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SUPERRADIATION OF CLASSICAL OSCILLATORS AT CONSTANT PUMPING

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The dependence of the generation efficiency in the superradiance regime of an ensemble of oscillators on the pump energy input rate, as well as on the characteristics of the change in the pump phase, is studied. It is assumed that the pumping phase can vary both in time and in space. The most efficient pumping method remains the mode with slow energy input into the system, in comparison with the characteristic generation time, and the nature of the distribution of the field phase along the system can be either random or with a certain spatial period, but slightly changing with time.

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INTRODUCTION

Starting with famous work of R.N. Dicke [1] the researchers became interested in the processes of generation of oscillations in superradiance regimes. This mode of radiation is usually realized in open systems, when high-frequency energy is removed from the system.

Generation of radiation in waveguide and resonator systems. Energy output is typical for real electronic devices. Essentially, these are dissipative regimes of generation in waveguide resonator systems such as a traveling wave tube. These regimes corresponded to the case of the interaction of the electromagnetic field of the waveguide with emitting particles, most often with moving beam electrons [2–4]. Since the system is open, the field leaves the waveguide, which for short systems is equivalent to dissipative processes of a distributed type. The peculiarity of such modes is that the field damping decrement in such a waveguide or resonator (without taking into account emitters or oscillators) may turn out to be greater than the growth increment of the generation process in the presence of these active elements. That is, in such regimes of generation or amplification there is no threshold for the development of the instability. In addition to the case of moving radiators in the waveguide, it was possible to consider the case of the interaction of a system of both stationary and moving[1] oscillators with the field of an open resonator. In this case, the oscillators did not interact directly with each other, but only through the resonator field common to the system. The mechanism of phase synchronization of the radiation of such oscillators by a waveguide-resonator field was discussed in [7–9].

If this resonator is filled with an active medium, a dissipative waveguide excitation regime is also possible, in which quantum oscillators interact only with the resonator field, and there is no direct interaction between them [10].

The superradiance regime discussed for quantum emitters [1, 11, 12] ensured the interaction of the emitters with each other due to the overlap of the wave functions of the particles. In the classical case, such an interaction occurred due to the electromagnetic fields of particles [13, 14].

Similar regimes are also realized for quantum oscillators that interact with each other both with their nearest neighbours and with all other oscillators in the system in the absence of an initial waveguide or resonator field. Moreover, due to the rather large distances between the particles, the interaction between them most often occurs only due to their own electromagnetic fields. The mode of superradiance in the same resonator, when quantum oscillators interact only with each other, was also discussed. It is important to note that at a relatively low density of oscillators, their wave functions do not overlap. Quantum oscillators at their low density can affect each other only by electromagnetic fields of their own radiation [10].

Similarity of dissipative regimes of generation and superradiance. Most problems of field amplification and generation by emitters and oscillators were considered in open waveguide or resonator systems. The particles interacted only with the waveguide or resonator field. Therefore, it was not clear how the results would change if we go to the superradiance regime, when each emitter or oscillator radiates by itself and the integral field is the sum of these individual fields.

Attempts to compare regimes of dissipative instability and superradiance in open systems were undertaken in [15, 16]. Along the way, it was discovered that the systems of equations describing the interaction of electron beams rotating in a magnetic field with the fields of a waveguide at cyclotron resonances, when simplified, were reduced to the descriptions of the interaction of oscillators considered in the above papers [17]. This circumstance indicated the existence of a common phase synchronization mechanism for all these cases.

A comparison was made of the dissipative mode of field generation in an open system, a resonator, with the superradiance mode in the same resonator uniformly filled with immobile excited classical and quantum oscillators. The analysis showed that the process increments and the maximum achievable field amplitudes of these two regimes practically coincide [10, 13]. The behavior of a system of oscillators similar

[1] For example, in the cyclotron resonance regime [5], where new effects were discovered [6].

to the case studied earlier [13] was considered, taking into account the possibility of longitudinal motion of oscillators along the resonator due to the action of the Lorentz force [18]. This mode is interesting for gaseous media,

However, studies [12, 13], were carried out for the processes of relaxation of the initial perturbation with a sufficiently large stored energy (in fact, the Cauchy problem). In practice, external pumping is often used, which makes it possible to restore the energy of the system. Therefore, in this paper, we will focus on the problem of field generation by a system of classical oscillators in the superradiance regime under external pumping conditions.

The purpose of this work is to study the conditions for continuous generation of oscillations by a system of classical oscillators in the superradiance regime with external pumping. Let us also discuss how different external pumping regimes affect the efficiency of oscillation generation.

