ELECTRODYNAMICS

https://doi.org/10.46813/2022-141-132

INSTABILITY OF THE SYSTEM OF CHARGED PARTICLES IN THE EXTERNAL CONSTANT MAGNETIC FIELD

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The results of studying the features of the dynamics of a system of charged particles (electrons) in an external magnetic field are described. The considered model practically coincides with the model of an ideal plasma. The electrons of such a plasma rotate at the cyclotron frequency. The rotation of charged particles leads to the emission of electromagnetic waves. The field strength of these radiated waves is very low. Therefore, usually these fields are neglected. In this work, these fields are taken into account. It is shown that with an increase in the oscillator density, oscillatory instability can develop. The dynamics of phase synchronization of these oscillators is traced. The conditions are found under which the oscillatory instability can be suppressed. Key words: oscillators, phase synchronization, plasma, instability.

PACS: 05.45.Xt; 52.55.-s; 52.55.Dy

INTRODUCTION

The question of the dynamics of a system of a large number of charged particles arises in many branches of physics. First of all, this is in plasma physics. It is known that if charged particles are in a certain potential, then they become oscillators. Moreover, the frequency of these oscillators and the relationship between them are determined by the type of potential in which they are located. An analysis of the dynamics of such a system of oscillators is presented, for example, in [1] and [2]. In this case, it turns out that the dynamics of such a system is always oscillatory and does not depend on the number of charged particles. However, if the connection between the oscillators has a different physical nature, then the dynamics can radically change. In particular, it can be unstable.

Thus, in [3–6], under the assumption that the connection between identical oscillators has some other physical nature, it was shown that the collective frequency of a system of such oscillators depends significantly on the number of oscillators. Moreover, the dynamics of such a system can become unstable. This instability was called oscillatory instability. In this paper, we consider the dynamics of a system of charged particles that are in an external constant magnetic field. In practice, we are talking about the dynamics of particles in the ideal plasma model. Particles (electrons) rotate in a magnetic field. Moreover, the frequency of these rotations does not depend (in the nonrelativistic case) on the electron distribution function and is equal to the cyclotron frequency ($\omega_H = eH/mc$, H – is the strength of the external magnetic field). The rotation of electrons in a magnetic field leads to their emission. This radiation acts on neighboring particles and is a specific physical mechanism of communication between oscillators. In the usual, accepted physical picture, the phases of these fields are random; therefore it is believed that these fields compensate each other. However, as shown in this work, the presence of such fields, such a connection between the oscillators can lead to their phase synchronization. All oscillators start to move in the same phase. The presence of such phased particle dynamics leads to the fact that the frequency of collective dynamics begins to decrease with an increase in the number of particles (oscillators). After reaching a certain critical number of oscillators, such a system becomes unstable. We will consider the possibility of such dynamics in this work.

The next (second) section describes the dynamics of a system of charged oscillators under the assumption that their phases are synchronized. The connection between the oscillators is the radiation field of each of the oscillators. It is shown that with an increase in the number of oscillators, the frequency of collective oscillations decreases. When a certain critical number of oscillators is reached, the system becomes unstable. In the third section, it is shown that the presence of coupling for a sufficiently large number of oscillators actually leads to their phase synchronization. Section four shows that sufficiently intense random fields can suppress the synchronization process.

1. DINAMICS OF PHASED OSCILLATORS

Consider the motion of particles with a charge in an external magnetic field directed along the axis *z*: $\vec{H}_0 = \{0,0,H\}$. In such a field, the particles rotate around the lines of force of the magnetic field. They move with acceleration and radiate. This radiation acts on neighboring particles. As a result, such particles interact with each other. For definiteness, we will assume that the considered ensemble of particles simulates ideal plasma. In this case, the Coulomb interaction of particles can be neglected. We will assume that the interaction is carried out only with the help of the fields that the particles emit during rotation. Moreover, the radiation frequencies of all particles are equal to the cyclotron rotation frequency of particles in an external magnetic field. Note that this frequency does not depend (in the nonrelativistic case) on the particle velocities. In this section, we will also assume that the radiation fields coincide not only the frequencies, but also the phases. Phase synchronization conditions are discussed in the next section. The electric field strength of the radiation of one particle in the vicinity of another particle is determined by the formula:

$$\vec{E} = -\frac{e \cdot \dot{\vec{v}}}{c^2 R} \; . \tag{1}$$

Here e – is the charge of the particle; $\dot{v}$ – is the acceleration of the particle during its rotation; c – is the speed of light; R – is the distance between the particles.

