https://doi.org/10.46813/2023-143-032

INFLUENCE OF ION VISCOSITY ON THE DISTRIBUTION OF PARAMETERS IN THE SHEATH AT THE BOUNDARY OF A STATIONARY WEAKLY IONIZED STRONGLY NONISOTHERMAL PLASMA

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A stationary weakly ionized highly nonisothermal plasma is considered in the hydrodynamic approximation. Taking into account the effects of ionization, recharging, and a self-consistent field, the effect of ion viscosity on the distribution of plasma discharge parameters in the sheath was investigated. Distributions of hydrodynamicion velocity and ion density, electron density, and self-consistent field potential in the sheath were obtained.

PACS: 52.40.Kh, 52.27.Cm, 52.25.Ya

INTRODUCTION

The interaction of plasma with surfaces that contact this plasma is an important area of research into the stationary state of a gas discharge. The negative potential of a wall with respect to the plasma, which arises due to the high mobility of electrons, attracts ions and repels electrons. Thus, a region of a positive space charge is formed near the wall and this space charges creens the neutral plasma from the negatively charged wall. Under these conditions, it is convenient to separate the plasma into two parts. The first part is the main plasma volume with characteristic size L (plasma size) where the quasi-neutrality condition is satisfied almost exactly. This part will be called below the quasi-neutrality region. The second part is a narrow region near the wall where ions are mainly concentrated, and the quasi-neutrality condition is not satisfied. This part will be called below the sheath. The characteristic width of the she at his in the range from several to several tens of Debye-Hückel screening radius $r_{De} = (T_e/(4\pi e^2 n_{e0}))^{1/2}$, where T_e – electron temperature, e – the charge of the electron, and n_{e0} – hydrodynamic density of electrons in the middle of the plasma. Typically, the value of r_{De} is small compared to other characteristic quantities, such as the plasma size L or the mean free paths of ions due to ionization, charge exchange, or collisions.

Notable early works devoted to studies of stationary plasma include [1-4]. In [1], it was assumed for the first time that the velocity of ions is determined by a static self-consistent electric field created by the balance of electric charges of electrons and ions. Based on this, an integral equation for the distribution of the plasma-sheath potential for different geometries, ion mean free paths and ionization methods was obtained. The solution of this equation in the case of a short ion mean free path in a cylinder with an ions generation proportional to the electron density gave the same potential distribution as found by Schottky [2, 3] for a positive column using the theory of ambipolar diffusion. As Bohm showed [4], the formation of a stationary layer of space charge was possible only under the condition that ions entered the region of the sheath at a speed larger than the ion-sound speed, $\upsilon_s = \upsilon_B = (T_e/m_i)^{1/2}$. This condition was obtained in the case of cold ions ($T_i = 0$) and without taking the viscosity into account. At $T_i \neq 0$ Bohm's speed is equal to $\upsilon_B = \upsilon_s(1 + \tau)^{1/2}$, where $\tau = T_i/T_e \ll 1$. Consequently, the ions are preliminarily accelerated by a self-consistent electric field in the quasi-neutrality region. If the ions move towards the plasma-confining surfaces under the action of a self-consistent electric field, there must be a potential maximum in the plasma. In the case of plane walls that will be considered in the paper, the symmetry dictates that this maximum is located in a plane in the middle of the plasma. It is convenient to choose the origin of coordinates $x = 0$ in this plane. Then, the dielectric walls that confine the plasma are located at $x = \pm L$ (Fig. 1).

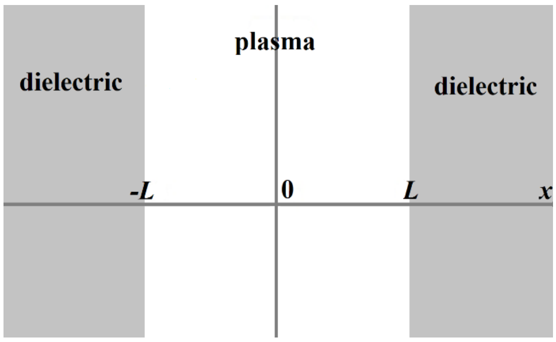

Fig. 1. Schematic layout of the plasma layer and the dielectric, confining the plasma, concerning the x coordinate

Further studies of the stationary state of plasma generated a large number of papers. Numerous references can be found, for example, in the works of Riemann [5, 6].