1. SYSTEM OF EQUATIONS WITH PUMPING INCLUDED

In [13] the equations for changing the amplitude of oscillators and the growth of their total field are given. A system of oscillators was considered: oscillators are located along the Ox axis, oscillations occur along the Oy axis. At the initial moment, oscillators have a given oscillation amplitude and randomly distributed phases (the Cauchy problem). In numerical experiments, oscillation phase synchronization was observed, accompanied by an increase in the field. Then came the period of the fall of the total energy of the system of oscillators, the violation of their synchronization and the damping of the field. It is of interest to study long-term generation in such a model, which should take place with a constant energy pumping, namely, with the restoration of a certain value of the oscillator amplitude. For modeling, we used a system of equations describing the behavior of classical oscillators from [13] with the addition of a pumping mechanism (the last term in the second equation):

$$\mathrm{E}(\mathrm{Z},\tau)=\frac{2}{N}\sum_{s=1}^{N}\mathrm{A}_s\cdot e^{i2\pi|\mathrm{Z}-\mathrm{Z}_s|},$$

$$\frac{d\,\mathrm{A}_j}{d\tau}=\frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2\mathrm{A}_j-\frac{1}{N}\sum_{s=1}^{N}\mathrm{A}_s\cdot e^{i2\pi|\mathrm{Z}-\mathrm{Z}_s|}-\mathrm{E}_0\cdot e^{2\pi i\mathrm{Z}_j}+\gamma\left(\left|\mathrm{A}_0\right|e^{i\cdot\delta_{j\tau}}-\mathrm{A}_j\right). \quad (1)$$

Here $\gamma=\gamma_0^2/\delta_D=\pi e^2M/mc$; $\mathrm{E}=eE/m\omega\gamma a_0$; $\mathrm{A}=A/a_0$; $\tau=\gamma t$; $k_0z=2\pi Z$; $\alpha=3k_0^2a_0^2\omega/4\gamma$; $\gamma_0^2=\pi e^2n_0/m=\omega_{pe}^2/4$; $M=b\cdot n_0$, b – length considered area in the longitudinal direction, n_0 – density particles per unit volume; E_0 – the amplitude of the external field running in the positive direction of the Z axis. Pumping is represented in (1) by the term $\gamma\left(\left|\mathrm{A}_0\right|e^{i\cdot\delta_{j\tau}}-\mathrm{A}_j\right)$, where γ – pace pumping; $|\mathrm{A}_0|$ – module primary amplitude supported by pumping; $\delta_{j\tau}$ – a phase that varies randomly from zero to 2π. In the general case, the phase can change randomly in time and space.

2. TIME-NON-CHANGING PUMPING

The following options are selected. The number of particles N = 300, $\alpha=1$, the amplitude of the additional external field that initiates the process of superradiance , E_0 =0.05. At the initial moment of time, the oscillators are uniformly distributed along the system, the amplitude modules of the oscillators are equal to unity $\left|\mathrm{A}_j(0)\right|=\left|\mathrm{A}_0\right|=1$, their phases $\psi_j(0)$ have random values in the range $0-2\pi$ ($\mathrm{A}_j=\left|\mathrm{A}_j(\tau)\right|e^{i\psi_j(\tau)}$).

Random distribution of the pump phase in the area. Let the electromagnetic pump phase $\delta_{j\tau}$ is randomly distributed along the system along the Ox axis. Changes in the pump phase for fixed points in space are chosen from zero to 2π and do not change in time. Calculations were carried out for this case at various values of the pumping rate γ .

Figs. 1a, 1b shows the time dependences of the average oscillator energy $|A|_{ev}^2=\frac{1}{N}\sum\left|A_j\right|^2$ (see Fig. 1a) and the field at the left and right ends of the system, as well as the maximum field in the system (see Fig. 1b) in the absence of pumping (γ =0).

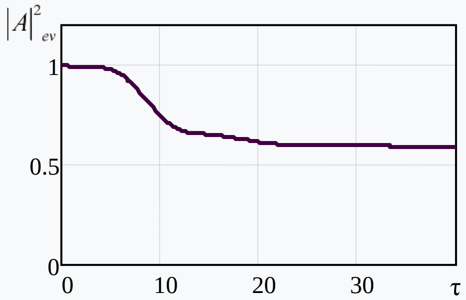

Fig. 1a. Time dependence of the average oscillator energy in the absence of pumping

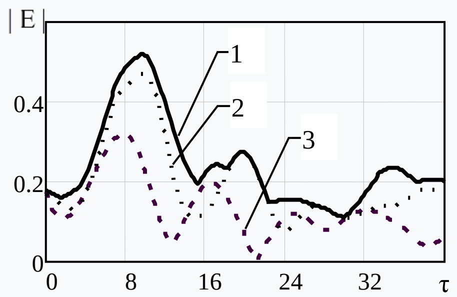

Fig. 1b. Time dependence of the field in the absence of pumping: 1 – the maximum field in the system; 2 – at the right end; 3 – at the left end

As can be seen from Figs. 1a, 1b, due to the synchronization of the oscillators, there is a surge in the energy field and then a slow process of damping the oscillations.

