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TRIVELPIECE-GOULD MODES OF WAVEGUIDE PARTIALLY FILLED WITH NON-NEUTRAL PLASMA. PART 2

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The frequency spectra of electron eigenmodes with the azimuthal number m =1 of a waveguide partially filled with non-neutral plasma are determined numerically within the entire allowable range of electron densities and magnetic field strengths, for various values of the degree of charge neutralization and finite values of the longitudinal wave vector.

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INTRODUCTION

In previous paper [1], the behavior of the frequencies of electron eigenmodes with an azimuthal number m=+1 of a waveguide partially filled with neutral plasma was analyzed. In present paper, which is a continuation of [1], we analyze the behavior of the modes of a waveguide partially filled with non-neutral plasma. All results in paper [1] and in present one are presented in the form of dependences on the para- meter $q=2\omega_{pe}^2/\omega_{ce}^2$ (ω_{pe} and ω_{ce} are cyclotron frequency of electrons). It is an important parameter for non-neutral plasma, which, together with the charge neutralization coefficient f, determines the characteristic plasma frequencies, and the eigenmode frequencies.

1. EIGEN MODES OF WAVEGUIDE PARTIALLY FILLED WITH NON-NEUTRAL PLASMA

The solutions of the dispersion equation (5) in paper [1] for the frequencies ω of the eigenmodes of a waveguide partially filled with non-neutral plasma are shown in Fig. 1 within the entire allowable range of parameter q values ($0 \le q \le q_{max}=1/(1-f)$) for different values of the neutralization coefficients f ($0 \le q \le 0.75$) and fixed values of parameters k_zR_p=1 and R_p/R_w=0.5. Solutions for neutral plasma (f=1) are presented and described in paper [1]. When comparing the behavior of modes, one must remember that at different f intervals of allowable values q vary greatly.

1.1. FAMILIES OF SOLUTIONS

As well as in the case of neutral plasma, solutions of dispersion equation (5) in [1] in the case of non-neutral plasma form four families of modes: SUH (slow upper hybrid), SLH (slow lower hybrid), FLH (fast lower hybrid), and FUH (fast upper hybrid). They are located in areas where oscillations are bulk: $T^2>0$. Here $T^2 \equiv -\varepsilon_3/\varepsilon_1$, ε_1 and ε_3 are the components of the dielectric permeability tensor of cold electron plasma. The boundaries of the areas are shown in Fig. 1 by solid lines. In accordance with the correspondence (1) and results (19), presented in [1], the mode frequencies ω' of these four families lie in the following intervals:

$$\begin{aligned}
&\text{the SLH family: } \left(-\min\left(\omega_{pe},|\Omega_e|\right)<\omega'<0\right),\\
&\text{the FLH family: } \left(0<\omega'<\min\left(\omega_{pe},|\Omega_e|\right)\right), \qquad (1)\\
&\text{the SUH family: } \left(-\omega_{UH}^{NNP}<\omega'<-\max\left(\omega_{pe},|\Omega_e|\right)\right),\\
&\text{the FUH family: } \left(\max\left(\omega_{pe},|\Omega_e|\right)<\omega'<\omega_{UH}^{NNP}\right).
\end{aligned}$$

In (1) $\omega'=\omega-m\omega^e_{rot}$ is the mode frequency in a rotating frame of reference, ω^e_{rot} is the electron rotation frequency in crossed fields, $\omega_{UH}^{NNP}=\sqrt{\omega_{pe}^2+\Omega_e^2}$ is the hybrid frequency of non-neutral plasma,

$$\Omega_e=\operatorname{sgn}(e)|\omega_{ce}|\left[1-q(1-f)\right]^{1/2}<0$$

is the "modified" electron cyclotron frequency.

The various radial modes of the families of upper hybrid modes (both SUH and FUH) of non-neutral plasma are located very close to each other and to the hybrid frequency with a Doppler shift $m\omega^e_{rot}\pm\omega_{UH}^{NNP}$. They remain in its small vicinity as the degree of filling of the waveguide with plasma $(R_p/R_w)^2$ and the parameter k_zR_p change. All these curves merge into one curve on the scale of Fig. 1.

As can be seen from Fig, for values of the neutralization coefficient lying in the interval $0 \le f \le 0.75$, the frequencies of the SUH family modes, being negative, decrease in absolute value with increasing the parameter q. When $f \approx 0.75$, their frequencies remain close to $-|\omega_{ce}|$ ($\omega/\omega_{ce} \approx -1$) throughout the entire allowable range of q ($0 \le q \le q_{max}=1/(1-f)$). If $f>0.75$ frequencies of the SUH modes, while remaining negative, increase in absolute value (see Fig. 2,a in [1]).