In the majority of works, the ions viscosity is considered a small parameter and disregarded in the ions motion equation. For example, [7] gives the condition when effects associated with viscosity can be neglected in the transport equations: $\upsilon_i \ll \nu L_\upsilon$, where υ_i is the hydrodynamic velocity of ions, ν is the frequency of collisions, L_υ is the characteristic scale of change in the hydrodynamic velocity. It is questionable that $\upsilon_i \ll \nu L_\upsilon$ is satisfied in the sheath even when this condition holds true in the quasi-neutrality region.

In [8], in the case of a stationary, weakly ionized, strongly noni so thermal plasma, estimates were obtained on how the viscosity of ions affects solutions of the system of hydrodynamic equations, taking into account the effects of ionization, charge exchange, and the self-consistent field together with the Poisson equation in the entire plasma volume. Solutions to this system of equations were also obtained taking into account the viscosity of the ions but only in the quasi-neutrality region.

The present work is a continuation of [8]. It investigates the solution of the system of the above equations in the sheath at the plasma boundary, taking into account the viscosity of the ions. The temperature of electrons and ions and the density of neutral particles (hydrogen) are assumed to be constant. An original numerical algorithm was developed to solve a system of three differential equations, two of which are of the second order and the third one is of the first order, by the Cauchy method for the initial values problem, taking into account the condition on the right boundary of the solution domain. This method is an alternative to solving the boundary value problem for eigenfunctions and eigenvalues. The position of the wall was determined by the condition of the equality of electron and ion fluxes.

The work is organized as follows. Section 2 describes the formulation of the problem and derives the basic equations. Section 3 is devoted to the description of the quasi-neutral approximation for solving the main system of equations. Section 4 describes the procedure for solving the basic equations in the sheath and presents the main results. Section 5 presents the conclusions of this research work.

1. BASIC EQUATIONS

To solve the problem of the stationary distribution of plasma parameters in gas discharges, we will use the hydrodynamic approximation. This approach can be used when the macroscopic parameters of the plasma, such as the hydrodynamic velocity v and density n of the particles, change rather slowly in space and time. Namely, the characteristic distances at which the values of macroscopic quantities change are much larger than the mean free path [9]. This approach is also valid in the case of a collision less plasma if the thermal motion of particles can be neglected, that is, the plasma must be sufficiently cold [10]. However, even if these conditions are not fulfilled, the hydrodynamic approach can be used for a qualitative analysis of plasma parameters.

To calculate the effect of viscosity on the distribution of plasma parameters, we will use the stationary system of the hydrodynamic equations of continuity and motion, complemented by the Poisson equation. We consider the one-dimensional case for dimensionless variables $\upsilon = \upsilon_i/\upsilon_s$, $n = n_i/n_{e0}$, $\Phi = e\varphi/T_e$ taking into account the effects of ionization, charge exchange, and a self-consistent electric field, where υ_i, n_i – hydrodynamic velocity and density of ions, φ – the self-consistent electric field potential. We assume that the electron density is determined by the Boltzmann formula $n_e = n_{e0}\exp(e\varphi/T_e)$.

$$v v' = -\Phi' - \left(\nu + \alpha \frac{e^{\Phi}}{n}\right) v - \tau \frac{n'}{n} + \frac{4}{3}\overline{\eta} v'', \quad (1)$$

$$n' v + n v' = \alpha e^{\Phi}, \quad (2)$$

$$\Phi'' = e^{\Phi} - n, \quad (3)$$

where $\alpha = \alpha_e/\omega_{pi}$, $\nu = \nu_{ex}/\omega_{pi}$, $\overline{\eta} = 4\overline{\eta}_i/(3m_i n_i)$, α, α_e, ν, ν_{ex}, $\overline{\eta}$, and $\overline{\eta}_i$ – dimensionless and dimensional frequencies of electron impact ionization and charge exchange of ions on hydrogen atoms and kinematic viscosity coefficients of ions. The prime denotes the derivative with respect to the dimensionless coordinate x/r_{De}. A detailed derivation of the system of equations (1)-(3) and a description of the hydrodynamic coefficients are given in the work [8].