Consider first the interaction of two particles. We will consider only the motion of particles transverse to the magnetic field. The dynamics of one of the particles can be described by the following equation:

$$m\ddot{\vec{r}} = \frac{e}{c}\left[\vec{r}\vec{H}_0\right] - \frac{e^2\dot{\vec{v}}}{c^2 R}; \;\; \ddot{\vec{r}} = \frac{e}{mc}\left[\vec{r}\vec{H}_0\right] - \frac{e^2\dot{\vec{v}}}{mc^2 R}. \tag{2}$$

It is convenient to write this vector equation in the form of a system of equations for the transverse components of the velocities of one of the particles (for example, the first):

$$\dot{v}_{x1} = \omega_H v_{y1} - \mu \cdot \dot{v}_{x2}, \;\; \dot{v}_{y1} = -\omega_H v_{x1} - \mu \cdot \dot{v}_{y2}, \tag{3}$$

where $\omega_H = eH/mc$ – is the circular frequency of rotation of a particle in a magnetic field, $\mu = e^2/Rmc^2$ – the effect of the second particle on the dynamics of the first particle (the coupling coefficient between particles).

Similar systems of equations can be written for the dynamics of the second particle. If there are many particles, then the equations that describe the dynamics of the velocity components can be represented as (differentiating the system of equations (3) and taking into account the system (3) itself), one can obtain the following system of equations:

$$\ddot{v}_k + v_k = 2\sum_{j \neq k} \mu_j v_j \; . \tag{4}$$

In equation (4), the dependent variable v_k determines either the x or the y velocity component of the k-th (any) particle. In addition, a new time has been introduced $\tau = \omega_H t$. Coupling coefficients, which are on the right side of equation (4) under the sign of the sum, differ from each other only in the distance between the particles R_j. Note that with a large number of oscillators (N >>1) and small coupling coefficients (μ_j << 1), the right-hand side of equations (4) is the same for all oscillators. In this case, equations (4) can be significantly simplified:

$$\ddot{v}_k + \left(1 - 2\sum_{j=1}^{N} \mu_j\right) v_k = 0 \; . \tag{5}$$

The instability condition takes the form:

$$2\sum_{j=1}^{N} \mu_j > 1 \, . \tag{6}$$

Condition (6) can be rewritten for the particle density. Indeed, the total number of interacting particles is equal to the particle density multiplied by the volume occupied by the interacting particles ($N = nV$). The volume of the spherical layer of radius r and thickness dr is equal to $V = 4\pi r^2 dr$. Using these data, the left side of inequality (6) can be rewritten:

$$2\sum_{j=1}^{N} \mu_j = 8\pi n \int_0^{R_{max}} \frac{r^2 dr}{r} \frac{e^2}{mc^2} = 4\pi \frac{e^2}{mc^2} n R_{max}^2 = \mu N. \tag{7}$$

And condition (6) takes the form:

$$n > 3 \cdot 10^{11} / R_{\max}^2 \; . \tag{8}$$

2. PHASE SYNCHRONIZATION OF CHARGED OSCILLATOR

The results obtained above (for particles) are valid when the phases of the oscillators are synchronized. This synchronization can be done with an external field. For example, with electron-cyclotron heating of plasma. Synchronization can be implemented and self-consistent. This is exactly the process we will consider. The equations of motion of two interacting particles are written out above (system of equations (4)). If there are many particles, then the equations can be written in the form

$$\dot{v}_{xk} = \omega_H v_{yk} - \sum_{j=1}^{N} \varepsilon_{kj} \cdot cos\,\psi_j \;\; , \tag{9}$$

$$\dot{v}_{yk} = -\omega_H \cdot v_{xk} - \sum_{j=1}^{N} \varepsilon_{kj} \cdot sin\,\psi_j \; . \tag{10}$$

Here $\varepsilon_{kj} = \mu_{kj} A_j \omega_H$; $\mu_j = e^2 / R_{kj} mc^2$; $\psi_j = \omega_H t + \varphi_j$.

