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DO THE DISPERSION PROPERTIES OF ELECTROMAGNETIC SURFACE WAVES AT THE SHARP BOUNDARY PLASMA-METAL IN SLAB VOIGT GEOMETRY REPRESENT THE LIMITING CASE OF THOSE FOR THE INTERFACE OF TWO PLASMAS?

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Electromagnetic surface waves in Voigt geometry are considered to resolve the contradiction between two classical studies. One investigated the waves at the boundary of two plasmas. These waves were reported not to propagate with frequencies below the ion cyclotron frequency. The other approach studied the waves at the metal-plasma interface. Dispersion properties of the waves with frequencies below the ion cyclotron frequency were investigated.

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INTRODUCTION

The comprehensive review of history of studying electromagnetic surface waves (SWs) by John A. Polo Jr. and Akhlesh Lakhtakia can be found in [1, 2]. They overviewed the period from Zenneck [3] and Sommerfeld [4] who studied SWs at the planar interface between a dielectric and a conductor to O. Takayama with co-authors [5] who experimentally demonstrated the existence of Dyakonov waves propagating along the planar interface of an isotropic dielectric and a uniaxial dielectric [6]. The interest in electromagnetic waves of surface type is explained by their wide scope of applications in the field of plasma electronics [7 - 12], plasma-antenna systems [13 - 17], magnetic confinement fusion plasmas [18 - 21], nano-technologies [22 - 25], plasma production [26 - 29], etc.

The propagation of slow SWs (their phase velocity is much smaller than the speed of light in vacuum) along a plane boundary of two plasmas, along a plane plasma layer and along a plane vacuum layer in a plasma were studied for the first time by Abdel-Shahid and Pakhomov [30]. The angle between an external static magnetic field and the wave propagation was assumed to be arbitrary. Dispersion equations were derived and investigated for electrostatic SWs including ion effects. However, they discussed only certain limiting cases. No details of the effect of the plasma particle density ratio in the two media were given. In addition, the influence of ions was poorly considered.

Studying the SW dispersion properties in Voigt geometry is of practical interest, since SW propagation perpendicularly to an external static magnetic field, which in turn is parallel to the plasma interface, often happens under experimental conditions. Along with this, a 90 degree angle between the wavevector and the external static magnetic field provides analytical tractability of the problem.

Detailed investigation of SW propagation along a sharp interface between two plasma-like media in Voigt geometry was presented by Uberoi and Rao [31]. The assumption of sharp interface is used in many boundary value problems, and is valid if the width of the inhomogeneous transient layer is much smaller than the depth of SW penetration into the plasmas [32]. Despite Kaufman was noted by Uberoi and Rao to be the first to study SW dispersion properties in this case [33], some differences were pointed out in [31] between the two papers. First, Kaufman did not take into account any ion effects. Second, the transcendental dispersion relation was discussed by Kaufman in the (ω_{ce}, ω) plane, whereas it was investigated by Uberoi and Rao in the parameter-space. This provided the advantage of uncovering details of the effects of variation of both external static magnetic field strength and plasma particle densities on wave propagation. Third, the influence of the difference of the plasma particle density in the two media on electrostatic oscillations was not discussed by Kaufman.

It is especially significant for the following to note that Uberoi and Rao found the lowest frequency below which SWs cannot propagate along a sharp interface between two plasma-like media in Voigt geometry. This frequency was determined as ion cyclotron frequency ω_{ci} or some other definite frequency $\omega_4 > \omega_{ci}$ depending on the plasma particle densities.

Azarenkov and Ostrikov overviewed the results of studying magnetoplasma SWs at the interface between a plasma-like medium and a metal in Voigt geometry in [34]. In the introduction, when describing the interface, the authors underlined that the consideration was limited “to the situations where the conductivities of the media in contact differed significantly $(\sigma_1 < \sigma_2)$”. (It should be emphasized that at this step, it looked precisely like the description of SW propagation along a sharp interface between two plasma-like media in Voigt geometry presented by Uberoi and Rao). In this case, the plasma properties of the medium with the higher conductivity were stated to be such that they could be neglected. The influence of this medium on the wave propagation was considered as that realized through the corresponding boundary conditions. Just the medium with the larger conductivity σ_1 was called a “metal”. This was justified as follows. Metals usually possess a significantly higher conductivity than semimetals, semiconductors, and gas-discharge plasmas.

