\right](f_2 - f_{-2}) - \frac{h_\theta \upsilon_\| \sin\theta}{2(R_0 + r\cos\theta)}(f_2 + f_{-2}) -$$
$$-i\frac{h_\varphi \upsilon_\perp}{4}\left[h_\varphi \frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\varphi}\right) - \frac{h_\theta R_0}{r(R_0 + r\cos\theta)}\right]\hat{V}(f_2 - f_{-2}) =$$
$$= -\frac{\tilde{q}}{M}\left\{E_3 \frac{\partial F_0}{\partial \upsilon_\|} + \frac{E_1}{2}\frac{\partial\left(F_1 + F_{-1}\right)}{\partial \upsilon_\perp} + \frac{H_2}{2c}\hat{V}\left(F_1 + F_{-1}\right) + \right.$$
$$+\frac{1}{2\upsilon_\perp}\left(E_1 - \frac{\upsilon_\|}{c}H_2\right)\left(F_1 + F_{-1}\right) - i\frac{E_2}{2}\frac{\partial\left(F_1 - F_{-1}\right)}{\partial \upsilon_\perp} +$$
$$\left. + i\frac{H_1}{2c}\hat{V}\left(F_1 - F_{-1}\right) - \frac{i}{2\upsilon_\perp}\left(E_2 + \frac{\upsilon_\|}{c}H_1\right)\left(F_1 - F_{-1}\right)\right\}.$$

The influence of drift effects on the plasma particle distribution functions is described by the harmonics $f_{\pm 1}(r,\theta,v_\|,v_\perp)$ and $f_{\pm 2}(r,\theta,v_\|,v_\perp)$, connected with $f_0(r,\theta,v_\|,v_\perp)$ in first-order magnetization parameters as

$$f_1 - f_{-1} = i\frac{\upsilon_\perp}{\Omega}\frac{\partial f_0}{\partial r} - i\frac{h_\theta^2 \upsilon_\|}{r\Omega}\hat{V} f_0 - i\frac{h_\varphi^2 \upsilon_\|}{R_0 \Omega_0}\cos\theta \hat{V} f_0 +$$
$$+ i\frac{\tilde{q}E_1}{M\Omega}\frac{\partial F_0}{\partial \upsilon_\perp} + 2i\frac{\tilde{q}H_2 \upsilon_\perp \upsilon_0}{Mc\Omega \upsilon_T^2}F_0 ,$$

$$f_1 + f_{-1} = -\frac{\upsilon_\perp h_\varphi}{r\Omega}\frac{\partial f_0}{\partial \theta} + i\frac{\upsilon_\perp h_\theta n}{R_0 \Omega_0} f_0 - \frac{h_\varphi \upsilon_\|}{R_0 \Omega_0}\sin\theta \hat{V} f_0 -$$
$$-\frac{\tilde{q}E_2}{M\Omega}\frac{\partial F_0}{\partial \upsilon_\perp} + 2\frac{\tilde{q}H_1 \upsilon_\perp \upsilon_0}{Mc\Omega \upsilon_T^2}F_0 ,$$

$$f_2 - f_{-2} = i\frac{h_\theta \upsilon_\perp \sin\theta}{4R_0 \Omega_0}\hat{V} f_0 , \quad (24)$$

$$f_2 + f_{-2} = -i\frac{h_\varphi \upsilon_\perp}{4\Omega}\left[h_\varphi \frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\varphi}\right) - \frac{h_\theta R_0}{r(R_0 + r\cos\theta)}\right]\hat{V} f_0 .$$