The frequencies of the FUH family modes lie in the area of positive frequencies. For all values of f they increase with increasing parameter q.

As it is clearly seen from Fig. 1, the frequencies of the upper hybrid modes of the SUH and FUH families with an azimuth number m=+1 do not fall into the area of low (ion) frequencies and do not cross zero frequency for any values of the neutralization coefficient $0 \le f \le 1$.This was also the case when the wave- guide was completely filled with non-neutral plasma [2, 3]. The frequencies of the SUH mode are closest to zero at $q=q_{max}$ and f=0 (see Fig. 1,d). Their frequencies are equal here

$\omega/|\omega_{ce}| = (1...2^{1/2})/2 \approx -0.2$. As parameter f increases, the frequencies of SUH modes move away from zero frequency.

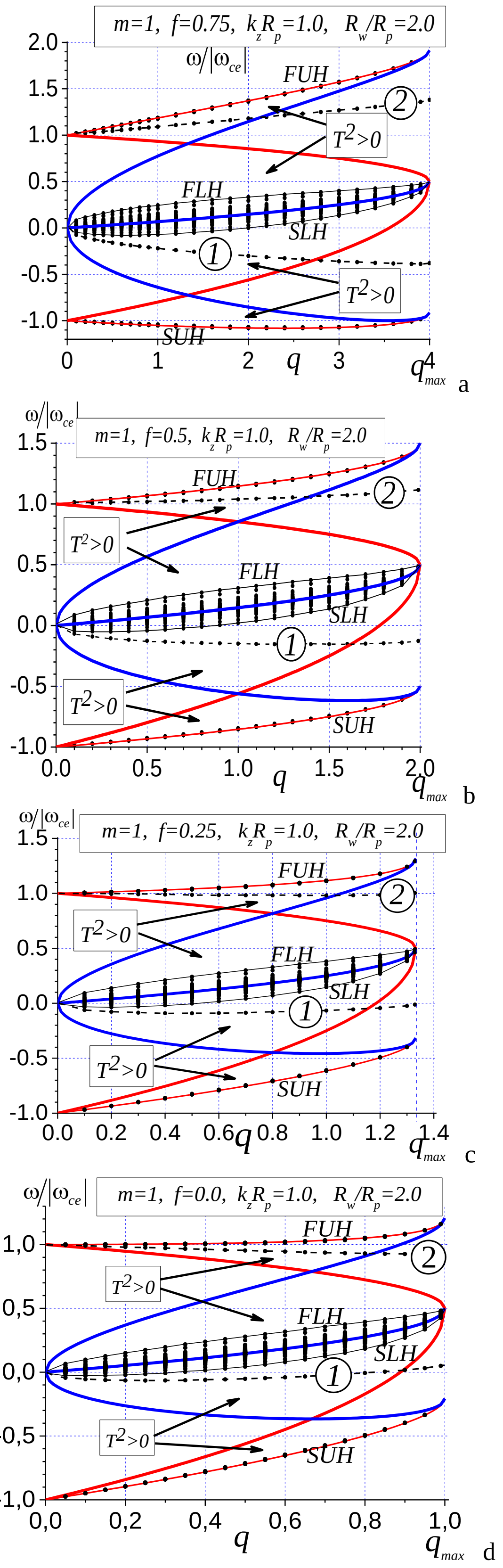

Fig. 1. Normalized frequencies $\omega/|\omega_{ce}|$ of eigenmodes of a waveguide partially filled with plasma depending on the parameter q at various values of the neutralization coefficient (f): f = 0.75 (a); 0.5 (b); 0.25 (c); 0 (d)

The families of lower hybrid modes SLH and FLH are located in the vicinity of the frequency $\omega = m\omega^{e}_{rot}$. In Fig. 1, this frequency dependence is indicated by a thick solid line inside the area where the SLH and FLH families are located. Their radial modes are not as closely spaced as the upper hybrid modes. As the parameter k_zR_p decreases, the mode frequencies approach the frequency $m\omega^{e}_{rot}$ ($\omega \to m\omega^{e}_{rot}$ when $k_zR_p \to 0$). The modes of the SLH and FLH families weakly depend on the degree of plasma filling of the waveguide $(R_p/R_w)^2$(for a fixed value of k_zR_p). This factor leads to a slight asymmetry of the SLH and FLH modes with respect to frequency $m\omega^{e}_{rot}$. The modes are shifted towards positive frequencies. Note that the above-analyzed upper hybrid modes SUH and FUH are also shifted, but to a much lesser extent.