In equations (9) and (10), the phase shift between the components of the particle velocity is taken into account. The second terms on the right-hand side of these equations appeared as a result of taking into account the radiation of particles that move in a circle. For what follows, it is convenient to go over to dimensionless time. The system of equations (9), (10) will be rewritten:

$$\dot{v}_{xk} = v_{yk} - \sum_{j=1}^{N} \frac{\varepsilon_{kj}}{\omega_H} \cdot cos\,\psi_j \; ; \tag{11}$$

$$\dot{v}_{yk} = -v_{xk} - \sum_{j=1}^{N} \frac{\varepsilon_{kj}}{\omega_H} \cdot sin\,\psi_j \;\; .$$

If the radiation fields are neglected ($\varepsilon_j = 0$), then the solutions of this system have the form:

$$v_x = A\sin(\tau + \varphi); v_y = A cos(\tau + \varphi);$$
$$\{A, \varphi\} = const \; . \tag{11a}$$

Taking into account the radiation fields ($\varepsilon_j \neq 0$) leads to:

$$\{A, \varphi\} \neq const \;\; ; \tag{12}$$

$$\begin{aligned} \dot{v}_{x,k} &= A_k\, cos(\tau + \varphi_k)\left[1 + \dot{\varphi}_k\right] + \dot{A} sin(\tau + \varphi_k) = \\ &= A_k\, cos(\tau + \varphi_k) + \sum_{j \neq k} \varepsilon_j\, sin(\tau + \varphi_j). \end{aligned} \tag{13}$$

The expression (13) multiplies by $cos(\tau + \varphi_k)$ and average. As a result, one can obtain equations that describe the dynamics of the phases:

$$\dot{\varphi}_k = \frac{1}{A_k} \sum_{j \neq k} \frac{\varepsilon_{kj}}{\omega_H} sin(\varphi_j - \varphi_k) \; . \tag{14}$$

Here $\varepsilon_{kj} = \mu_{kj} A_j \omega_H$. Taken into account that $\langle exp(i\varphi) \rangle = \frac{1}{N} \sum_{j \neq k}^{N} exp(i\varphi_j)$. Then the equations for determining the phases of two arbitrary particles can be obtained from the equations:

$$\dot{\varphi}_k = \varepsilon_N \left\{ \begin{bmatrix} \cos(\varphi_k - \varphi_l)\langle \sin(\varphi_j - \varphi_l)\rangle - \\ -\sin(\varphi_k - \varphi_l)\langle \cos(\varphi_j - \varphi_l)\rangle \end{bmatrix} \right\}, \quad (15)$$

$$\dot{\varphi}_l = \varepsilon_N \left\{ \begin{bmatrix} \cos(\varphi_l - \varphi_k)\langle \sin(\varphi_j - \varphi_k)\rangle - \\ -\sin(\varphi_l - \varphi_k)\langle \cos(\varphi_j - \varphi_k)\rangle \end{bmatrix} \right\}.$$

Equations defining the distances between the phases of arbitrary particles will be useful:

$$\dot{\Delta}_{kl} = \varepsilon_N \left\{ \begin{matrix} \cos(\varphi_k - \varphi_l) \begin{bmatrix} \langle \sin(\varphi_j - \varphi_l)\rangle + \\ \langle \sin(\varphi_j - \varphi_k)\rangle \end{bmatrix} - \\ -\sin(\varphi_k - \varphi_l) \begin{bmatrix} \langle \cos(\varphi_j - \varphi_l)\rangle + \\ + \langle \cos(\varphi_j - \varphi_k)\rangle \end{bmatrix} \end{matrix} \right\}. \quad (16)$$

Here $\Delta_{kl} = \varphi_k - \varphi_l$. Analytical conclusions:

1. If the phases are (initially) evenly spaced in the range from 0 to 2π, then all average values are equal to zero. This means that such a configuration (such a distribution) of the phases does not lead to a change in the distance between the phases. Synchronization is missing.