As one can see the exact drift-kinetic equation for f_0, after substituting Eqs. (24) into Eq. (23), is complicated, having four partial derivatives in $r,\theta,v_\|,v_\perp$:

$$-i\omega f_0 + \frac{\partial\theta}{\partial t}\frac{\partial f_0}{\partial\theta} + i\frac{\partial\varphi}{\partial t} n f_0 - \frac{\partial r}{\partial t}\frac{\partial f_0}{\partial r} - \frac{\partial V}{\partial t}\left(\upsilon_\perp \frac{\partial f_0}{\partial \upsilon_\|} - \upsilon_\| \frac{\partial f_0}{\partial \upsilon_\perp}\right) =$$
$$= Q(E_1, E_2, E_3) , \quad (25)$$

where

$$\frac{\partial\theta}{\partial t} = \frac{h_\theta \upsilon_\|}{r} - \frac{h_\varphi h_\theta^2}{2r^2 \Omega_0}\left(2\upsilon_\|^2 - \upsilon_\perp^2\right) - \frac{h_\varphi \cos\theta}{2rR_0 \Omega_0}\left(2\upsilon_\|^2 + \upsilon_\perp^2\right),$$

$$\frac{\partial\varphi}{\partial t} = \frac{h_\varphi \upsilon_\|}{R_0 + r\cos\theta} + \frac{\upsilon_\perp^2}{2R_0 \Omega_0}\frac{\partial h_\theta}{\partial r} + \frac{h_\theta\left(2h_\theta^2 \upsilon_\|^2 + h_\varphi^2 \upsilon_\perp^2\right)}{2r(R_0 + r\cos\theta)\Omega_0} +$$
$$+\frac{h_\theta \cos\theta\left(2\upsilon_\|^2 + \upsilon_\perp^2\right)}{2R_0 \Omega_0 (R_0 + r\cos\theta)} ,$$

$$\frac{\partial r}{\partial t} = \frac{h_\varphi \sin\theta}{2R_0 \Omega_0}\left(2\upsilon_\|^2 + \upsilon_\perp^2\right), \quad (26)$$

$$\frac{\partial V}{\partial t} = \frac{h_\theta \upsilon_\perp \sin\theta}{2(R_0 + r\cos\theta)} - \frac{h_\varphi h_\theta^2 \upsilon_\perp \upsilon_\| \sin\theta}{2r(R_0 + r\cos\theta)\Omega_0} ,$$

$$Q = 2\frac{\tilde{q}(\upsilon_\| - \upsilon_0)}{M\upsilon_T^2}F_M E_3 - \frac{\tilde{q}F_M}{M\Omega}\left[\kappa_n - \kappa_T\left(\frac{3}{2} - \frac{\upsilon_\perp^2 + \upsilon_\|^2}{\upsilon_T^2}\right)\right]\times$$
$$\times\left(E_2 + \frac{\upsilon_\|}{c}H_1\right) - \frac{\tilde{q}\left(2h_\theta^2 \upsilon_\|^2 + h_\varphi^2 \upsilon_\perp^2\right)}{Mr\Omega \upsilon_T^2}F_M E_2 - \quad (27)$$
$$-\frac{\tilde{q}\cos\theta F_M}{MR_0 \Omega_0 \upsilon_T^2}\left(2h_\varphi^2 \upsilon_\|^2 + h_\theta^2 \upsilon_\perp^2 + \upsilon_\perp^2\right)E_2 - \frac{\tilde{q}\upsilon_\perp^2 F_M}{M\Omega \upsilon_T^2}\frac{\partial E_2}{\partial r} -$$
$$-\frac{\tilde{q}h_\varphi \sin\theta}{MR_0 \Omega_0 \upsilon_T^2}\left(2\upsilon_\|^2 + \upsilon_\perp^2\right)F_M E_1 + \frac{\tilde{q}\upsilon_\perp^2 F_M}{M\Omega \upsilon_T^2}\left(\frac{h_\varphi}{r}\frac{\partial E_1}{\partial\theta} - \frac{i h_\theta n E_1}{R_0 + r\cos\theta}\right).$$

It should be noted that the right-hand side of Eq. (25) is written in Eq. (27) for the case when the influence of the equilibrium current on the magnetic drift effects (proportional to Ω^{-1}) can be neglected. In contrast with initial Vlasov equations, where the plasma particle distribution functions depend on the three velocity variables ($\upsilon_\|,\upsilon_\perp,\sigma$), the drift-kinetic equations are written for the particle distribution functions averaged over the gyrophase angle σ in velocity space, depending only the $\upsilon_\|$ and $\upsilon_\perp$ velocities relative to $\mathbf{H}_0$. As a result, the drift-kinetic equations are simpler and more convenient for solutions in the low-frequency range.

