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TRIVELPISE-GOULD MODES OF WAVEGUIDE PARTIALLY FILLED WITH NEUTRAL OR NON-NEUTRAL PLASMA

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The frequency spectra of eigenmodes with azimuthal number $m = 1$ are determined by numerical methods for a waveguide partially filled with neutral or non-neutral plasma. Calculations are performed within the entire permissible range of electron density or magnetic field strengths, for different values of the degree of charge neutralization and the longitudinal wave vector. Areas of existence of bulk and surface electronic modes of waveguide are also defined. The results are presented in the form of dependences on the parameter $q = 2\omega_{pe}^2/\omega_{ce}^2$.

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INTRODUCTION

The Trivelpeace-Gould (**TG**) modes [1] – the eigenmodes of a cylindrical waveguide completely or partially filled with neutral plasma in a magnetic field – are well-known. The behavior of the frequencies of the TG modes of a plasma at rest and moving along a magnetic field is determined. The motion of plasma electrons leads to a radical rearrangement of the frequency spectrum of TG modes observed in the laboratory frame of reference. Due to the Doppler shift, the modes traveling towards the motion can become low-frequency. In this case, an electron-ion instability is possible due to the relative motion of electrons and ions (of the Buneman instability type).

TG modes are also known in non-neutral plasma [2], in which, along with the longitudinal magnetic field, there is radial electric field due to the uncompensated space charge of the particles of the plasma itself. Under the action of crossed fields, charged particles drift in azimuth. Moreover, electrons drift faster than ions. The relative motion of electrons and ions is a common property of non-neutral plasma. Usually, high-frequency electronic modes due to the Doppler shift can fall into the area of low (ionic) frequencies. In this case, Cherenkov [3] or cyclotron resonance between the electron mode and ions and the development of electron-ion instability are possible. The instability due to the relative azimuth motion of the plasma components must be universal in non-neutral plasma. The dispersion equations for the electron modes of neutral and non-neutral plasmas and their solutions coincide with the substitutions [2]:

$$\omega \to \omega', \; \omega_{ce} \to \Omega_e . \qquad (1)$$

In (1) ω is the mode frequency in the laboratory frame of reference, $\omega' = \omega - m\omega^e_{rot}$ is the frequency in the frame rotating with frequency ω^e_{rot}, m is the azimuth number, $\omega^e_{rot} = (-\omega_{ce}+\Omega_e)/2 > 0$ is the "slow" electron rotation frequency in crossed fields, $\omega_{ce}=eB/(m_ec)<0$, $e < 0$ and m_e are the charge and mass of the electron, $\Omega_e = \mathrm{sgn}(e)[\omega_{ce}^2-4eE_r/(m_er)]^{1/2}$ is the "modified" cyclotron frequency of electron in crossed fields, the radial electric field E_r is due to the uncompensated plasma space charge $E_r = (m_e/2e)\omega_{pe}^2r(1-f) < 0$, $f = n_i/n_e$ is the charge neutralization coefficient, $n_{e,i}$=const are the electron and ion densities, $\omega_{pe}^2=4\pi e^2n_e/m_e$ is the square of the Langmuir electron frequency, r is the distance to the waveguide axis.

Having reduced the problem of determining the spectra of non-neutral plasma to relations (1), this was often limited. However, for comparison with experiment, the frequencies of non-neutral plasma modes just in the laboratory frame are of interest, and, as can be seen from [1], the motion of the plasma has a significant effect on the shape of the spectra.