System of differential equations (1)-(3) considering the viscosity of the ions is a fifth-order nonlinear system for the unknown functions. It must be supplemented by the boundary conditions. For reasons of symmetry, at the center of the plasma, the following is true

$$v(0) = v''(0) = n'(0) = \Phi'(0) = 0. \quad (4)$$

Also, we can assume that $\Phi(0) = 0$ because the potential is defined up to a constant. One more boundary condition is the condition at the plasma-wall boundary: the hydrodynamic ions flux is equal to the electrons flux in the direction of the coordinate x growth. Additionally, it is assumed that the electrons are distributed according to the Maxwell-Boltzmann distribution, and there are no effects of reflection from the wall and electron emission on the wall [11]

$$\Gamma(L) = n(L)v(L) = \sqrt{m_i/(2\pi m_e)}\exp(\Phi(L)). \quad (5)$$

Thus, we have a system of fifth-order equations with five boundary conditions, i.e., the problem of determining the eigenvalues. For example, if L, T_e, T_i, and n_n are given, then stationary gas discharge is possible at a certain valuen_{e0} that is determined by the solution of the system.

Finding eigenfunctions and eigenvalues of a nonlinear system (1)-(3) is a rather difficult problem. Therefore, an alternative approach is used. We take boundary conditions (4) as initial conditions, supplement them with an arbitrary initial value for n_{e0}, and integrate system (1)-(3) in the direction of positive x, that is, we solve the Cauchy problem (see, for example, [6], [12], where the case of cold ions was considered).

2. QUASI-NEUTRAL APPROXIMATION

We use the smallness of the second derivative of the potential in equation (3): $\Phi'' \ll n$, e^{Φ}. Let's call this approach the “quasi-neutral approximation”. In the 1st iteration ($\Phi'' = 0$), from equation (3) we get

$$n_1 = e^{\Phi_1}, \ \Phi_1 = \ln n_1, \ \Phi_1' = \frac{n_1'}{n_1}, \quad (6)$$

$$\Phi_1'' = \frac{n_1''}{n_1} - \left(\frac{n_1'}{n_1}\right)^2. \quad (7)$$

Expressions (6) are then substituted in equations (1) and (2), which take the form

$$v_1 v_1' = -(1+\tau)\frac{n_1'}{n_1} - (\nu + \alpha) v_1 + \frac{4}{3}\overline{\eta} v_1'', \quad (8)$$

$$n_1' v_1 + n_1 v_1' = \alpha n_1. \quad (9)$$

In the 1st iteration, we consider $\Phi''=0$ and by solving equations (6), (8) and (9) we find the solutions υ_1, n_1 and Φ_1. In the 2nd iteration, $\Phi''=\Phi_1''$ is determined by expression (7) and equations (6), (8) and (9) take the form

$$n_2 = e^{\Phi_2} - \Phi_1'', \qquad \Phi_2 = \ln(n_2 + \Phi_1''),$$

$$\Phi_2' = \frac{n_2' + \Phi_1'''}{n_2 + \Phi_1''}, \tag{10}$$

$$v_2 v_2' = -\frac{n_2' + \Phi_1'''}{n_2 + \Phi_1''} - \left(\nu + \alpha\frac{n_2 + \Phi_1''}{n_2}\right) v_2 - \tau\frac{n_2'}{n_2} + \\ + \frac{4}{3}\bar{\eta} v_2'', \tag{11}$$

$$n_2' v_2 + n_2 v_2' = \alpha(n_2 + \Phi_1''). \tag{12}$$

It is convenient to find the initial values of the plasma parameters at the origin of coordinates using expansions of the plasma parameters near this point

$$\Phi = (a_0 + a_1\bar{x}^2 + ..)\bar{x}^2, v = (b_0 + b_1\bar{x}^2 + ..)\bar{x}, \\ n = c_0 + c_1\bar{x}^2 + .., \tag{13}$$

where $\bar{x} = x/r_{De}$. We find coefficients of these expansions with the required accuracy by substituting (13) into (1)-(3) and equating terms with equal powers while considering the sufficient number of expansion terms. In [6], a similar expansion was used to overcome the problem of the singularity in the middle of the plasma in the region near $x = 0$ for the case of cold ions ($T_i = 0$).