3. TRAJECTORIES OF UNTRAPPED AND TRAPPED PARTICLES

The number of partial derivatives in Eq. (25) can be reduced after introducing the new conventional variables associated with the corresponding invariants of motion of charged particles in a considered plasma model. As usual, the conservation integrals (the motion invariants) should be connected with the particle energy ($\upsilon_\|^2 + \upsilon_\perp^2 = const$), magnetic moment ($\upsilon_\perp^2 / H_0 = const$), and, so-called, longitudinal invariant.

According to Eqs. (26), in the zeroth approximation in Larmor radius corrections, we can introduce the new variables υ and μ (nondimensional magnetic moment) instead of $\upsilon_\parallel$ and $\upsilon_\perp$ as

$$\upsilon = \sqrt{\upsilon_\parallel^2 + \upsilon_\perp^2}\,, \quad \mu = \frac{\upsilon_\perp^2}{\upsilon_\parallel^2 + \upsilon_\perp^2}\left(1 + \varepsilon\cos\theta\right), \quad (28)$$

where $\varepsilon = r / R_0$ is the inverse aspect ratio of a torus.

Since the tokamak magnetic field $\mathbf{H}_0$ is nonuniform and has a minimum, all plasma particles should be separated into two groups of, so-called, untrapped and trapped particles. Such a separation [8, 12, 13] can be done by the inequalities for μ and θ:

$$0 \le \mu \le 1 - \varepsilon\,, \quad -\pi \le \theta \le \pi \ \text{– untrapped particles,}$$

$$1 - \varepsilon \le \mu \le 1 + \varepsilon\,, \quad -\theta_t \le \theta \le \theta_t \ \text{– trapped particles,}$$

analyzing the condition $\upsilon_{\parallel,s} = s\upsilon\sqrt{1 - \mu(1 - \varepsilon\cos\theta)} = 0$, where $s = \pm 1$ distinguishing the positive and negative parallel velocity relative to $\mathbf{H}_0$. Here the stop (reflection, turning) points of trapped particles are defined as

$$\pm\theta_t = \pm\arccos\left(\frac{\mu - 1}{\varepsilon\mu}\right). \quad (29)$$

However, in the first approximation in the Larmor radius corrections, we should take into account that the toroidal drift of charged particles leads to the deflection of their trajectories from magnetic surfaces. By the characteristical equations for $\partial\theta / \partial t$ and $\partial r / \partial t$ in Eqs. (26) we can define the radial coordinate of untrapped and trapped particles, moving along the $\mathbf{H}_0$-field lines, respectively, as $r_{u,s} = r + \tilde{r}_{u,s}(r,\theta)$ and $r_{t,s} = r + \tilde{r}_{t,s}(r,\theta)$. Here r is the radius of the considered magnetic surface,

$$\tilde{r}_{u,s}(r,\theta) = -\frac{q(r)}{\Omega_0(r)}\int_{-\pi}^{\theta}\frac{\upsilon_{\parallel,s}^2(r,\eta) + 0{,}5\upsilon_\perp^2(r,\eta)}{\upsilon_{\parallel,s}(r,\eta)}\sin\eta\, d\eta\,, \quad (30)$$

$$\tilde{r}_{t,s}(r,\theta) = -\frac{q(r)}{\Omega_0(r)}\int_{-\theta_t}^{\theta}\frac{\upsilon_{\parallel,s}^2(r,\eta) + 0{,}5\upsilon_\perp^2(r,\eta)}{\upsilon_{\parallel,s}(r,\eta)}\sin\eta\, d\eta\,, \quad (31)$$

where, under $\varepsilon \ll 1$,

$$\upsilon_{\parallel,s}(r,\eta) = s\upsilon\sqrt{1 - \mu\left(1 - \frac{r}{R_0}\cos\eta\right)}\,, \quad (32)$$

$$\upsilon_\perp^2(r,\eta) = \mu\upsilon^2\left(1 - \frac{r}{R_0}\cos\eta\right). \quad (33)$$

Projections of the typical guiding-center trajectories of untrapped and trapped particles on the transverse cross-section of the moderate magnetic surfaces in tokamaks, r=*const* (dashed circle lines, r = 0.7a), are plotted in Figs. 2 and 3, respectively.