The dispersion equation for the electron modes of non-neutral plasma was investigated in various limiting cases. For waves propagating across the magnetic field ($k_z = 0$), the dispersion equation is simplified and admits analytical solutions, one of which (diocotron mode) falls into the region of low (ionic) frequencies [2, 3]. In rare articles, in which the frequencies of TG modes were determined at $k_z \neq 0$, the consideration was carried out in the region of low frequencies, rarefied plasma, and in the absence of ions:

$$\omega, \; \omega^e_{rot} \ll \omega_{pe} \ll |\omega_{ce}|, \quad f = 0 . \qquad (2)$$

We note in this connection the articles [4, 5]. It was shown in [4] that the diocotron and lower hybrid modes reach zero frequencies already at small values k_z and become negative at large values k_z. The results obtained are valid only in a small range of parameter values (2) and do not give an idea of the behavior of modes outside this range. The behavior of the upper hybrid modes was not analyzed in [4, 5], assuming that their frequencies are outside the area (2). However, due to the Doppler shift, the upper hybrid modes of non-neutral plasma can also fall into the area of low (ionic) frequencies and lead to electron–ion instability.

In the limiting case of a waveguide completely filled with non-neutral plasma ($R_p = R_w$, where $R_{p,w}$ are the plasma and waveguide radii), the dispersion equation is also simplified. Its solutions are found analytically (see [2]). The solutions define four families of electronic modes: "fast" (FLH) and "slow" (SLH) lower hybrid modes and "fast" (FUH) and "slow" (SUH) upper hybrid modes. In [6, 7], the dependences of the frequencies of these modes on the parameter

$$q \equiv 2\,\omega_{pe}^2 \big/ \omega_{ce}^2 \,. \tag{3}$$

were built. It was shown that, due to the Doppler shift, long-wavelength ($k_z R_p << 1$) lower hybrid SLH modes with an azimuthal number $m \geq 1$ fall into the low-frequency area and reach zero frequency at small values of the parameter q. Upper hybrid SUH modes with an azimuth number $m \geq 2$ fall into the low frequency region at large values of q ($q \lesssim 1$). SUH modes with an azimuth number $m = 1$ do not fall into the low frequency area for any allowable values of the parameter q. This excludes the possibility of excitation of electron-ion instabilities of the SUH mode with $m = 1$ in a waveguide completely filled with non-neutral plasma.

However, long-wavelength low-frequency oscillations with an azimuthal number $m = 1$ are still observed in experiments with non-neutral plasma (see, for example, [8–13]). Apparently, there is instability of long-wavelength along the magnetic field low-frequency oscillations in a negative radial electric field (directed to the axis) with a significant degree of neutralization and partial filling of the waveguide with plasma:

$$\omega \sim \omega_{ci} << \left|\omega_{ce}\right|, \omega_{pe}\,; \quad k_z R_w << 1; \quad f \leq 1;$$

$$q \lesssim \left(1 - f\right)^{-1}; \quad R_p < R_{\rm w} \;. \tag{4}$$

Only the lowest radial mode is excited. The excitation is fast. This presumably means that $\gamma \gtrsim \omega_{ci} \sim \omega$.

Different authors attracted different mechanisms of instability excitation (see reviews in [13, 14], but even now, decades later, the nature of the instability has not been reliably clarified.

Can the relative motion of electrons and ions of non-neutral plasma be the cause of the low-frequency instabilities observed in the above-mentioned [8-13] and other similar experiments? For this, in any case, it is necessary that the eigenmodes of oscillations with the azimuthal number $m = 1$ fall into the region of low (ionic) frequencies at the values of the plasma parameters (4).

Let us find out whether there are eigenmodes with an azimuthal number $m = 1$ in the low-frequency region when the waveguide is partially filled with non-neutral plasma $R_p/R_{\rm w} < 1$ and at different values of charge neutralization coefficients f.

In this paper, the frequency spectra ω of eigenmodes with the azimuth number $m = 1$ of a waveguide partially filled with non-neutral plasma ($R_p < R_w$) are numerically determined from the dispersion equation over the entire admissible range of parameter q (3), various values of the charge neutralization coefficient, $0 \leq f \leq 1$, and finite values of the longitudinal wave vector k_z . Ions are taken into account only as a neutralizing background (in f). The areas on the parameter plane (ω, q) are also determined, where the electron modes are bulk in the plasma, and where they are surface. The results were presented in [14]. In present paper the studies of the behavior of neutral plasma modes are presented.

