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RELATIVISTIC KINETICS AND HYDRODYNAMICS OF HOT COLLISIONAL PLASMAS

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In the paper, relativistic equations of local hydrodynamics for the laboratory fusion plasmas are obtained. Relativistic effects in the physics of electron transport appear primarily because of macroscopic features of relativistic thermodynamic equilibrium given by the Maxwell-Jüttner distribution function, and the characteristic velocity of plasma flow is significantly small: $V \ll v_{te} < c$. We propose an approach in which the plasma electrons are treated as fully relativistic and the hydrodynamic flow is treated in the weakly relativistic approximation. For convenience, the obtained relativistic effects are divided between “quasi-relativistic” terms, which in the nonrelativistic limit coincide with well-known expressions, and fully relativistic terms, which disappear at$c \to \infty$. The considered mixed approach can be useful for construction of transport models for numerical studies of both astrophysical objects and hot fusion plasma.

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INTRODUCTION

Relativistic effects in astrophysical objects and fusion plasmas do not necessarily require extremely high temperatures and energies. They appear to be non-negligible even for electronic temperatures T_e of the order of tens keV, i.e. when $T_e \ll m_e c^2$. Relativistic effects in kinetics, hydrodynamics and transport physics in collisional plasmas appear due to a macroscopic features of relativistic thermodynamic equilibrium given by the Maxwell-Jüttner distribution function (or relativistic Maxwellian) [1]. In fusion devices such as ITER [2, 3] and DEMO [4], where electron temperatures must reach several tens of keV, relativistic effects for electron transport become noticeable. The same is true for aneutronic fusion reactors, where the expected electron temperature should be about 50...70 keV and above [5-9].

It has recently been shown [10, 11] that relativistic effects can modify electron transport, making the fluxes noticeably different from those calculated in the nonrelativistic limit for both tokamaks and stellarators. At the same time, virtually all transport codes developed to date for modeling fusion reactor scenarios are based on a nonrelativistic approach.

Usually, in the literature devoted to relativistic kinetics and MHD of plasmas the covariant formalism with the 4-vectors is applied [12, 13]. This is the most general and straightforward way to obtain the transport and MHD equations with conservation of Lorentz invariance [14, 15]. Usually, this formalism is applied to describe astrophysical objects. However, for the problems, where the Lorentz invariance is of low importance, the kinetics is considered in the same way as in the non-relativistic limit [10, 11, 16-20].

The present work is focused on description of transport processes in a hot collisional plasmas with relativistic electrons and macroscopic flows with characteristic velocities $V \ll v_{te}$. The main goal is to derive the equations of local hydrodynamics in the weakly relativistic approach with respect to the mean flow, i.e. neglecting the terms of the order $V^3/(c^2 u_{te})$, $V^4/(c^2 u_{te}^2)$ and above, while the thermal effects involving plasma electrons are described as fully relativistic. The final equations are mathematically similar to the non-relativistic ones and have a transparent physical interpretation.

FIRST MOMENTS IN THE REST FRAME

First, it is convenient to write a relativistic kinetic equation for the electron distribution functionf_e in divergent form and without 4-vectors,

$$\frac{\partial f_e}{\partial t} + \frac{\partial}{\partial x_k}(v_k f_e) + \frac{\partial}{\partial u_k}(\dot{u}_k f_e) = C_e(f_e) , \qquad (1)$$

where $\dot{x_k} = v_k$ is the velocity with $k = 1,2,3$, $u_k = v_k \gamma$ is the momentum per unit mass with $\gamma = \sqrt{1 + u^2/c^2}$ as the relativistic factor, and $m\dot{u}_k = eE_k + \frac{e}{c}[\boldsymbol{v} \times \boldsymbol{B}]_k$ is the force with electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$, respectively. Here and below, the standard rule of summation over the repetitive indexes is supposed. The operator $C_e(f_e)$ describes the collisions of electrons with themselves and ions, i.e. $C_e(f_e) = C_{ee}(f_e) + C_{ei}(f_e)$, where ions are considered non-relativistic.