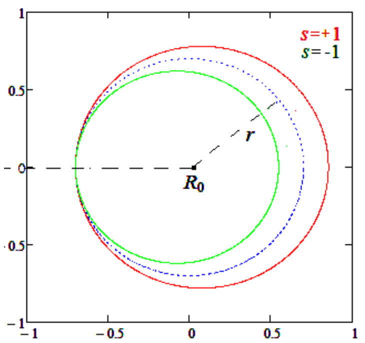

Fig. 2. The trajectories of the untrapped particles in an axisymmetric tokamak with circular magnetic surfaces

As for positively charged untrapped particles (ions), moving along the $\mathbf{H}_0$-field lines, with s=+1, due to magnetic drift they are shifted (red line) to a region of a weaker magnetic field (i.e., outward from the magnetic surface). While ions moving against a $\mathbf{H}_0$-field, with s=-1, drift to a region of a stronger magnetic field (i.e., inside the magnetic surface, green line). Both trajectories in Fig. 2 are plotted for untrapped ions, starting at the inner part of the magnetic surface, r=*const*, under $\varepsilon = 0.18$, $\mu = 0.5$ and Larmor radius $\upsilon / \Omega_0 = 0.04$ cm.

The main feature of the drift deflections of both the untrapped and trapped particles is that they are determined by the particle Larmor rotation in the poloidal magnetic field $\bar{H}_{0\theta}(r)$, since $\Omega_0\varepsilon / q \equiv \Omega_{0\theta}$, where $\Omega_{0\theta} = \tilde{q}\bar{H}_{0\theta} / (Mc)$ is the Larmor (cyclotron) frequency of charged particles in the $\bar{H}_{0\theta}$-field, that depends significantly on r. After integration in Eq. (30):

$$\tilde{r}_{u,s}(r,\theta) = \frac{s\upsilon}{3\Omega_{0\theta}}\left[\left(\frac{4}{\mu} - 1 + \frac{r}{R_0}\cos\theta\right)\sqrt{1 - \mu\left(1 - \frac{r}{R_0}\cos\theta\right)} - \left(\frac{4}{\mu} - 1 - \frac{r}{R_0}\right)\sqrt{1 - \mu\left(1 + \frac{r}{R_0}\right)}\right]. \quad (34)$$

As a result, the maximal deflection of untrapped particles should take place, in our notation, at the external part of the considered magnetic surface, i.e., at $\theta = 0$:

$$r_{u,s}^{\max}(r) = \tilde{r}_{u,s}(r,0) = \frac{s\upsilon}{3\Omega_{0\theta}}\left[\left(\frac{4}{\mu} - 1 + \frac{r}{R_0}\right)\sqrt{1 - \mu\left(1 - \frac{r}{R_0}\right)} - \left(\frac{4}{\mu} - 1 - \frac{r}{R_0}\right)\sqrt{1 - \mu\left(1 + \frac{r}{R_0}\right)}\right]. \quad (35)$$

It should be noted that the features of the drift trajectories of negatively charged untrapped electrons are opposite with respect to ions, i.e., the field-aligned electrons, with s=+1, drift into the inner part of the magnetic surfaces and vice versa.