The dispersion relation presented in [34] was noted to be obtained earlier by Seshadri [35]. However, a detailed analysis of the propagation conditions and the frequency ranges of SW existence were reported in [34] to be absent in [35].

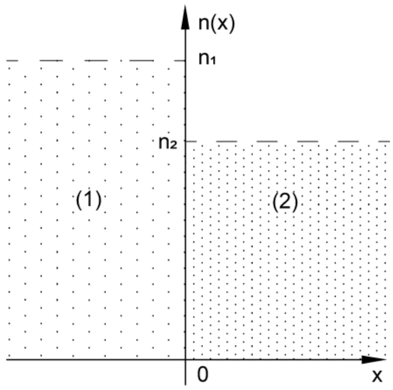
From the analysis of the dispersion relation, Azarenkov and Ostrikov made conclusions about the frequency ranges where SWs can propagate. In particular and what is especially important for the following, if SWs propagate along the y-axis (Figure), i.e. with positive wavenumber, $k_2 > 0$, then SW propagation was shown to be possible in the frequency range $\omega < \omega_{ci}$ (and also $\omega > \omega_{UH}$ with ω_{UH} being the upper hybrid frequency).

The specific objective of the present paper is to get an understanding of the reasons for the following contradiction. On one hand, the sharp boundary metal-plasma is an obvious limiting case of the general model, which is the interface of two magnetoactive plasmas. On the other hand, direct precise studies of the SW dispersion properties in these two cases result in conclusions, which contradict each other.

The paper is arranged as follows. The motivation of the study is provided in the present Introduction. The model of the plasma1-plasma2 structure under consideration here, and basic assumptions are described in Section 1. The spatial distribution of the wave fields is given and the dispersion relation is derived and treated analytically in Section 2. Finally, the obtained results are discussed in the Conclusions.

1. STATEMENT OF THE PROBLEM

Both groups: Uberoi and Rao [31], as well as Azarenkov and Ostrikov [34], considered the following structure (see Figure). A plasma with the plasma particle density n_2 is placed in the half space $x > 0$. The (y-z)-plane $x = 0$ presents the interface between two media along which an external static magnetic field is directed, $\vec{B}_0 || \vec{z}$. A plasma with the larger particle density $n_1 > n_2$ is placed in the half space $x < 0$. In the case of a metal-plasma interface, this plasma particle density is assumed to be infinite, $n_1 \gg n_2$.



Schematic of the problem

2. ANALYTICAL TREATMENT OF THE PROBLEM

SWs are considered to propagate perpendicular to $\vec{B}_0$ along the (y-z)-plane,

$$H_z(\vec{r}, t) = A exp(k_1 x + i k_y y - i\omega t), \text{ if } x < 0; \quad (1)$$

$$H_z(\vec{r}, t) = B exp(-k_2 x + i k_y y - i\omega t), \text{ if } x > 0; \quad (2)$$

$$E_y(x) = A \frac{i(N_1 + \mu_1 N_y)}{N_{\perp 1}^2} exp(k_1 x + i k_y y - i\omega t), \text{ if } x < 0; \quad (3)$$

$$E_y(x) = B \frac{i(\mu_2 N_y - N_2)}{N_{\perp 2}^2} exp(-k_2 x + i k_y y - i\omega t), \text{ if } x > 0. \quad (4)$$

In (1)-(4), A and B are the constants of integration, $k_{1,2}^{-1}$ are the depths of SW penetration into the respective plasmas, $k_{1,2}^2 = k_{\perp 1,2}^2 + k_y^2$, where k_y is the wavenumber, $\mu_{1,2} = \varepsilon_2(n_{1,2})/\varepsilon_1(n_{1,2})$,
$k_{\perp 1,2}^2 = k^2 N_{\perp 1,2}^2$, $N_{\perp 1,2}^2 = \varepsilon_1(n_{1,2})(\mu_{1,2}^2 - 1) > 0$,
$k = \omega/c$ is the vacuum wavenumber, ω is the angular wavefrequency, c is the speed of light in vacuum, and ε_1 and ε_2 are the components of the plasma permittivity tensor,

$$\varepsilon_1 = 1 - \sum_\alpha \frac{\Omega_\alpha^2}{\omega^2 - \omega_\alpha^2}, \ \varepsilon_2 = -\sum_\alpha \frac{\Omega_\alpha^2 \omega_\alpha}{\omega(\omega^2 - \omega_\alpha^2)}. \quad (5)$$