Fig. 2 show the same dependences at low (γ =0.1) and high (γ =1) pump rates.

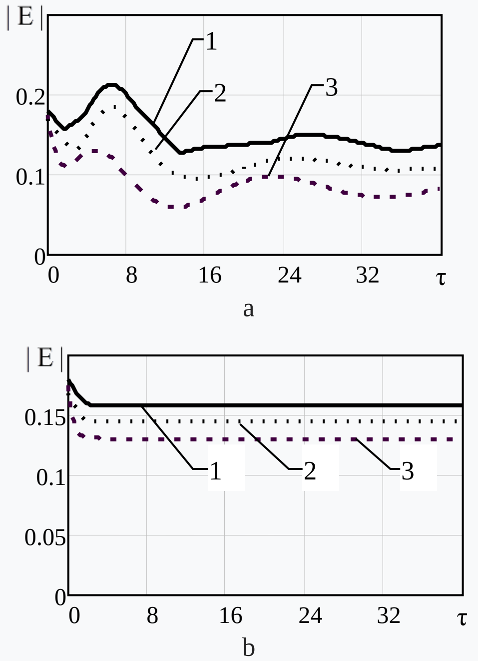

Fig. 2. Time dependence of the field at different pumping rates: 1 – the maximum field in the system; 2 – field at the right end; 3 – at the left end. Pumping rates: a – $\gamma = 0.1$*, b –* $\gamma = 1$

As can be seen from Figs. 2 and 3, pumping with random phases along the system prevents synchronization of the oscillators, as a result of which the field strength remains at the "noise" level. In this case, at a low pumping rate, a faster decrease in the average oscillator amplitude than without pumping is observed. At a higher pump level, the average amplitude stabilizes at a fairly high level, but, as already noted, this does not lead to an increase in the field due to synchronization suppression.

3. THE PUMPING PHASE IS CONSTANT IN SPACE AND RANDOM IN TIME.

For generality, let us also consider the case where the pumping phase is constant along the system, but varies randomly in time.

Let the pumping phase be $\delta_{j\tau}$ constant along the system (and hence at any time it is the same for all oscillators) but randomly vary in time in the range from zero to 2π. Calculations were carried out for this case at different values of the pumping rate γ and different periods of random phase change Δτ.

Fig. 3 shows the time dependence of the average oscillator energy at a low (γ=0.1) pumping rate and different periods of random variation of the pumping phase Δτ. Fig. 4 shows the fields in the system for the same conditions.

It follows from Figs. 3,a and 4,a that, at a pumping phase that is constant both in time and in space, the system is observed to enter a stationary regime (τ>100) with a fairly high level of average amplitude and maximum field. The field level testifies to the emerging effective phase synchronization of individual oscillators. An increase in the frequency of phase change leads to stochatization of processes with a decrease in the average amplitude of the oscillators and the field in the system (see Fig. 3,a,b and Fig. 4,a,b).

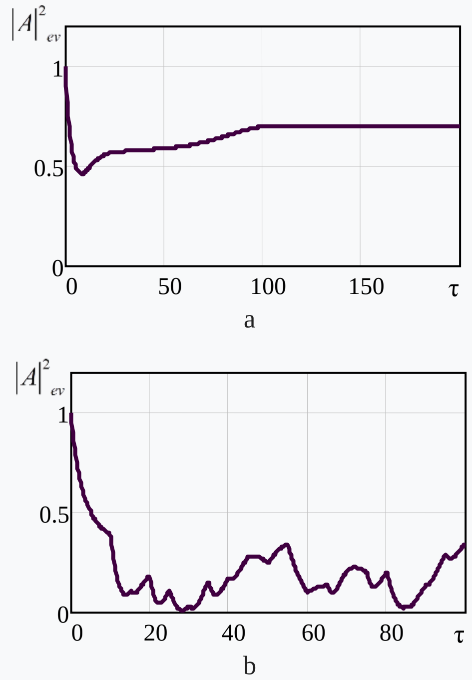

Fig. 3. Time dependence of the average energy of oscillators at $\gamma = 0.1$ *and different periods of pump phase change: a – Δτ =∞, b – Δτ =5*