All radial modes of the SLH family fall into the area of low (ion) frequencies and reach zero frequency (see Fig. 1), just as it was in the case of waveguide completely filled with plasma [2, 3]. Long-wavelength modes ($k_zR_p << 1$) fall into the low-frequency area at small values of q ($q<<1$). Shorter wavelength modes – at larger values q.

1.2. DIOCOTRON AND CYCLOTRON MODES

In addition to four families of solutions, there are two more modes indicated in Fig. 1 with dotted lines and the numbers “1” and “2”. They are located near to the SLH and FUH families, but separately from them, and depend noticeably differently on the calculation parameters. As can be seen from Fig. 1, when $q \to 0$ they approach and merge with the corresponding family. This demonstrates the relationship between modes “1” and “2” with the SLH and FUH families. Note, modes “1” and “2” are bulk at sufficiently small q. They become surface at larger q.

Mode "1" is a diocotron mode with a finite value of the longitudinal wave vector k_z (the term was introduced in [4]). Fig. 1 show the results of calculations for a not small value k_z ($k_zR_p=1$). With a decrease in the parameter k_zR_p from 1 (at a constant degree of filling the waveguide with plasma, $(R_p/R_w)^2=const$), mode “1” shifts upward towards the SLH family. The frequencies of these modes, in turn, approach the frequency $m\omega^{e}_{rot}$. However, when $k_zR_p \to 0$ (and fixed $q \neq 0$) the frequency of mode “1” ω_1 does not tend to $m\omega^{e}_{rot}$, as do the frequencies of all modes of the SLH family. The frequency ω_1 tends to the value determined by (3).

With an increase in the charge neutralization coefficient f from 0 to 1, mode “1” shifts towards negative frequencies. When $f>0.25$, the mode frequency ω_1 is negative within the entire allowable range of change q (for the numerical values of the parameters chosen in the calculations).

With an increase in the degree of filling of the waveguide with non-neutral plasma ($(R_p/R_w)^2 \to 1$), mode “1” closely approaches the SLH mode family and merges with it, becoming its lowest radial mode [4].

In the long-wavelength approximation ($k_z=0$), the solutions of the dispersion equation are found analytically. Frequencies of modes “1” and “2” are given by expression (2.9.8) in [5]. At $m=1$, $f=0$ and $q<<<1$ the frequency of mode “1” (diocotron mode) ω_1

has a well-known expression: $\omega_1=\omega_d=(R_p/R_w)^2\omega^e_{rot}$. It can be shown that for small $k_zR_p\neq 0$ and arbitrary R_p/R_w, $f\neq 0$, the expression for the frequency of the diocotron mode ω_1 takes the following form:

$$\omega_1 = (q/4)|\omega_{ce}|\left\{\left[(R_p/R_w)^2 - f\right] - (k_z R_p)^2\right\} \qquad (3)$$

(k_zR_p<<1, q<<q_{max}). For k_zR_p=0, the expression for the frequency ω_1 is given in [6] (Sec.VII). For f=0 it is given in [5]. When $f\neq 0$, the correction to the frequency due to k_zR_p, retains the same form as for f=0. The correction is negative and does not depend on f and q.

As follows from (3), as well as from Figs. 1, 2, the sign of the frequency ω_1 and, in general, the behavior of the diocotron mode is determined by three parameters (k_zR_p, R_p/R_w, f). When $(R_p/R_w)^2>f$ frequency (3) increases with growth q (see Fig. 2,a). It reaches zero at

$$q = q_{res} = (k_z R_p)^2 \Big/ \left[(R_p/R_w)^2 - f\right], \qquad (4)$$

and becomes positive at larger values q. When $(R_p/R_w)^2<f$ (see Fig. 2,c), the frequency decreases with increasing q. It is negative within the entire range of change q and does not reach zero at $q\neq 0$. When $(R_p/R_w)^2\approx f$, the frequency ω_1 remains close to zero within the entire range of allowable values q (see Fig. 2,b).

Numerical calculations according to dispersion equation (5) in paper [1] for small values of k_zR_p, presented in Fig. 2 showed behavior of mode "1" close to expression (3) within the entire range of allowable values q, except for very small values of q.

Note when k_z=0 and f=0, the frequency ω_1 of the diocotron mode is positive ($\omega_1=\omega_d>0$). When k_zR_p=1 the frequency of mode "1" ω_1 is negative in most of the interval of change of q and f(see Fig. 1). The change of the frequency sign of the diocotron mode with increasing k_zR_p was discovered in [4]. With the values of the parameters (R_p/R_w=0.5, $q=8\cdot 10^{-4}$) chosen in [4], this occurs at a very small value of $k_zR_p\approx 0.015$.