In (5), Ω_α and ω_α are the plasma and cyclotron frequency of the particle of species α: $\alpha = i$ for ions, and $\alpha = e$ for electrons, respectively. The presentations (1) and (2) satisfy the boundary conditions at $x \to \pm\infty$, where the SW fields vanish.

In the ion cyclotron frequency region, if the plasmas are sufficiently dense so that $\Omega_{i1,2}^2 \gg \omega_i^2$, the observable $N_{\perp 1,2}^2 \approx -\Omega_{i1,2}^2/\omega_i^2$. The waves are assumed to be of surface nature in both plasmas, which implies $k_{1,2}^2 > 0$. This condition can be treated as follows. SWs can propagate with sufficiently short wavelengths only, such that $c^2 k_y^2 > \Omega_{i1}^2$.

The boundary conditions at the interface, $x = 0$, consist in the continuity of the tangential wave electric and magnetic fields. Their application provides the following dispersion relation [31]:

$$(\mu_2 N_y - N_2) = (N_{\perp 2}^2 / N_{\perp 1}^2)(\mu_1 N_y + N_1). \quad (6)$$

In the ion cyclotron frequency range, where the contradiction is found, SWs propagate with positive wavenumber, $k_y > 0$ [31, 34]. In the limiting case $c^2 k_y^2 \gg \Omega_{i1}^2 > \Omega_{i2}^2 \gg \omega_i^2$, which was demonstrated in [31, 34] to correspond to the upper limit of the frequency range, one can derive the following asymptotic solution to the dispersion relation (6):

$$\omega = \omega_i \left(1 + 2\frac{n_2}{n_1} - \frac{\Omega_{i2}^2}{2c^2 k_y^2} - \frac{n_2}{n_1} \frac{\Omega_{i1}^2 + \Omega_{i2}^2}{2c^2 k_y^2}\right). \quad (7)$$

It is clear from eq. (7) that in the short wavelength limit, the SW eigenfrequency is larger than the ion cyclotron frequency, if $n_2 \neq 0$.

In the limiting case of metal-plasma boundary, $n_1 \gg n_2$, one has $|N_{\perp 1}^2| \gg |N_{\perp 2}^2|, N_y^2$. Despite the observable N_1 in this limit is imaginary and large, $N_1 \approx i|N_{\perp 1}|$, the r.h.s. of eq. (6) can be neglected due to the presence of the larger denominator $N_{\perp 1}^2$. Then eq. (6) reads [32, 34, 35]:

$$\mu_2 N_y - N_2 = 0. \quad (8)$$

Its solution was presented in [35] in terms of phase velocity as a function of the frequency, $\omega/k_y = c/\sqrt{\varepsilon_1}$. Such a representation of the solution of the dispersion relation is justified since the expression for the eigenfrequency as a function of the wavenumber is much more cumbersome:

$$\omega = 0.5\Big(c^2k_y^2 + \omega_i^2 + \Omega_{i2}^2 - \\ - \sqrt{\left(c^2k_y^2 + \omega_i^2 + \Omega_{i2}^2\right)^2 - 4c^2k_y^2\omega_i^2}\Big). \quad (9)$$

However, the short wavelength limit of eq. (9) is much more simple,

$$\omega = \omega_i[1 - \Omega_{i2}^2/(2c^2k_y^2)]. \quad (10)$$

This result cannot be obtained from eq. (7) in the limit $n_1 \gg n_2$, which assumes, in particular, $\Omega_{i1}^2 \gg c^2k_y^2$, since the opposite assumption is already applied to derive eq. (7).

CONCLUSIONS

The conclusions of studying the dispersion properties of electromagnetic surface waves at the boundary of two plasmas [31] and at the metal-plasma interface [34] in Voigt geometry are found to contradict each other in the frequency range below the ion cyclotron frequency. The contradiction is valid despite the fact that the structure metal-plasma is the natural limiting case of the boundary of two plasmas in which the two plasma particle densities differ significantly. The contradiction is analyzed and explained.