1. DISPERSION EQUATION. CALCULATION MODEL

The dispersion equation for the potential modes of a waveguide partially filled with a homogeneous rigidly rotating non–neutral plasma has the well–known form [2]

$$k_z R_p \frac{K_m\left(k_z R_w\right) I'_m\left(k_z R_p\right) - K'_m\left(k_z R_p\right) I_m\left(k_z R_w\right)}{K_m\left(k_z R_w\right) I_m\left(k_z R_p\right) - K_m\left(k_z R_p\right) I_m\left(k_z R_w\right)} =$$

$$= \varepsilon_1 T k_z R_p \frac{J'_m\left(T k_z R_p\right)}{J_m\left(T k_z R_p\right)} + m\varepsilon_2 \,. \tag{5}$$

In (5) J_m is the Bessel function of the first kind of order m, I_m, K_m are the modified Bessel functions, J'_m, I'_m, K'_m are their derivatives with respect to the entire argument,

$$T^2 \equiv -\frac{\varepsilon_3}{\varepsilon_1} \,. \tag{6}$$

This definition differs from the definition given in [1, 2] by the value k_z^2. The components of the dielectric permeability tensor of a cold electron plasma ε_1, ε_2, ε_3 have the form

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \tag{7}$$

$$\varepsilon_1 = 1 - \frac{\omega_{pe}^2}{\nu^2}\,, \qquad \varepsilon_2 = \frac{\omega_{pe}^2 \Omega_e}{\nu^2 \omega'}\,, \qquad \varepsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega'^2}\,,$$

where $\nu^2 \equiv \omega'^2 - \Omega_e^2$. When the waveguide is uniformly filled with electrons and ions, the combination E_r/r, frequency Ω_e and other frequencies in (5), (7) do not depend on the radius r.

In the theory of non-neutral plasma, the parameter q plays an important role [2, 3]. Along with f it determines the ratio of the electric and magnetic components of the Lorentz force:

$$4eE_r \big/ \left(m_e r \omega_{ce}^2\right) = q\left(1 - f\right) < 1\,, \tag{8}$$

and, therefore, determines all the characteristic frequencies of the plasma components: the "modified" cyclotron frequency Ω_α ($\alpha = e,i$) and the rotation frequency ω^α_{rot} (see (11), (12)). Only in the range of values q

$$0 < q < q_{\max} \equiv 1 \big/ \left(1 - f\right) \tag{9}$$

the finite motion of electrons along the radius is realized and the equilibrium state of the ensemble of electrons, and, hence, of the entire plasma, is possible.

In theoretical studies, the behavior of the frequencies ω of eigen plasma oscillations is usually represented as a dependence on the longitudinal wave vector k_z. At the same time, the dependences of eigen plasma frequencies on external fields, plasma density are measured in the experiment, i.e. actually on the parameter q, while k_z remains unchanged and is often undefined or incorrectly estimated. It is the parameter q that is controlled and changed in the experiment. The dependences of frequencies on a parameter q are more suitable for the purpose of comparison theory with experiment. In accordance with these remarks, we consider the quantity q as an independent variable and present the calculation results as dependences on this variable. All quantities included in the dispersion equation (5) were expressed as functions of q. All frequencies were normalized to $|\omega_{ce}|$:

$$\Omega_e/|\omega_{ce}| = \text{sgn}(e)\left[1-q(1-f)\right]^{1/2} < 0\,, \quad (10)$$

$$\omega^e_{rot}/|\omega_{ce}| = (1/2)\,\text{sgn}(e)\left\{-1+\left[1-q(1-f)\right]^{1/2}\right\}, \quad (11)$$

$$x' \equiv \omega'/|\omega_{ce}|\,, \quad (12)$$

$$x \equiv \omega/|\omega_{ce}| = x' + (m/2)\,\text{sgn}(e)\left\{-1+\left[1-q(1-f)\right]^{1/2}\right\}. \quad (13)$$