Using the quasi-neutral approximation for the system of equations (1)-(3) without the viscosity was necessary to overcome the singularity at the point $\upsilon = \upsilon_{Ti}$. When the viscosity was included in consideration, the order of equation (1) increased, and this singularity was eliminated. However, the necessity of using the quasi-neutral approximation remained.

The following values of dimensional and dimensionless quantities were used in the calculations: $n_{e0} = 10^{10}$ cm^{-3}, $T_e = 2$ eV, $T_i = 0.1$ eV, $n_n = 10^{14}$ cm^{-3}, $\upsilon_{Te} = 5.93 \cdot 10^7$ cm/c, $\upsilon_{Ti} = 3.09 \cdot 10^5$ cm/c, $\upsilon_s = 1.38 \cdot 10^6$ cm/c, $r_{De} = 1.05 \cdot 10^{-2}$ cm, $\omega_{pi} = 1.32 \cdot 10^8$ c^{-1}, $\alpha_e = 1.39 \cdot 10^3$ c^{-1}, $\nu_{ex} = 2.74 \cdot 10^5$ c^{-1}, $\nu_{Ci} = 1.49 \cdot 10^5$ c^{-1}, $\bar{\eta}_i = 2.25 \cdot 10^5$ cm^2/c, $\alpha = 1.05 \cdot 10^{-5}$, $\nu = 2.08 \cdot 10^{-3}$, $\nu_C = 1.13 \cdot 10^{-3}$, $\bar{\eta} = 15.5$. Solutions of the systems of equations (8), (9) using (6) and (11), (12) using (7) and (10) taking into account the viscosity were obtained in the work [8]. Viscosity influences the variation of plasma parameters in the quasi-neutrality region very little and produces only a slight increase of the size of this region. Fig. 2 shows the dependences of the dimensionless hydrodynamic ions velocity υ_2, the densities of ions n_2 and electrons n_{e2}/n_{e0}, the potential of the self-consistent electric field – Φ_2, the flow of ions $n_2\upsilon_2$ in the second iteration of the quasi-neutral approximation and the hydrodynamic density of ions n_1 in the first iteration of the quasi-neutral approximation on the coordinate x at the plasma boundary in the case when the viscosity is taken into account. Dependencies in Fig. 2 are limited when the condition (5) on the wall ($L_q \approx 105.857$ cm) is satisfied. It should be noted that deviation of the quasi-neutral solution $n_2$from the exact solution n of the system of equations (1)-(3) increases with an increase of the space charge $(n_2 - n_{e2}/n_{e0})$. As is known, the beginning of the sheath and the boundary of the region of applicability of the quasi-neutral approximation are defined by the point in space where the fraction of the space charge becomes noticeable: $(n_2 - n_{e2}/n_{e0})/n_2 \approx 0.01$. In Fig. 2, this condition corresponds to the point $x \approx 105.5$ cm. Therefore, when $x \gtrsim 105.5$ cm, the dependencies in this figure are qualitative only and are different from the exact solutions of the system of equations (1)-(3).

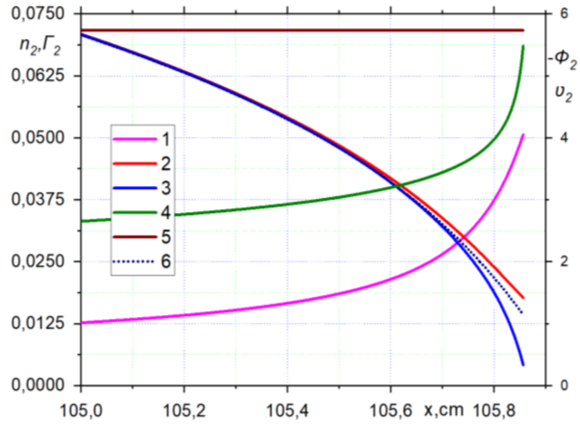

Fig. 2. Dependencies of the dimensionless hydrodynamic ions velocity υ_2 (1), the densities of ions n_2 (2) and electrons n_{e2}/n_{e0} (3), the potential of the self-consistent electric field – Φ_2 (4), the flow of ions $\Gamma_2 = n_2\upsilon_2$ (5) in the second iteration of the quasi-neutral approximation and the dimensionless hydrodynamic density of ions n_1 (6) in the first iteration of this approximation on the coordinate x at the plasma boundary, taking the viscosity into account

3. SOLUTION OF THE SYSTEM OF BASIC EQUATIONS IN THE SHEATH

Without taking the viscosity into account, the system of equations (1)-(3) has a singularity at the point $\upsilon = \upsilon_{Ti}$. Therefore, from the middle of the plasma ($x = 0$) to the point $\upsilon = \upsilon_s$, the problem was solved by the Cauchy method with the help of the quasi-neutral approximation. The quasi-neutral solutions in the second iteration n_2, υ_2 and Φ_2 were obtained.