Simple asymptotic expressions for SW eigenfrequencies are derived in limiting cases. Dispersion properties of SWs propagating in Voigt geometry along the sharp slab interface metal-plasma are shown to be the limiting case of those in the general case of the plane boundary of two plasmas with finite particle densities. The contradiction is explained as follows. Uberoi and Rao did carry out tremendous analyses of the dispersion relation (6) in regard of its root existence [31]. However, they concentrated on the comparison of their research on the case of a plasma-vacuum interface rather than a metal-plasma boundary. Specifically, the plasma particle density in the left half space n_1 was assumed to be fixed, while that in the right half space varied in the range $0 < n_2 < n_1$. The limiting case $n_2 \ll n_1$ was reached by considering the right half space as a vacuum rather than considering the left half space as a metal. That is why Uberoi and Rao did not deal with the problem under consideration here, as well as they did not study the disappearance of SWs in the limit $n_2 \to n_1$. Nevertheless, their paper [31] is very valuable.

From the mathematical point of view, the contradiction is explained by applying two limiting transitions in different orders in two cases, which provide different results. From the physical point of view, the answer to the question, which is put in the title of the paper, is as follows. First, an increase of the plasma particle density is accompanied by a decrease of the characteristic spatial scale of the wave: either the wavelength or the depth of wave penetration into the plasma. This decrease is restricted by the limits of application of the magnetohydrodynamic approach explored in the present paper, $k_1\rho_{Li} \ll 1$ (here ρ_{Li} is the ion Larmor radius). Second, the plasma particle density can be infinite only in theory. In practice, the density is limited by values of that in metals, which is of the order of 10^{29} m^{-3}. Third, when studying the full dispersion relation (6) one has to be sure that the wave is of surface nature in both plasmas. This imposes restrictions on the wavenumber, $k_y^2 > \Omega_{i1}^2/c^2$. When considering the metal-plasma interface one does not take care of the wave nature in the metal: whether it has surface or oscillating characteristics. This makes it possible to include the range $\Omega_{i2}^2/c^2 < k_y^2 < \Omega_{i1}^2/c^2$ into the consideration. Such a broadening of the wavenumber range can be explained by the presence of significant dissipation in a metal, which can provide a very small depth of the wave penetration into the metal. Fourth, only when the plasma particle densities differ significantly, $n_1 \gg n_2$, which is usually considered as metal-plasma interface, the dispersion relation (6) can be reduced to eq. (8). This means that SW dispersion properties at a sharp interface between two plasma-like media are determined mostly by n_2 (the smaller plasma particle density) while n_1 introduces negligibly small correction to SW eigenfrequency.

The latter conclusion (about the leading role of the plasma with smaller particle density in determining the SW dispersion properties) was reported also for electromagnetic SWs propagating along the sharp interface between two plasmas in Voigt geometry in circular waveguides [36]. However, the contradiction discussed in the present paper was not observed there, since magnetohydrodynamic SWs do not propagate in circular waveguides with frequencies below the lower hybrid frequency [12].

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ЧИ Є ДИСПЕРСІЙНІ ВЛАСТИВОСТІ ЕЛЕКТРОМАГНІТНИХ ПОВЕРХНЕВНИХ ХВИЛЬ НА РІЗКІЙ ПЛОСКІЙ МЕЖІ ПЛАЗМА-МЕТАЛ У ГЕОМЕТРІЇ ФОЙГТА ГРАНИЧНИМ ВИПАДКОМ ЦИХ ХВИЛЬ НА МЕЖІ ДВОХ ПЛАЗМОВИХ СЕРЕДОВИЩ?

І.О. Гірка, М. Тумм

Розглянуто електромагнітні поверхневі хвилі в геометрії Фойгта з метою усунути протиріччя між двома класичними дослідженнями. Одне – досліджувало хвилі на межі двох плазмових середовищ, і було показано, що хвилі не поширюються на частотах, нижче іонної циклотронної. Інше – вивчало хвилі на межі метал-плазма, і було досліджено дисперсійні властивості хвиль, нижче іонної циклотронної частоти.