In order to derive the equations for such values as the mean flow velocity, density and temperature of plasma electrons, it is natural to assume that plasma is very close to the thermodynamical equilibrium given by the “drifting” Maxwell-Jüttner distribution function,

$$f_{e0} = C_{MJ} \frac{n_e}{\pi^{3/2} u_{te}^3} \exp\left(-\mu\gamma_0 \left[\gamma - \frac{1}{\gamma_0} - \frac{V_k u_k}{c^2}\right]\right), \quad (2)$$

where n_e is the density of electrons measured in the rest frame which moves with mean flow velocity $\boldsymbol{V}$, $\gamma_0 = 1/\sqrt{1 - V^2/c^2}$ is the relativistic dilation factor, $u_{te} \equiv p_{te}/m_e = \sqrt{2T_e/m_e}$ is the thermal momentum per unit mass (formally, u_{te} coincides with the thermal velocity in non-relativistic limit, but is not limited by speed of light), T_e is the electron temperature and $\mu = \frac{m_e c^2}{T_e} > 1$ (typically, μ > 10 for fusion plasmas). The normalizing coefficient equals

$$C_{MJ} = \sqrt{\frac{\pi}{2\mu}} \frac{e^{-\mu}}{K_2(\mu)} = 1 - \frac{15}{8\mu} + \frac{345}{128\mu^2} +, \quad (3)$$

with $K_n(\mu)$ as the modified Bessel function of second kind of the n-th order.

While T_e is assumed here to be arbitrary high (with only natural limitation $T_e < m_e c^2$, just to exclude a generation of the electron-positrons pairs), the mean velocity satisfies the conditions $V/u_{te} \ll 1$ and $V^2/c^2 \ll 1$. The last condition makes possible to apply the weakly relativistic approach with respect to flow,

$$\gamma_0 = 1/\sqrt{1 - V^2/c^2} \simeq 1 + V^2/2c^2, \quad (4)$$

and reduce f_{e0} to

$$f_{e0} \simeq C_{MJ} \frac{n_e}{\pi^{\frac{3}{2}} u_{te}^3} \exp\left[-\mu\left(\gamma - 1 - \frac{V_k u_k}{c^2}\right) - \frac{m_e V^2}{2T_e}\right]. \quad (5)$$

The form of representations of f_{e0} in Eqs. (2) and (5) with coefficient given by Eq. (3) is chosen in such a way that the limit of f_{e0} (which is the classical drifting Maxwellian) when $c \to \infty$would be the most obvious.

Now we will adapt to our notations the definitions given by other authors; see [12, 13, 16]. In order to obtain the equations for density, momentum and energy, one needs to integrate kinetic equation Eq. (1) with the corresponding weight functions: 1, $m_e u_k$ and $m_e c^2(\gamma - 1)$, respectively. For that, following to the algorithm of Braginskii [21], the Lorentz transformation from the local coordinate system to the rest frame is required, where$\boldsymbol{V} = 0$ and $\gamma_0 = 1$. The variables that correspond to the rest frame are labeled by prime. For compactness, let us introduce the notations: $\langle F \rangle = (1/n_e) \int F f_e d^3 u$ and $\langle F' \rangle = (1/n_e) \int F' f_e' d^3 u'$. Evidently, that in the rest frame $\langle 1 \rangle = 1$and$\langle v_k' \rangle = 0$.