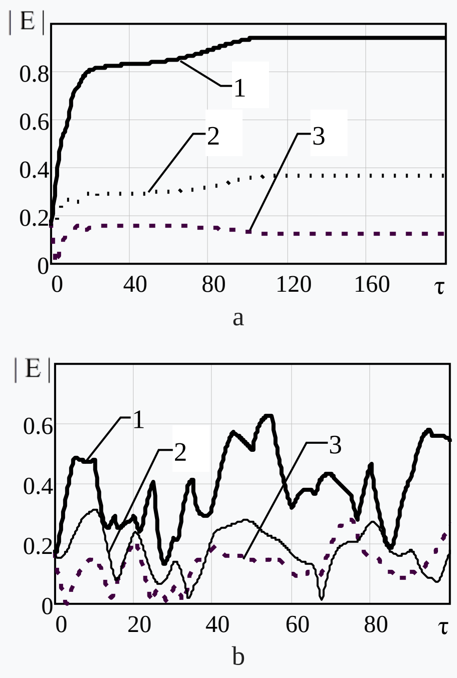

Fig. 4. Time dependence of the field at the left and right ends of the system and also the maximum field in the system at $\gamma = 0.1$ *and different periods of pump phase change: a – Δτ =∞, b – Δτ =5*

Increasing the pumping rate γ does not change the qualitative picture. At Δτ = ∞, the stationary regime is also observed, but in a shorter time and at higher levels of the average amplitude and field (τ>5 at γ=1). Similarly, increasing the phase change frequency also leads to stochatization in a shorter time with increasing pumping rate γ.

CONCLUSIONS

The chosen pump control mechanism limits the growth of oscillator amplitudes to a certain value. More important, however, is the change in the pump phase. Simulation of these processes is quite complicated, therefore, only separate regimes of the phase behavior of the pump were chosen. Considering these modes of pump phase shifts, one can get an idea of how a change in the pump phase affects the efficiency of field generation.

In the first case, there are no phase changes in time, but this phase is different at each point along the length of the system and, generally speaking, is randomly chosen. In the case of a low pump energy injection rate (γ<0.1), the characteristic time of which is much shoter than the characteristic time of development of the generation process, the field amplitudes in the system in the absence of pumping turn out to be half as in Fig. 1b. A high injection rate ($\gamma = 1$) keeps the amplitudes of the oscillators sufficiently large, but the amplitude of the excited field even slightly decreases in comparison with the previous case. At high pump injection rates, the behavior of the systems does not change qualitatively.

A somewhat different dynamics is observed when the phase of the pump field is constant along the system, but varies randomly with time. Let us compare these regimes with the case when no changes in the time of the pump phase occur at all. By the way, it can be seen that the oscillator amplitudes remain significant and the field, at least within the system, reaches values that are almost twice as large as in the case of no pumping. However, these significant fields are observed inside the system; at its ends, the field is several times smaller.

If the pump phase changes are quite rare, that is, the time of these changes is many times longer than the characteristic time of the generation process, then significant field values can also be observed inside the system. At the ends of the system, the same tendency of a noticeable decrease in the field amplitude is observed. With frequent changes in the pump phase, when the period of change is shorter than the characteristic lasing time, the lasing efficiency decreases, and the lasing regime acquires a different character. The period of a noticeable change in the field is an order of magnitude longer than the characteristic lasing time, and quasi-periodic field oscillations are also observed with a period close to the period of the change in the pump phase.

Thus, the most efficient method of pumping remains the regime with slow energy input into the system (γ<0.1), and the nature of the distribution of the field phase along the system can be both random and with a certain spatial period, but slightly changing with time.

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НАДВИПРОМІНЮВАННЯ КЛАСИЧНИХ ОСЦИЛЯТОРІВ ПРИ ПОСТІЙНОМУ НАКАЧУВАННІ

Євген В. Поклонський, Станіслав О. Тоткал

Досліджено залежність ефективності генерації в режимі надвипромінювання ансамблю осциляторів від швидкості введення енергії накачування, а також від характеристик зміни фази накачування. Передбачається, що фаза накачування може змінюватися як від часу, так і в просторі. Найбільш ефективним способом накачування залишається режим з повільним введенням енергії в систему, порівняно з характерним часом генерації, причому характер розподілу фази поля вздовж системи може бути як випадковим, так і з певним просторовим періодом, що змінюється з часом.