At the point $\upsilon = \upsilon_s$, the quasi-neutral solution has a singularity. After passing the singularity point $\upsilon = \upsilon_{Ti}$ at the point $\upsilon_{Ti} < \upsilon < \upsilon_s$ (the joining point), these solutions n_2, υ_2, Φ_2, and Φ_2' were used as initial conditions for solving the system of equations (1)-(3) without the last term in (1). In this way, the solution n of the system of equations (1)-(3) was obtained from the joining point to the plasma boundary without taking into account the viscosity [8]. However, it should be understood that the mathematical solutions of systems of equations without viscosity n_2 and n always differ by some value $n_2 - n$. This value $n_2 - n \ll |\Phi_1''|$ is small in the quasi-neutrality region and increases as the plasma boundary is approached. For a smoother joining, one can take into account the difference Δn in the initial conditions of the system of equations (1)-(3) at the joining point without taking into account the viscosity. Any iteration of the quasi-neutral approximation can be chosen as an approximate solution. Then, these initial conditions can be written as:

$$n = n_2 + \Delta n \quad (14)$$

and from the system of equations (1) - (3)

$$v = v_2 + \Delta v \approx \frac{n_2 v_2}{n_2+\Delta n}, \quad (15)$$

$$\Phi = \Phi_2 + \Delta\Phi \approx \ln(e^{\Phi_2} + \Delta n). \quad (16)$$

When obtaining (15) and (16), we neglected changes in the derivatives of the plasma parameters when the density changes by Δn at the joining point.

Our calculations have shown that the system of equations (1)-(3) is stable against a change in the value of Δn in the initial conditions (14)-(16). When Δn is taken into account, the solution of the system of equations (1)-(3) without taking into account viscosity is shifted by a distance $x(n_2+\Delta n) - x(n_2)$, which is much smaller than the size of the sheath. Therefore, for calculations without taking into account the viscosity [8] in the initial conditions of the system of equations (1)-(3), the value $\Delta n = 0$ was chosen at the joining point.

As already noted above, taking into account the viscosity increases the order of the equation of motion of ions (1). This leads to the situation when two second-order equations appear in the system (1)-(3). The eigenfunctions of these equations are exponential functions. In addition, the singularity disappears from systems (1)-(3), (8)-(9), and (11)-(12). In this case, the system of equations (1)-(3) is unstable against a change in the value of Δn in the initial conditions (14)-(16). Fig. 3 shows the dependences of the difference n_2 - n of the dimensionless hydrodynamic ion densities, which were obtained in the second iteration of the quasi-neutral approximation and when solving the system of equations (1)-(3), on x coordinate for different values of Δn in the initial conditions on different scales. The blue line corresponds to the solution with $\Delta n \approx 4.079 \cdot 10^{-6}$ and $L_1 \approx 104.29$ cm. In this case, as L is approached (condition (5) is satisfied), the ion density sharply increases. The green line corresponds to the solution with $\Delta n \approx 4.086 \cdot 10^{-6}$ and $L_2 \approx 104.25$ cm. Here, when approaching L (condition (5) is not satisfied), the potential and density of electrons $\to \infty$.These two solutions are not physical because their graphical dependences cannot pass near the nominal boundary of the quasi-neutrality region ($x \approx 105.5$ cm).The magenta line corresponds to a solution with $\Delta n \approx 4.081 \cdot 10^{-6}$ and $L_3 \approx 105.833$ cm (condition (5) is satisfied).This solution can be a possible solution of the system of equations (1)-(3). Unfortunately, it is impossible to find the unique "correct" solution of the system of equations (1)-(3) with this formulation of the problem. Boundary condition (5) determines the potential $\Phi(L)$ and the hydrodynamic ion flux $n(L)\upsilon(L) \approx const(L)$ at the plasma boundary uniquely, but there is a certain arbitrariness in the choice of $n(L)$ and $\upsilon(L)$. Therefore, the system of equations (1)-(3) can have many possible solutions in this case. The physically meaningful solutions can be defined as follows: near the nominal boundary of the quasi-neutrality region ($x \approx 105.5$ cm), the graphic dependences of the solutions of the system of equations (1)-(3) on x coordinate should not differ much from the second iteration of the quasi-neutral solution.