For Maxwell-Jüttner distribution function, the relation between the total relativistic energy and temperature is well known [12],

$$\mathcal{E}_{total} = n_e m_e c^2 \langle \gamma' \rangle = n_e \left(m_e c^2 \frac{K_3(\mu)}{K_2(\mu)} - T_e\right). \quad (6)$$

Alternatively, the internal thermal energy Eq. (6) can be represented in different form [10],

$$W \equiv n_e m_e c^2 \langle \gamma' - 1 \rangle = \left(\frac{3}{2} + \mathcal{R}\right) n_e T_e, \quad (7)$$

which reminds the classical expression, where $\mathcal{R}$ is the relativistic correction term,

$$\mathcal{R} = \mu\left(\frac{K_3(\mu)}{K_2(\mu)} - 1\right) - \frac{5}{2} = \frac{15}{8\mu} - \frac{15}{8\mu^2} + \frac{135}{128\mu^3} + \quad (8)$$

Here, Eqs. (7) and (8) give a quasi-classical form for energy. Similarly, also the heat flux can be defined, which, however, is equal in the rest frame to the energy flux,

$$q_k = n_e m_e c^2 \langle (\gamma' - 1) v_k' \rangle, \quad (9)$$

which is also related to the averaged momentum as follows,

$$n_e m_e \langle u_k' \rangle = \frac{1}{c^2} q_k. \quad (10)$$

It is useful to mention that the moment in Eq. (10) represents a purely relativistic effect and is equal to zero in the classical limit, while the heat flux Eq. (9) is "quasi-classical" in the above sense. Indeed, for $c \to \infty m_e c^2(\gamma - 1) \to v^2/v_{te}^2$, and the values u_k' and v_k' become indistinguishable, while $\langle v_k' \rangle$ =0.

The next required moment is the momentum flux,

$$n_e m_e \langle v_k' u_j' \rangle = p_e \delta_{kj} + \pi_{kj}, \quad (11)$$

which, similarly to the non-relativistic representation, decomposes into hydrostatic scalar pressurep_e,

$$p_e = \frac{1}{3} n_e m_e \langle \frac{u'^2}{\gamma'} \rangle = n_e T_e, \quad (12)$$

and (traceless) viscous stress tensorπ_{kj},

$$\pi_{kj} = n_e m_e \langle v_k' u_j' \rangle - p_e \delta_{kj}. \quad (13)$$

The moments related to the collisional operator are also required. Since the conservation laws of momentum and energy in Coulomb collisions of electrons with themselves are satisfied automatically, only the contribution from electron-ion collisions survives in integration. Then, by definition, electron-ion collisional friction force is the following:

$$R_k^{ei} = \int m_e u_k' C_{ei}(f_e')\, d^3 u' \quad (14)$$

Similarly, the stress tensor generated by the electron-ion collisions can be defined as

$$F_{kj}^{ei} = \int m_e v_k' u_j' C_{ei}(f_e') d^3 u'. \quad (15)$$

The collisional rate of the heat-flux generation is, respectively,

$$G_k^{ei} = \int m_e c^2 (\gamma' - 1) v_k' C_{ei}(f_e') d^3 u'. \quad (16)$$

The rate of collisional energy exchange between relativistic electrons and classical ions is:

$$P^{ei} = \int m_e c^2 (\gamma' - 1) C_{ei}(f_e') d^3 u'. \quad (17)$$

In this case, only the dominant part of the energy exchange is taken into account for the calculation, i.e. in Eq. (17) both electrons and ions distribution functions are assumed to be equilibrium (Maxwellian for ions and Maxwell-Jüttner for electrons, but with their own temperatures). Then, the result of integration can be presented as follows [22],

$$P^{ei} = P^{ei}_{(cl)} C_{MJ} \left(1 + \frac{2}{\mu} + \frac{2}{\mu^2}\right), \quad (18)$$

where $P^{ei}_{(cl)}$ is the classical (non-relativistic) electron-ion energy exchange rate [23],

$$P^{ei}_{(cl)} = -\frac{4}{\sqrt{\pi}} \nu_{e0} \frac{m_e}{m_i} n_i Z_i^2 (T_e - T_i) \propto -\frac{T_e - T_i}{T_e^{\frac{3}{2}}}. \quad (19)$$

In somewhat different form Eq. (18) was obtained also in [14, 16].