The dependences of the components of the tensor (7) on q take the form:

$$\varepsilon_1 = 1-\frac{q/2}{x'^2-1+q(1-f)}\,, \quad (14)$$

$$\varepsilon_2 = \frac{q/2}{x'^2-1+q(1-f)}\cdot\frac{\text{sgn}(e)\left[1-q(1-f)\right]^{1/2}}{x'}\,, \quad (15)$$

$$\varepsilon_3 = 1-\frac{q/2}{x'^2}\,. \quad (16)$$

The parameters of the problem are

$$m\,,\ f\,,\ k_z R_p\,,\ R_p/R_w < 1\,. \quad (17)$$

For unambiguity, we consider the azimuth number to be positive (m>0). Frequencies x', x can take positive and negative values. In the calculations presented in this paper, the parameters (17) set equal: m = +1, k_zR_p = 1, R_p/R_w = 1/2. The derivatives of the Bessel functions J'_m, I'_m, K'_m in (5) were replaced by their expressions using recurrent formulas [15].

The calculations were carried out within the entire range of allowable values of q (9). In the process of solving the dispersion equation (5), the eigen frequencies in the rotating frame of reference x' (12) were determined, and from it the frequencies x (13) in the laboratory frame of reference were determined.

2. AREAS OF EXISTENCE OF BULK AND SURFACE MODES

Let us determine the areas on the plane of parameters x, q, in which the modes in the plasma are bulk or surface. This is determined by the sign of the quantity T^2 (6). Modes that are located in the region where $T^2 > 0$, are bulk. The dependence of the mode potential on the radius in plasma has the form $J_m(Tk_zr)$. Modes located in the region where $T^2 < 0$ are surface modes. The dependence of the mode potential on the radius in plasma has the form $I_m(|T|k_zr)$. The boundaries between the regions are located where the value T^2 changes sign, and this is determined by the sign reversal of the tensor components ε_1 and ε_3.

In variables (x'^2, q), the shape of the boundaries has a particularly simple form (Fig. 1). Areas where $T^2 > 0$ and modes are bulk are shaded and indicated by arrows. In a neutral plasma (f = 1) these are two bands, in non-neutral plasma (f < 1) these are two triangular areas that touch at the value

$$q = q_t \equiv 2/(3-2f)\,. \quad (18)$$

The equality $\omega'^2=\omega_{pe}^2=\Omega_e^2$ holds at the point of touch [2]. In other areas, we have $T^2<0$, here the modes are surface.

As can be seen from (14), (16), the component ε_1 depends on x'^2, f the component depends only on x'^2, and does not depend on f. On the plane of variables (x'^2, q), the components ε_1, ε_3 do not depend on the characteristics of the wave and the degree of filling of the waveguide (m, k_zR_w, k_zR_p). The position of the boundaries does not depend on them either. This is also true for the boundaries on the plane (x', q).

On the plane of variables (x, q), the position of the boundaries between the areas of bulk and surface modes for different values of the neutralization coefficient f is shown in Fig. 2. There are four regions where the modes are bulk (T^2>0). They are indicated by arrows. In other regions, the modes are surface. The non-neutral plasma components ε_1, ε_3 and the position of the boundaries depend on the azimuthal number m, but still do not depend on the degree of filling of the waveguide and the longitudinal wave vector (k_zR_w, k_zR_p). For a neutral plasma (f = 1), the position of the interfaces does not depend on the azimuth number m either.

The solutions of the dispersion equation (5) are also shown in (see Fig. 2) for neutral and non-neutral plasmas.

3. MODES OF WAVEGUIDE PARTIALLY FILLED WITH NEUTRAL PLASMA

On Fig. 2,a the solutions of equation (5) for neutral plasma (f=1) depending on the variable q are shown. Solutions exist for all values q from the interval $0 < q < \infty$. On Fig. 2,a shows solutions in the interval $0 < q < 10$.