By setting Δn at the joining point between the blue and green lines (Fig. 3), the dichotomy method can be used to obtain a solution with any boundary values of the hydrodynamic ions density $n(L)$ or ions velocity $\upsilon(L)$ and the corresponding value of L.

The joining of the solutions of the system of equations (1)-(3) and the second iteration of the quasi-neutral approximation was carried out at the point $x \approx 103.52$ cm. This point was chosen from the condition that the number of decimal places in the variable Δn is sufficient to obtain solutions with different values of hydrodynamic ions density $n(L)$ or velocity $\upsilon(L)$ by the dichotomy method. That is, if the value of this point is taken less, then 32 decimal places may not be enough to reach solutions with the desired $n(L)$ or $\upsilon(L)$. In the chosen case, x = 9850 r_{De}.

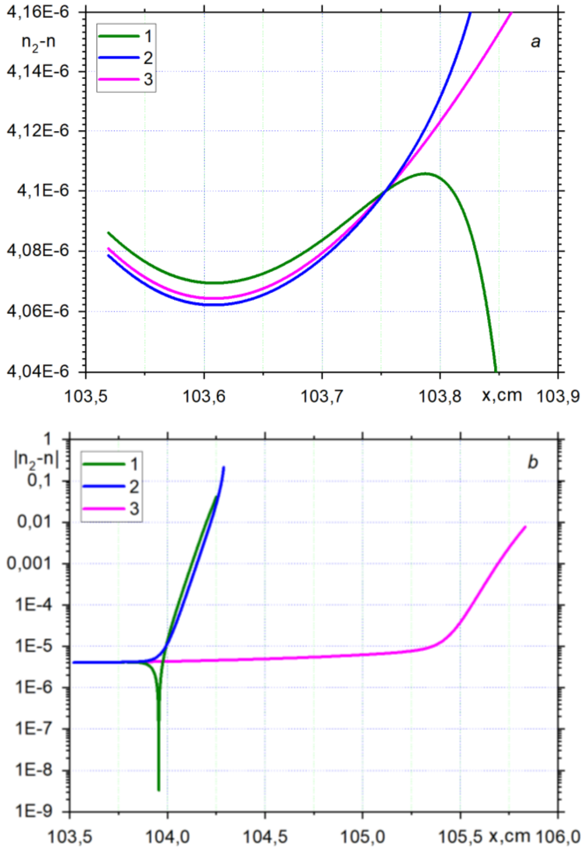

Fig. 3. Dependences of the difference n_2 - n of the dimensionless hydrodynamic ion densities, which were obtained in the second iteration of the quasi-neutral approximation and when solving the system of equations (1)-(3), on x coordinate for different values of Δn in different scales (a, b)

As an example, let us consider the behavior of plasma parameters for three cases when the graphical dependences are slightly different from the second iteration of the quasi-neutral solution near the point $x \approx 105.5$ cm.

Fig. 4 shows the dependences of the ratio of the fraction of the dimensionless space charge $-\Phi''$ to the dimensionless electron density e^{Φ} on the distance $L - x$ to the edge of the plasma for solving the system of equations (1)-(3) for various plasma sizes, taking the viscosity into account, L_3, $L_4 \approx 105.869$ cm, $L_5 \approx 105.920$ cm and without viscosity $L_0 \approx 104.593$ cm. In the absence of viscosity, the size of the sheath is ≈ 0.335 cm. Taking the viscosity into account, for the chosen solutions this value increases to 0.34...0.44 cm (≲15 %). In this case, the plasma size L increases by ~1 %.