HYDRODYNAMIC EQUATIONS IN LABORATORY FRAME

For integration in the local coordinate system, a Lorentz invariance of 4-momentum volume has to be taken into account, that can be written in our notations as $d^3u/\gamma = d^3u'/\gamma'$. The Lorentz transformation of the momentum and energy from the local coordinate system into the rest frame [24] can be reformulated in our notations as following,

$$u_k = \gamma_0 \gamma' V_k + u'_k + (\gamma_0 - 1)\frac{V_k V_j}{V^2} u'_j,$$
$$\gamma = \gamma_0 \left(\gamma' + \frac{V_j u'_j}{c^2}\right). \quad (20)$$

Relations in Eq. (20) are precise. However, below we will apply a weakly relativistic approach with respect to V; see Eq. (4). In the local frame, where$V \neq 0$, the moments get an additional contributions related to the mean flow, which are accounted in the weakly relativistic approach, neglecting the terms of order $V^3/(c^2 u_{te})$, $V^4/(c^2 u_{te}^2)$ and above.

It is convenient to represent all moments as a sum of two parts: "quasi-classical" contribution and the term of purely relativistic correction that completely disappear in a non-relativistic limit $c \to \infty$. Thus, it was found more appropriate to group the relativistic correction terms with the formal factor $1/\mu$.

Direct integration of Eq. (1) requires two lowest moments for the local coordinate system, $n_e\langle 1 \rangle = \gamma_0 n_e$ and $n_e\langle v_k \rangle = \gamma_0 n_e \mathrm{V}_k \equiv \gamma_0 \Gamma_k$, which correspond to density and particles flux, respectively. Here we accounted that $\langle v'_k \rangle = 0$. From that, the continuity equation can be obtained,

$$\frac{\partial}{\partial t}(\gamma_0 n_e) + \frac{\partial}{\partial x_k}(\gamma_0 \Gamma_k) = 0. \quad (21)$$

Note that formally this equation has exactly the same form as in a fully relativistic approach. The weakly relativistic expansion Eq. (4) is supposed, but not applied directly here for compactness.

The next equation is the momentum balance that has to be obtained integrating Eq. (1) weighed by $m_e u_k$. Here, both the momentum and the momentum flux are required. It can be shown that the momentum Eq. (10) in the rest frame can be represented as

$$n_e m_e \langle u_k \rangle = n_e m_e \left(V_k + \delta U_k^{(r)}\right), \quad (22)$$

where the additional term, which has the meaning of relativistic correction for momentum per particle of unit mass, is

$$\delta U_k^{(r)} = \frac{1}{\mu}\left[\left(\frac{5}{2} + \mathcal{R}\right) V_k + \frac{1}{p_e}\left(\pi_{kj} V_j + q_k\right)\right]. \quad (23)$$

Here, the hydrostatic pressure p_e and viscous stress tensor π_{ij} are given by Eqs. (12) and (13), and the terms of order $V^3/(c^2 u_{te})$ and above are neglected.

Similarly, the flux of momentum Eq. (11) in the rest frame can be represented as

$$n_e m_e \langle v_k u_j \rangle = \Pi_{kj} + \delta \Pi_{kj}^{(r)}, \quad (24)$$

with the lowest "quasi-classical" term formally coinciding with the non-relativistic definition [23],

$$\Pi_{kj} = p_e \delta_{kj} + \pi_{kj} + n_e m_e V_k V_j, \quad (25)$$

while and the correction term is

$$\delta \Pi_{kj}^{(r)} = \frac{1}{c^2}\left[q_k V_j + q_j V_k + \left(\frac{5}{2} + \mathcal{R}\right) p_e V_k V_j + \frac{1}{2}\left(\pi_{kl} V_j + \pi_{jl} V_k\right) V_l\right]. \quad (26)$$

Using Eq. (23), it may be convenient to rewrite the relativistic correction term Eq. (26) as following,

$$\delta \Pi_{kj}^{(r)} = \frac{1}{c^2}\left[p_e \delta U_k^{(r)} V_j + q_j V_k + \frac{1}{2}\left(\pi_{kl} V_j + \pi_{jl} V_k\right) V_l\right]. \quad (27)$$