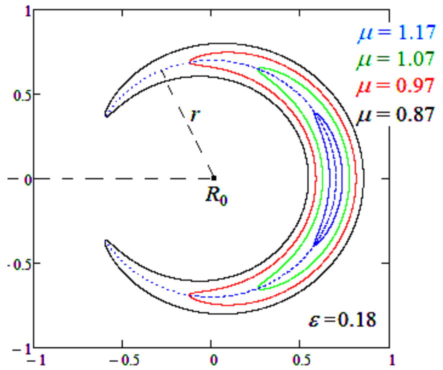

Fig. 3. The trajectories of the trapped particles in an axisymmetric tokamak with circular magnetic surfaces

In contrast to untrapped particles, the trajectories of the trapped particles have the ‘banana’-forms. Oscillating between the stop-points, both the positively and negatively charged trapped particles change the sign of parallel velocity, $s = \pm 1$, during one bounce period. As a result, the banana-orbit widths of the trapped ions and electrons are doubled due to their drift both inward from the surface for *s*=+1 and outward for *s*=-1.

The deflection of trapped particles from the magnetic surface can be determined by a simple expression after integration in Eq. (31):

$$\tilde{r}_{t,s}(r,\theta) = \frac{s\upsilon}{3\Omega_{0\theta}}\left(\frac{4}{\mu} - 1 + \frac{r}{R_0}\cos\theta\right)\sqrt{1-\mu\left(1-\frac{r}{R_0}\right)}. \quad (36)$$

Thus, the maximal deflection of the trapped particles $r_{t,s}^{\max}(r) = \tilde{r}_{t,s}(r,0)$ (i.e., half of their maximum orbit-width in the equatorial plane of the torus, at $\theta = 0$) is estimated by

$$r_{t,s}^{\max}(r) = \frac{s\upsilon}{3\Omega_{0\theta}}\left(\frac{4}{\mu} - 1 + \frac{r}{R_0}\right)\sqrt{1-\mu\left(1-\frac{r}{R_0}\right)}. \quad (37)$$

The banana trajectories of trapped particles in Fig. 3 are plotted at different levels of the nondimensional magnetic moment μ for particles starting at stop-points on the magnetic surface, shown as a dashed line. The banana sizes and the values of the stop-points of trapped particles depend substantially on μ, according to Eq. (29). As can be seen, strongly trapped particles (under large μ) have smaller sizes and orbit-widths.

CONCLUSIONS

The pressureless 2D toroidal current-carrying plasma model has been described to develop the kinetic theory of low-frequency oscillations in axisymmetric tokamaks with circular magnetic surfaces and large aspect ratios. The steady-state distribution function of plasma electrons and equilibrium magnetic field are self-consistent, satisfying Maxwell's equations.

If the toroidal magnetic field is changed to longitudinal cylindrical *z*-projection, $\bar{H}_{0\varphi} \to \bar{H}_{0z}$, $h_\varphi \to h_z$, and $R_0 \to \infty$, our 2D toroidal model is transformed into a cylindrical magnetized current-carrying plasma model in the helical magnetic field.

The drift-kinetic equations for the perturbed distribution functions of the trapped and untrapped (passing, circulating) particles are derived accounting for the magnetic drift effects in the first-order over the magnetization parameters, proportional to the Larmor radius gyration of ions and electrons moving along the equilibrium magnetic field lines.

The characteristical equations in the drift-kinetic equations allow us to estimate the finite orbit-widths of the 'banana'-trajectories of both the trapped and untrapped particles. Analytical expressions are derived for the particle deflections from the magnetic surfaces.

Since the toroidal drift deflections of untrapped and trapped particles are defined by the poloidal magnetic field, the corresponding orbit-widths are much larger than their Larmor radius in $\mathbf{H}_0$-field.

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ДРЕЙФОВО-КІНЕТИЧНІ РІВНЯННЯ У ЗАМАГНІЧЕНІЙ ПЛАЗМІ ЗІ СТРУМОМ

М.І. Гришанов, М.О. Азарєнков

Кінетичні моделі замагніченої плазми зі струмом розроблені для вивчення впливу ефектів магнітного дрейфу на взаємодію хвиля-частинка у токамаках та циліндричних плазмових системах із гвинтовим магнітним полем. Отримано дрейфово-кінетичні рівняння для збурених функцій розподілу захоплених і пролітних частинок у двовимірній осесиметричній тороїдальній плазмі з урахуванням їх баунс-коливань і кінцевої ширини орбіт їхніх бананових траєкторій.