2. Let the phases be slightly irregular. In addition, we will assume that the synchronization process has started ($\varphi_k \to \varphi_l;\ \ \varphi_j \to \varphi_k$). In this case, to determine the dynamics of the change in the position of the phases in the right-hand side of equation (16), only the second term can be saved:

$$\dot{\Delta}_{kl} = -2\varepsilon_N \sin\Delta_{kl}. \quad (17)$$

Equation (17) has singular points with coordinates $\Delta_{kl} = \pi n$, n – an arbitrary integer. Moreover, only those for which n is even ($n = 2m$) will be stable. This means that in the presence of a non-uniform arrangement of phases, their synchronization will always be observed. All phases will tend to a certain stable position. Moreover, the speed of movement to a steady state is described by the function:

$$\Delta_{kl} = \Delta_{kl}(0)\exp(\pm 2\varepsilon_N \tau). \quad (18)$$

The upper sign determines the phase divergence from unstable stationary points, the lower sign determines the rate of decrease in the distance between the phases.

3. RESULTS OF NUMERICAL RESEARCH

The dynamics of the phases can be traced in more detail by numerical methods. For this, the system of equations (14) was solved numerically. We note right away that all the features described above were clearly observed in numerical studies. Moreover, to see the analytical conclusions formulated above, it was enough to consider 10 oscillators.

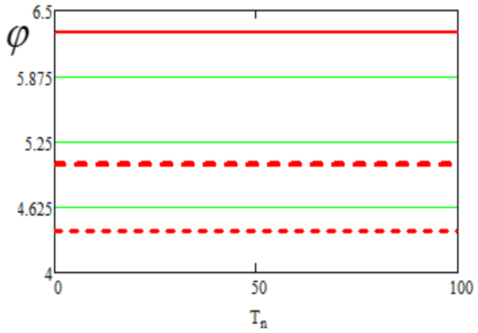

Fig. 1. All phases were initially evenly distributed in the phase range from 0 to 2π. During the entire counting time, the value of the phases of each oscillator did not change

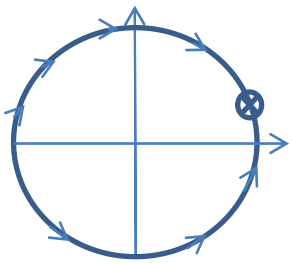

Fig. 2. The phases of all particles tend to one stable phase

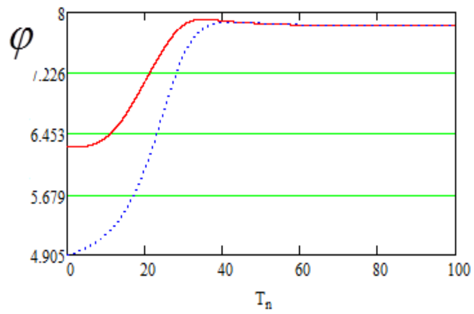

Fig. 3. Synchronizing the phases of the oscillators. The case when the phase of one of the oscillators violated the uniform distribution. The phases of the oscillators are presented, which are located on the lower half of the circle (see Fig. 2)

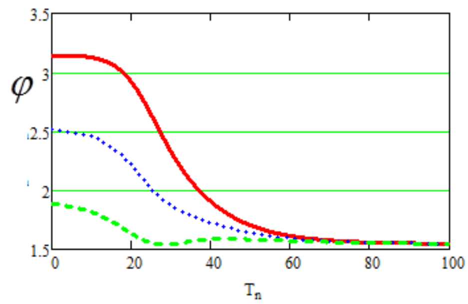

Fig. 4. Synchronizing the phases of the oscillators. The case when the phase of one of the oscillators violated the uniform distribution. The phases of the oscillators are shown, which are located on the upper half of the circle (see Fig. 2)