The collisional friction that required for momentum balance can also be represented in the similar form:

$$\int m_e u_k C^{ei}(f_e)\, d^3u = R_k^{ei} + \delta R_k^{ei(r)}, \quad (28)$$

with zero-order term equal to that defined in Eq. (14), while the relativistic correction is

$$\delta R_k^{ei(r)} = \frac{3V^2}{2c^2}\left(\frac{V_k V_j}{V^2} + \frac{1}{3}\delta_{kj}\right) R_j^{ei} + \frac{1}{c^2}\left(V_k P^{ei} + F_{kj}^{ei}\right). \quad (29)$$

Taking into account the terms, given by Eqs. (22-29), the momentum balance equation can be written as follows,

$$\frac{\partial}{\partial t}\left[n_e m_e \left(V_k + \delta U_k^{(r)}\right)\right] + \frac{\partial}{\partial x_j}\left(\Pi_{kj} + \delta \Pi_{kj}^{(r)}\right) =$$
$$= e n_e E_k + \frac{1}{c}[\boldsymbol{J} \times \boldsymbol{B}]_k + R_k^{ei} + \delta R_k^{ei(r)}. \quad (30)$$

Here, $\boldsymbol{J} = e n_e \boldsymbol{V} = e n_e(\boldsymbol{V}_e - \boldsymbol{V}_i)$ is the electron electric current that corresponds to the mean flow.

The lastshould be the energy balance equation, which should be obtained by integrating kinetic equation weighted by kinetic energy $m_e c^2(\gamma - 1)$. Processing as above and taking into account Eq. (7), we obtain

$$n_e m_e c^2 \langle \gamma - 1 \rangle = \left(\frac{3}{2} + \mathcal{R}\right) p_e + n_e \frac{m_e V^2}{2} + \delta \mathcal{E}^{(r)}, \quad (31)$$

with, respectively,

$$\delta\mathcal{E}^{(r)} = \frac{V^2}{c^2}\left(\frac{5}{2}+\mathcal{R}\right)p_e + \frac{1}{c^2}\left(\pi_{kj}V_j + q_k\right)V_k. \quad (32)$$

Additionally, comparing Eq. (32) and Eq. (23), one can find a useful relation,

$$\delta\mathcal{E}^{(r)} = \frac{1}{c^2}p_e\delta U_k^{(r)}V_k\,. \quad (33)$$

Here, the standard rule of summation over the repetitive indexes is supposed.

In the same way, the energy flux can be obtained,

$$n_e m_e c^2\langle(\gamma-1)v_k\rangle = Q_k + \delta Q_k^{(r)},$$

where "quasi-classical" part formally coincides with the classical definition,

$$Q_k = q_k + \left(\frac{5}{2}+\mathcal{R}\right)p_eV_k + \pi_{kj}V_j + n_e\frac{m_eV^2}{2}V_k, \quad (34)$$

while the term of pure relativistic correction is

$$\delta Q_k^{(r)} = \frac{2V^2}{c^2}\left[\left(\frac{5}{2}+\mathcal{R}\right)p_eV_k + \frac{1}{2}\left(\frac{V_kV_j}{V^2}+\delta_{kj}\right)\pi_{jl}V_l + \frac{3}{2}\left(\frac{V_kV_j}{V^2}+\frac{1}{3}\delta_{kj}\right)q_j\right]. \quad (35)$$

Note that the terms proportional to $m_eV^2/2$ in Eq. (31) and Eq. (34), which are related to the mean flow kinetic energy, can be excluded from the energy balance by simple manipulation and using the continuity equation Eq. (21). After that, the energy balance equation would describe only the balance of internal thermal energy.