The behavior of mode "1" at considerable value k_zR_p=1 (see Fig. 1) is not described by expression (3) and differs noticeably from the behavior of the same mode at small values k_zR_p (see Fig. 2). However, the trend of shifting the frequency of mode "1" towards negative frequencies with increasing f is observed both at small and at considerable values of k_zR_p.

Note that in both non-neutral and neutral plasmas, mode "1" crosses the line on which the electron cyclotron resonance condition is satisfied, which in crossed fields has the form $\omega'=\Omega_e$. This can be seen in Fig. 1 of this paper and Fig. 2 of paper [1]. Near the point of crossing the relation is satisfied: $\varepsilon_1\to\pm\infty$. Oscillations with such a frequency have a resonant effect on electrons, lead to an increase in their Larmor radius, and the linear theory, within the frame of which the dispersion equation (5) in [1] was obtained, is not applicable.

Mode "2" is located near the modes of FUH family. When k_z=0 its frequency is determined by expression (2.9.8) in [5]. When values of q are small but finite and $k_z\to 0$, the frequency of mode "2" ω_2 does not tend to the upper hybrid frequency with a Doppler shift $m\omega^e_{rot}+(\Omega_e^2+\omega_{pe}^2)^{1/2}$, as all modes of the FUH family do. When k_zR_p <<1 and q<<q_{max} the frequency of mode "2" is approximately equal to:

$$\frac{\omega_2}{|\omega_{ce}|} \approx 1 - \frac{q}{4}\left\{\left[\left(\frac{R_p}{R_w}\right)^2 - f\right] + \frac{1}{2}\left[\left(\frac{R_p}{R_w}\right)^2 - 1\right]^3 \left(\frac{k_z R_p}{2}\right)^2\right\}. \qquad (5)$$

The correction to the frequency ω_2 due to $(k_zR_p)^2$ is positive. When $f>(R_p/R_w)^2$, the frequency ω_2 is larger than $|\omega_{ce}|$, when $f<(R_p/R_w)^2$ it is less than $|\omega_{ce}|$.

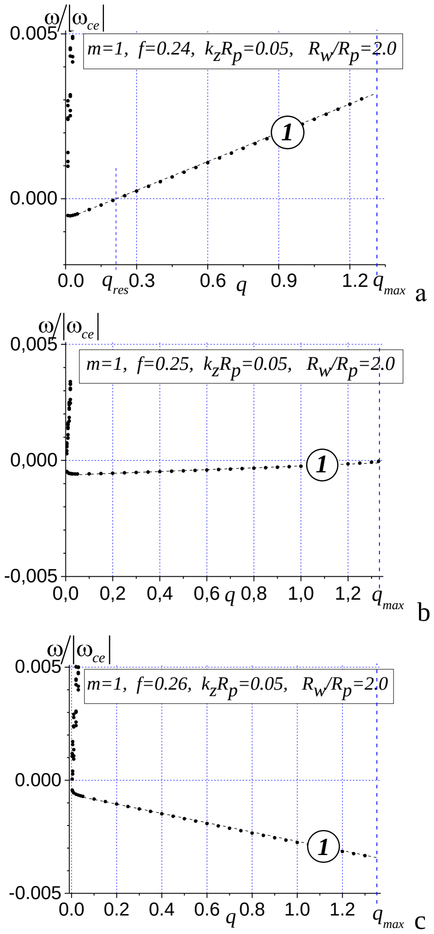

Fig. 2. Behavior of the long-wavelength mode "1" (k_zR_p<<1) at different values of the charge neutralization coefficient f close to the value $f\approx(R_p/R_w)^2$ (f=0.24 (a); f=0.25 (b); f=0.26 (c)). The value q_{res} in Fig. 2,a is determined by (4)

When the degree of filling of the waveguide with plasma decreases ($(R_p/R_w)^2\to 0$) and f=0 the frequency ω_2 approaches the cyclotron frequency $|\omega_{ce}|$. When the degree of filling increases ($(R_p/R_w)^2\to 1$), mode "2" moves away from the FUH family, its frequency ω_2 approaches the "fast" electron rotation frequency (equal to $|\Omega_e|+\omega^e_{rot}=|\omega_{ce}|-\omega^e_{rot}$), which is less than $|\omega_{ce}|$.