4. SUPPRESSION OF PHASE SINCHRONIZATION

Thus, in the general case, within the framework of the considered model, the system of charged particles becomes unstable when the number of oscillators exceeds the critical one. The question arises about the mechanisms of suppression of this instability. The first thought that comes up is that the presence of a sufficient level of fluctuations will not allow the phase synchronization process to take place. In general, this is a rather difficult problem to solve. However, estimates for the required value of fluctuations are easy to obtain. To do this, add to the right-hand side of the equation for the phase a term that describes the presence of random forces (additive random forces):

$$\dot{\varphi}_l = \varepsilon_{lN}\, Im\left\{ \begin{matrix} exp\left(-i\left(\varphi_l - \varphi_k\right)\right)\dfrac{1}{N} \\ \cdot \sum\limits_{j\neq l,k} exp\left(i\varphi_j - i\varphi_k\right) \end{matrix} \right\} + \xi(\tau). \qquad (19)$$

Here $\varepsilon_{lN} = \dfrac{1}{A_l}\sum\limits_{j\neq k,l}^{N} \mu_{lj} A_j$; $\mu_{lj} = e^2/R_{lj}mc^2$.

For simplicity, we will assume that the addition on the right-hand side of the system of equations (19) is a delta-correlated random function:

$$\langle \xi(t) \rangle = 0; \langle \xi(t)\xi(t_1) \rangle = D\delta\left(t - t_1\right). \qquad (20)$$

It is possible obtain such conditions that the iffusion process will suppress synchronization:

$$D > 4\pi^2\mu \cdot N . \qquad (21)$$

CONCLUSION

Above, we considered a simple model in which the elements of coupling between the oscillators are weak electromagnetic fields, which are excited by rotating electrons. Even in this case, at a sufficiently high density of oscillators, instability (oscillatory instability) may arise. A high probability of the occurrence of phase synchronization, and as a result, the occurrence of instability, arises when the ensemble of oscillators is subjected to an external synchronizing field. The mechanisms of such external synchronization are currently well studied (see, for example, [7–9]).

A few words should be said about plasma diamagnetism. One charged particle in a magnetic field is diamagnetic. However, in a plasma, the rotation phases of particles are random. As a result, the currents of these particles cancel each other out. The fields that are excited by these currents are also compensated. The intensity of the radiation field of one particle is very small. Therefore, at low plasma densities, the effect of these fields is insignificant. Plasma diamagnetism disappears within the framework of the model considered above. Thus, the above-described feature of the dynamics of plasma particles will mainly manifest itself at high plasma densities.

ACKNOWLEDGEMENTS

The authors are grateful to Professor Voitsenya V.S. for many helpful discussions of the results of the work and for drawing their attention to the existence of the “density limit” problem.

This work was supported by the target program "Plasma Physics and Plasma Electronics: Foundations and Applications" of the National Academy of Sciences of Ukraine (grant No. 0117U006867).

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Article received 09.09.2022

НЕСТІЙКІСТЬ СИСТЕМИ ЗАРЯДЖЕНИХ ЧАСТИНОК У ЗОВНІШНЬОМУ ПОСТІЙНОМУ МАГНІТНОМУ ПОЛІ

В.О. Буц, Д.М. Ваврів

Описано результати дослідження особливостей динаміки системи заряджених частинок (електронів) у зовнішньому магнітному полі. Розглянута модель практично збігається із моделлю ідеальної плазми. Електрони такої плазми обертаються із циклотронною частотою. Обертання заряджених частинок призводить до випромінювання електромагнітних хвиль. Напруженість поля цих хвиль, що випромінюються, дуже мала. Тому зазвичай цими полями нехтують. У цій роботі ці поля враховуються. Показано, що зі збільшенням густини осциляторів може розвиватися коливальна нестійкість. Простежена динаміка фазової синхронізації цих осциляторів.