3.1. FAMILIES OF SOLUTIONS

Four families of solutions are identified: families of fast (FLH) and slow (SLH) lower hybrid modes, families (FUH) of fast and slow (SUH) upper hybrid modes. These modes are bulk for all values of q.

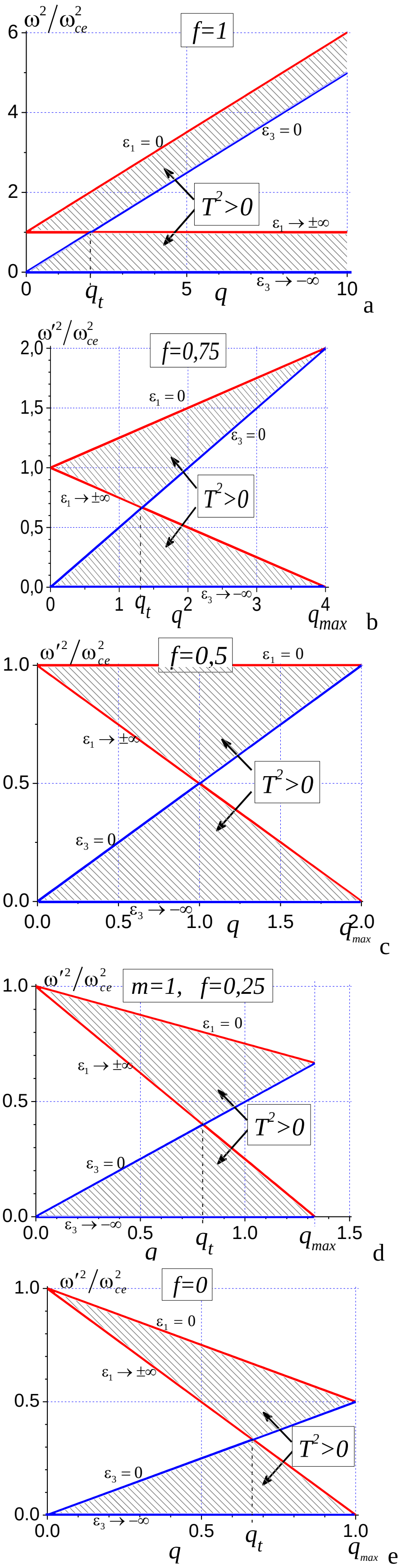

Fig. 1. Areas of existence of bulk (shaded) and surface modes on the parameter plane (ω'^2/ω_{ce}^2, q) for different values of the charge neutralization coefficient (f): f = 1(a); 0.75 (b); 0.5 (c); 0.25 (d); 0 (e). The quantities q_{max} and q_t are defined in (9) and (18)