From Fig. 1 it is seen that in both non-neutral and neutral plasmas mode "2" crosses the line on which the condition ε_3=0 is satisfied, i.e. the mode frequency in the frame of reference rotating with the frequency ω^e_{rot} coincides with the Langmuir frequency ($\omega'=\pm\omega_{pe}$).

It seems that mode "2" with azimuthal number m=1 and frequency (5) was observed under conditions q<<1 in experiments [7] and was called there the "cyclotron" mode. It remains unclear why, at m=1, FUH family modes were not observed, which, like the “2” mode, rotate in the positive direction, and whose frequencies are also close to $|\omega_{ce}|$, as well as SUH modes, which rotate in the negative direction and whose frequencies are close to $-|\omega_{ce}|$.

CONCLUSIONS

1. For a waveguide partially filled with non-neutral plasma, the behavior of eigenmode frequencies with an azimuthal number $m=1$ and a finite value of the longitudinal wave vector k_z is determined within the entire allowable range of parameter q values for various values of the neutralization coefficient f and fixed values of the parameters k_zR_p and (R_p/R_w).

Frequency dependences $\omega(q)$ are more suitable for the purposes of comparing theory with experiment than “traditional” dependences $\omega(k_z)$. The dependencies $\omega(q)$ turned out to be useful for the theory itself. They made it possible to demonstrate the relation between mode “1” and the family of lower hybrid modes SLH, and mode “2” with the family of upper hybrid modes FUH. Due to dependences $\omega(q)$ the areas of surface modes and bulk ones were determined. Areas have an extremely simple form. It is a consequence of the lucky choice of variables in which they are presented.

Eigenmodes form four families: two families of upper hybrid modes (SUH and FUH) and two families of lower hybrid modes (SLH and FLH). The modes of these families are bulk within the entire range of allowable values q.

There are also two modes that are located separately from these families – mode “1” (diocotron) and mode “2” (“cyclotron”). They are bulk at sufficiently small values of the parameter q and become surface at larger values.

2. The mode frequencies of the upper hybrid families SUH and FUH are extremely closely spaced and very close to the upper hybrid frequency with a Doppler shift, $\omega \approx m\omega^e_{rot} \pm \omega_{UH}^{NNP}$. These modes (with an azimuthal number $m=1$) do not fall into the low-frequency area either with complete or partial filling of the waveguide with plasma, for any values of the charge neutralization coefficient f and parameter q.

3. The frequencies of the lower hybrid modes of the SLH and FLH families are located in the vicinity of the frequency $\omega \approx m\omega^e_{rot}$. These modes fall into the area of low (ion) frequencies, as it was in the case of waveguide completely filled with non-neutral plasma.

4. Mode “1” (diocotron) with a finite value k_z in non-neutral plasma can fall into the low frequency area and reach zero frequency. It depends on the relationship between the filling degree of the waveguide $(R_p/R_w)^2$ and the charge neutralization coefficient of the plasma f. In the long-wavelength limit ($k_zR_p \ll 1$), an analytical expression for the frequency of the diocotron mode ω_1 is obtained. It is shown that diocotron mode reaches the zero of frequency if the waveguide is sufficiently filled with plasma, $(R_p/R_w)^2 > f$. Under the condition $(R_p/R_w)^2 \approx f$ long-wavelength ($k_zR_p \ll 1$) diocotron mode “1” lies in the low-frequency area ($\omega \lll \omega_{ce}$) within the entire allowable range of parameter q values.

5. Mode “2” (cyclotron) does not fall into the low frequency area. Its frequency ω_2 remains on the order of the cyclotron frequency $|\omega_{ce}|$ within the entire range of allowable values of the parameter q and for any values of the charge neutralization coefficient f. An analytical expression for the frequency of the cyclotron mode ω_2 is obtained. It is shown that under condition $(R_p/R_w)^2 < f$, the frequency of mode “2” is higher than the cyclotron frequency $|\omega_{ce}|$ and it increases with increasing q. When the opposite inequality is satisfied, the frequency is less $|\omega_{ce}|$ and it decreases with increasing q.

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МОДИ ТРАЙВЕЛПІСА-ГУЛДА ХВИЛЕВОДУ, ЧАСТКОВО ЗАПОВНЕНОГО ЗАРЯДЖЕНОЮ ПЛАЗМОЮ. ЧАСТИНА 2

Ю.М. Єлісеєв

Частотний спектр власних електронних мод з азимутальним числом m = 1, визначений чисельно для хвилеводу, частково заповненого зарядженою плазмою. Розрахунки виконані в межах усього допустимого діапазону густини електронів і напруженостей магнітного поля для різних значень ступеня зарядової нейтралізації і поздовжнього хвилевого вектора.