The last term to be considered is the collisional energy exchange, which can also be written as follows,

$$\int m_ec^2(\gamma-1)C^{ei}(f_e)\,d^3u = P^{ei} + R_k^{ei}V_k + \delta P^{ei(r)}, \quad (36)$$

whereP^{ei} is given by Eq. (18) with classical part given by Eq. (19), $R_k^{ei}$is given by Eq. (14), and the correction-term to relativistic flow is

$$\delta P^{ei(r)} = \frac{2V^2}{c^2}\left[P^{ei} + R_k^{ei}V_k + G_k^{ei}V_k + \frac{V_kV_j}{V^2}F_{kj}^{ei}\right]. \quad (37)$$

Here, G_k^{ei} and F_{kj}^{ei} are given by Eqs. (15) and (16), correspondingly.

Finally, taking into account the terms, given by Eqs. (31)-(37), the equation for the balance of thermal energy can be written in the following form:

$$\frac{\partial}{\partial t}\left[\left(\frac{3}{2}+\mathcal{R}\right)p_e + \delta\mathcal{E}^{(r)}\right] + \frac{\partial}{\partial x_j}\left(\mathrm{Q}_k + \delta\mathrm{Q}_k^{(r)}\right) = \\ = J_kE_k + P^{ei} + R_k^{ei}V_k + \delta P^{ei(r)}. \quad (38)$$

The equations Eqs. (21), (30), and (38) describe the collisional relativistic hydrodynamics, derived in the mixed approach, fully relativistic for the thermal electrons and weakly relativistic for the mean flow.

FURTHER STEPS

Since the final transport equations require, as in the classical treatment, a knowledge of only the few first moments and only in the rest frame, it is necessary to make a closure of the model. Here we will draw only the preliminary sketch of such closure, which itself is beyond the scope of the present paper.

The first step is to formulate and solve a linearized kinetic equation with thermodynamic forces on the right-hand side (gradients of plasma parameters and electric field). As it was shown in [25], where the relativistic effects in radial neoclassical fluxes for the toroidal systems were considered, the most adequate method for solving the linearized relativistic kinetic equation is to represent the solution in the form of a series of the generalized Laguerre polynomials of order $\alpha = 3/2 + \mathcal{R}$. For the low temperature limit, $T_e/mc^2 \to 0$, this representation comes to the classical form with the Sonine polynomials.

As a final step, we need to calculate the necessary moments of the distribution function using the obtained solution. To make the results most transparent, the obtained moments can be expanded into a $1/\mu$ series, retaining only the first relativistic correction term.

CONCLUSIONS

In this paper, the relativistic hydrodynamics of collisional hot plasmas is considered. Equations for first moments are obtained, which allow us to create a numerical transport model for the study of astrophysical and fusion hot plasmas.

The main point of the model is the use of "mixed" approach, when plasma electrons are described in fully relativistic approach and mean plasma flow is considered in weakly relativistic approach.

The equations obtained in the paper have been written in a form convenient enough for implementation of the relativistic approach into the transport codes which so far are based only on the classical approach. Such modification is necessary for the development and predictive investigation of the fusion reactor scenarios with hot plasmas and relativistic electrons.

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РЕЛЯТИВІСТСЬКІ РІВНЯННЯ ЛОКАЛЬНОЇ ГІДРОДИНАМІКИ З ПОВІЛЬНИМИ ПОТОКАМИ

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Отримано релятивістські рівняння локальної гідродинаміки для плазми лабораторного термоядерного синтезу. Релятивістські ефекти у фізиці транспорту електронів проявляються насамперед через макроскопічні особливості релятивістської термодинамічної рівноваги, яка задається функцією розподілу Максвелла-Ютнера, а характерна швидкість течії плазми є суттєво малою: $V \ll v_{te} < c$. Запропоновано підхід, у якому електрони плазми вважаються повністю релятивістськими, а гідродинамічна течія розглядається у слабкорелятивістському наближенні. Для зручності отримані релятивістські ефекти розділено між «квазірелятивістськими» членами, які в нерелятивістській межі збігаються з відомими виразами, та повністю релятивістськими членами, які зникають при $c \to \infty$. Розглянутий змішаний підхід може бути корисним при побудові транспортних моделей для чисельних досліджень як астрофізичних об'єктів, так і плазми гарячого термоядерного синтезу.