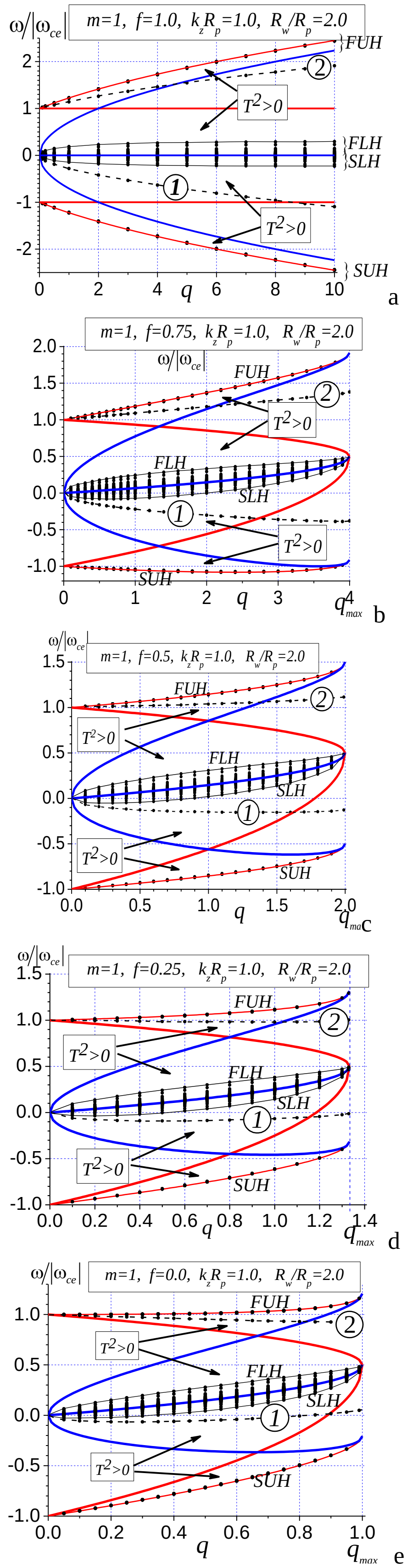

Fig. 2. Normalized frequencies $\omega/|\omega_{ce}|$ of eigenmodes of a waveguide partially filled with plasma depending on the parameter q at various values of the neutralization coefficient (f): f = 1 (a); 0.75 (b); 0.5 (c); 0.25 (d); 0 (e). The solid lines are the boundaries of the areas of existence of bulk and surface eigenmodes

In the areas where they are located, the inequality is fulfilled $T^2>0$. Their frequencies lie in the frequency intervals:

the SLH family $-\left(-\min\left(\omega_{pe},|\omega_{ce}|\right)<\omega<0\right)$,

the FLH family $-\left(0<\omega<\min\left(\omega_{pe},|\omega_{ce}|\right)\right)$, (19)

the SUH family $-\left(-\omega_{UH}<\omega<-\max\left(\omega_{pe},\omega_{ce}\right)\right)$,

the FUH family $-\left(\max\left(\omega_{pe},\omega_{ce}\right)<\omega<\omega_{UH}\right)$.

The value $\omega_{UH}=\sqrt{\omega_{pe}^2+\omega_{ce}^2}$ denotes the hybrid frequency of the neutral plasma.

For the parameters chosen for the calculation, the SUH family modes are located very close to each other and to the hybrid frequency $-\omega_{UH}$, while the FUH family modes are located very close to the frequency $+\omega_{UH}$. This situation takes place in a wide range of calculation parameters. The SLH and FLH modes are located much more "freely". The bandwidth where these modes are located increases with increasing k_z. Inside the band, the modes are irregularly distributed: the modes are closely spaced in pairs. Thus, the 1st and 2nd radial modes are located close to each other, the 3rd is farther from them. The 3rd and 4th modes are located close, and the 5th are farther from them, and so on. The same irregular arrangement of radial modes takes place in the families of upper hybrid modes FUH and SUH.

These four families are also present in a waveguide completely filled with plasma and lie in the same frequency intervals (19). The behavior of the waveguide modes for complete and partial filling with plasma is similar. With partial filling, the frequencies of the modes of the SLH and FLH families shift slightly upward, towards positive frequencies. In this case, the frequencies of the modes of the FUH and SUH families also shift towards positive frequencies, but negligibly little. The asymmetry in the location of the fast (F) and slow (S) modes is due to the last term on the right side of the dispersion equation (5) and is a demonstration of the plasma magnetoactivity.

3.2. DIOCOTRON AND CYCLOTRON MODES

When the waveguide is partially filled with plasma, along with the enumerated four families of modes, two modes appear, located beside, but separate from the families. They demonstrate a somewhat different frequency dependence on the parameter q.

The mode labeled "1" in Fig. 2,a is associated with the SLH mode family. In a neutral plasma, its frequency (and azimuthal phase velocity) is negative over the entire range of the parameter q. In non-neutral plasma ($f<1$) mode "1" is called "diocotron" mode. This term arose in the theory of non-neutral plasma several years after the publication of [1].