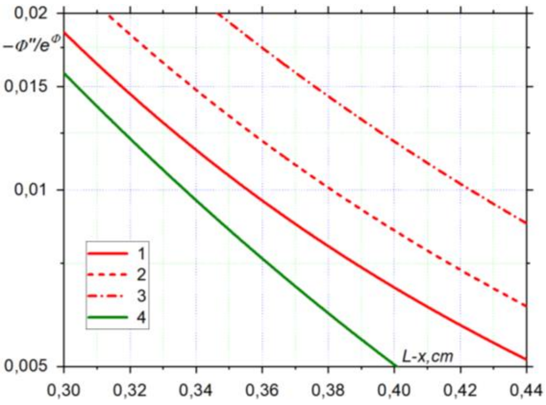

Fig. 4. Dependences of the ratio of the proportion of the dimensionless space charge – Φ'' to the dimensionless electrons density e^{Φ} on the distance L - x for solutions of the system of equations (1)-(3) for different plasma sizes, taking the viscosity into account L_3 (1), L_4 (2), L_5 (3) and without viscosity for L_0(4)

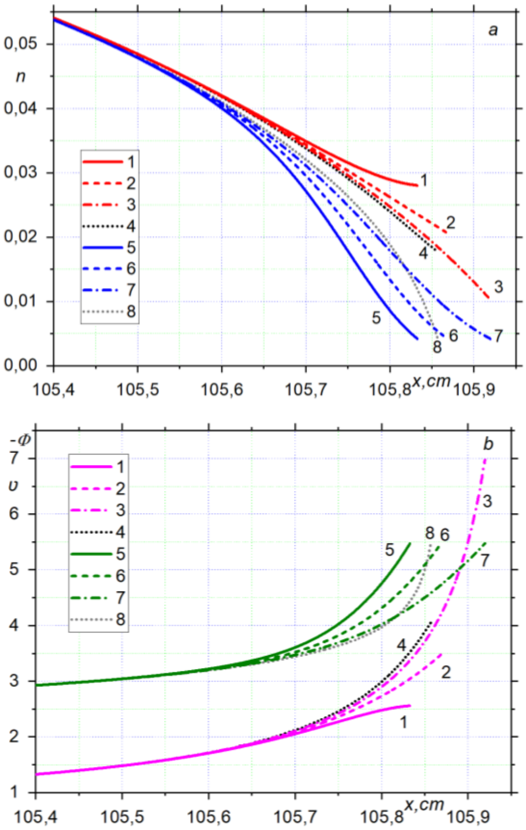

Fig. 5. Dependences of the dimensionless hydrodynamic densities of ions n (a, 1–4) and electrons n_e/n_{e0} (a, 5–8), the ions velocity υ (b, 1–4) and the potential of the self-consistent electric field – Φ (b, 5–8) on x coordinate for various plasma sizes L_3 (1.5), L_4 (2.6), L_5 (3.7) and the second iteration of the quasi-neutral approximation for L_q (4.8) taking the viscosity into account

Fig. 5 shows the dependences of the dimensionless hydrodynamic ions velocity υ, ions density n, electrons density n_e/n_{e0}, and self-consistent electric field potential – Φ on x coordinate for various values of the plasma size L_3, L_4, L_5 and the second iteration (L_q) of the quasi-neutral approximation when the viscosity is taken into account. Solutions of the system of equations (1)-(3), taking into account the viscosity for L_3, L_4, and L_5 at $x \approx 105.5$ cm, do not visually differ from the quasi-neutral solution. It should be understood that such a condition can be satisfied by an infinite number of solutions with different L.

In Fig. 6 the dependences of the dimensionless hydrodynamic velocities υ of ions, the densities of ions n and electrons n_e/n_{e0} and the potential of the self-consistent electric field – Φ on the coordinate $L - x$ in the sheath at the edge of the plasma are shown for cases with the viscosity for L_3, L_4, and L_5 and without it for L_0. The size of the sheath for various cases is visible in Fig. 4.