Mode "2" is related to the FUH family of modes. Its frequency is in the interval $|\omega_{ce}|<\omega_2<\omega_{UH}$ and is always positive.

At $q\to0$, modes "1" and "2" approach the SLH and FUH families, respectively, which demonstrates their connection with these families. At sufficiently small values q, these modes are bulk. With increasing of q, they go beyond the frequency ranges (19) and become surface modes.

When $k_zR_w\to0$ and q is finite, the frequency of mode "1" tends to a nonzero value, while all modes of the SLH family tend to zero. Similarly, the frequency of mode "2" tends to a value that differs from ω_{UH}, while all modes of the FUH family tend to ω_{UH}. This is a common feature of modes "1" and "2".

Calculations show that as the degree of filling of the waveguide with plasma increases ($R_p/R_w\to1-0$), mode "1" approaches the family of SLH modes. Mode "2" moves away from the FUH family of modes and approaches the frequency $|\omega_{ce}|$. In non-neutral plasma, mode "2" is called "cyclotron".

3.3. COMPARISON WITH THE RESULTS OF ARTICLE [1]

The potential modes of a waveguide partially filled with neutral plasma were studied in [1], where the dependences of natural frequencies on the longitudinal wave vector k_z were presented (see Fig. 8 in [1]). On Fig. 3 of present article are presented the solutions of Eq. (5) depending on k_z for the same values of the parameters as in Fig. 8 in [1], with the difference that in present article the azimuth number m is considered positive ($m=+1>0$), and the mode frequencies ω can be positive and negative. In [1], the frequencies were considered positive, and the azimuthal numbers m were considered positive and negative.

Comparison Fig. 8 [1] and Fig. 3 of present article shows that the modes located at the bottom of Fig. 8 marked "+1" and "–1" correspond in Fig. 3 to lowest radial modes of the FLH and SLH families, respectively (indicated by dotted lines). The modes located at the top of Fig. 8 [1], marked "n=+1" and "–1", correspond in Fig. 3 of present article to the modes, denoted by the numbers "2" and "1", respectively. The upper hybrid modes of the FUH and SUH families in Fig. 8 [1] are absent. They are located above the frequency range shown in Fig. 8 [1].

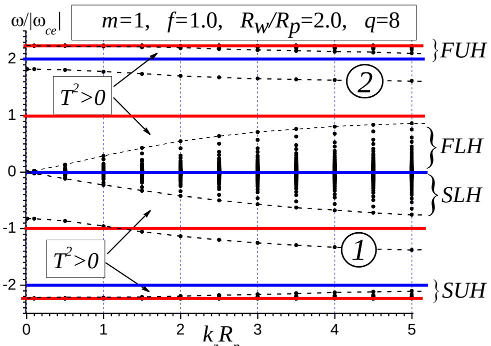

Fig. 3. Frequencies of the Trivelpiece-Gould modes of a waveguide partially filled with neutral plasma (f = 1). The calculation parameters (indicated at the top of the figure) are the same as in Fig. 8 in [1]

The behavior of the presented modes in both figures looks the same, but in Fig. 3, the frequencies ω are normalized to $|\omega_{ce}|$, while in Fig. 8 in [1], the normalization to ω_{pe} is erroneously indicated. The normalization of frequencies to $|\omega_{ce}|$ is correct. This misprint is repeated in some monographs.

Note that mode “2” within the entire range of values of parameter k_zR_p presented in Fig. 3, is a surface mode. Mode “1” is a surface mode for sufficiently large values of k_zR_p, approximately at k_zR_p>1.2. At smaller values of k_zR_p, mode “1” is bulk.

CONCLUSIONS

In present article the solutions are given of the dispersion equation for the potential eigenmodes of a waveguide partially filled with neutral or non-neutral plasma – the Trivelpiece-Gould modes. The frequencies are presented as dependences on the parameter q, which determines the equilibrium and stability of non-neutral plasma. For the purpose of comparing theory and experiment, such a representation is much more informative than the commonly used dependences of frequencies on wave vector k_z, because it gives an idea of the dependences of frequencies on external fields and plasma parameters that are controlled and changed in the experiment. The areas where the modes are surface and where are bulk are determined. They have an extremely simple form. The behavior of neutral plasma modes is analyzed.