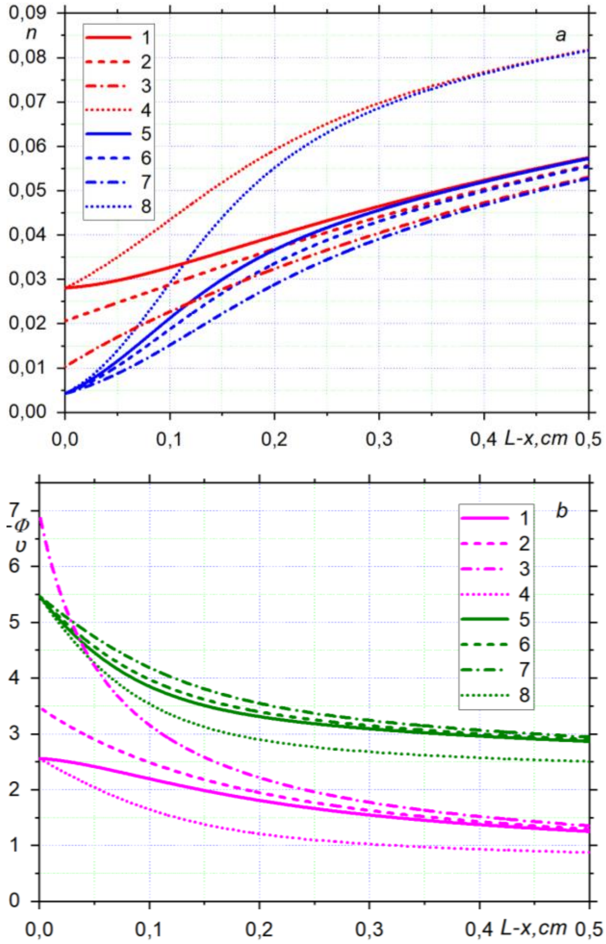

Fig. 6. Dependences of the dimensionless hydrodynamic densities of ions n (a, 1–4) and electrons n_e/n_{e0} (a, 5–8), the ions velocity υ (b, 1–4) and the potential of the self-constrained electric field – Φ (b, 5–8) on x coordinate with the viscosity for L_3 (1,5), L_4 (2,6), L_5 (3,7) and without the viscosity for L_0 (4,8)

Taking the viscosity into account increases the hydrodynamic velocity of ions at the boundary of the sheath from υ_s to $\approx 1.5\upsilon_s$ and makes the change in plasma parameters smoother.

CONCLUSIONS

As is known, in a stationary gas discharge near a surface with which the plasma interacts, a sheath is formed. This is because electrons, due to their greater mobility, charge the wall negatively relative to the rest of the plasma volume. The negative potential of the wall attracts the ions. As a result, a narrow region (the sheath) is formed that screens the surface from the rest of the plasma volume (the quasi-neutrality region). The sheath boundary can be determined from the condition $n_i - n_e \approx 0.01 n_e$. The size of the sheath can be $\approx 30...50\ r_{De}$.

The system of hydrodynamic equations of motion and continuity for ions together with the Poisson equation was solved by the Cauchy method. The values of the parameters at the center of the plasma were used as the initial conditions. The plasma boundary was determined from the condition of equality of the hydrodynamic ions flux and the kinetic electrons flux.

The paper proposes and applies a method for finding the parameters of a weakly ionized, strongly nonisothermal stationary plasma with $T_i \neq 0$, taking into account the viscosity of ions in the sheath. Calculations have shown that when the viscosity is taken into account, plots of the dependences of the plasma parameter on the spatial coordinate become smoother, the size of the sheath increases lightly ($\lesssim$15 %), and the hydrodynamic velocity of ions at the boundary of the sheath increases approximately by ~ 50 %.

ACKNOWLEDGEMENTS

In conclusion, the author expresses deep gratitude to Doctor of Physical and Mathematical Sciences Dmytro Leonidovich Grekov for constructive discussions and valuable advice during the work.

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Article received 13.12.2022

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ВПЛИВ В’ЯЗКОСТІ ІОНІВ НА РОЗПОДІЛ ПАРАМЕТРІВ У ПЕРЕХІДНОМУ ШАРІ НА МЕЖІ СТАЦІОНАРНОЇ СЛАБКО ІОНІЗОВАНОЇ СИЛЬНО НЕІЗОТЕРМІЧНОЇ ПЛАЗМИ

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Розглянуто стаціонарну слабко іонізовану сильно неізотермічну плазму в гідродинамічному наближенні. З урахуванням ефектів іонізації, перезарядки самоузгодженого поля досліджено вплив в’язкості іонів на розподіл параметрів плазмового розряду в перехідному шарі. Отримано розподіли гідродинамічних швидкості та густини іонів, густини електронів і потенціалу самоузгодженого поля у цьому шарі.