REFERENCES

1. A.W. Trivelpiece, R.W. Gould. Space charge waves in cylindrical plasma columns // *Journ. of Appl. Phys.* 1959, v. 30, p. 1784-1793.
2. R.C. Davidson. *Theory of Nonneutral Plasmas.* New York: “Benjamin”, 1974. M.: “Mir”, 1978, 215 p.
3. R.H. Levy, J.D. Daugherty, O. Buneman. Ion resonance instability in grossly nonneutral plasmas // *Phys. Fluids.* 1969, v. 12, p. 2616-2629.
4. S.A. Prasad, T.M. O’Neil. Waves in a cold pure electron plasma of finite length // *Phys. Fluids.* 1983, v. 26, p. 665-672.
5. R. Keinigs. Effect of magnetic field errors on low-frequency waves in a pure electron plasma // *Phys. Fluids.* 1981, v. 24, p. 860-863.
6. Yu.N. Yeliseyev. Oscillation spectrum of an electron gas with a small density fraction of ions // *Plasma Physics Reports* // 2010, v. 36, № 7, p. 563-582.
7. Yu.N. Yeliseyev. Stability of hybrid modes of a single-component electron plasma containing an admixture of background gas ions // *Plasma Physics Reports*. 2014, v. 40, № 5, p. 442-453.
8. M.A. Vlasov, Ye.I. Dobrokhotov, A.V. Zharinov // *Nuclear Fussion.* 1966, v. 6, p. 24.
9. M.V. Nezlin. *Dinamika puchkov v plasme.* M.: “Energoizdat”, 1982 (in Russian).
10. Y. Sakawa, C. Joshi. Growth and nonlinear evolution of the modified Simon-Hoh instability in an electron beam-produced plasma // *Phys. Plasmas*. 2000, v. 7, p. 1774-1780.
11. B.M. Annaratone, A. Escarguel, T. Lefevre, C. Rebont, N. Claire, F. Doveil // *Phys. Plasmas*. 2011, v. 18, p. 032108.
12. S. Jaeger. *Etude theorique et experimentale des instabilites basses frequences dans un plasma en champs magnetique et electrique croises:* Ph.D. thesis. Aix-Marseille Universite, 2010.
13. Pierre David. *Tomography in a linear magnetized plasma*: Ph.D. thesis. Aix-Marseille Universite, 2017.
14. Yu.N. Yeliseyev. Trivelpiece-Gould modes of waveguide partially filled with non-neutral plasma // *Proc. “XIII Workshop on Non-Neutral Plasmas*, Milano, Italy, 19-22 September 2022”. Book of abstract P.9
15. I.S. Gradshtejn, B.M. Ryzhik. *Tablitsi integralov, summ, ryadov i proizvedenij.* M.: “GIFML”, 1963, 1108 p. (in Russian).

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МОДИ ТРАЙВЕЛПІСА-ГУЛДА ХВИЛЕВОДА, ЧАСТКОВО ЗАПОВНЕНОГО ЗАРЯДЖЕНОЮ ПЛАЗМОЮ

Ю.М. Єлісеєв

Частотний спектр власних мод з азимутальним числом $m = 1$ визначається числовими методами для хвилеводу, частково заповненого нейтральною або зарядженою плазмою. Розрахунки виконуються в межах усього допустимого діапазону густини електронів і напруженостей магнітного поля для різних значень ступеня нейтралізації заряду і поздовжнього хвильового вектора. Також визначені області існування об'ємних та поверхневих електронних мод хвилеводу. Результати представлені у вигляді залежностей від параметра $q = 2\omega_{pe}^2/\omega_{ce}^2$.