https://doi.org/10.46813/2021-134-090

# INVESTIGATION OF THE BOUNDARY-VALUED PROBLEM ON RESONANCE MHD NON-UNIFORMITY BY INTEGRAL EQUATIONS USING

*Y.N. Oleksandrov, I.Sh. Nevliudov, O.O. Chala, I.B. Botsman, V.V. Nevliudova*
*Kharkiv National University of Radio Electronics, Kharkiv, Ukraine*
*E-mail: olena.chala@nure.ua*

Numerical simulation of interior field velocity is studied on the basis of the rigorous analytical solution of the boundary-valued magnetohydrodynamics problem on the sphere type non-uniformities. The basis for the analytical solution is the method of integral equations of linear magnetohydrodynamics. The analysis of the obtained results is carried out.

PACS: 52.65.Kj

## INTRODUCTION

It can be stated that the processes of simulation in different fields of physics are presently in the forefront of their claim. In this case the emphasis is on the numerical simulation which really performs well. In the electromagnetic field theory they are obtained, in particular, owing to the finite set of the strongest method's such as the moments method, averaging technique, elementary boundary, surface integral equations, boundary elements, Galerkin or Galerkin-Petrov method etc. [1]. But these results, nevertheless, are mainly concerned, on the one hand, only with the quantitative description of the considered processes and, on the other hand, they are simply little efficient in case of three-dimensional problems. But the analytical or semi-analytical methods, the integral equations methods being among them [2] making it possible to give a general picture of the phenomenon as a whole regardless of the problem dimensionality, less claimed at present continue to give the qualitative description.

Magnetohydrodynamics (MHD) non-uniformities representing a good theoretical model for description of diffraction phenomena of the real structures occurred in practice are considered in this work on its basis. It is possible to single out two fundamentally different directions of investigations into the MHD wave scattering on non-uniformities of plasma and magnetic field densities and plasma flow density around these non-uniformities with respect to their possible applications. On the one hand, they are MHD phenomena taking place in the ionosphere of the Earth and planets, in the atmosphere of the Sun, in the interplanetary and interstellar plasma and the phenomena directly associated with investigation into collapsing masses magnetic field, superstars nature etc. [3]. It becomes evident just here that, in consequence, the MHD non-uniformities appear when contacting with non-equilibrium processes giving rise to the anomalies in ionization distribution. On the other hand, these are phenomena connected with different techniques, in particular, with the laboratory plasma units. It is important both when investigating propagation of high-frequency plasma instabilities in great linear accelerators, and in the problems of resonance structures application to electrons acceleration etc. [4].

As it is well known the MHD description of plasma is of particular interest for the phenomena where the electric field can reach great values. This can be associated both with the polarization phenomena, and the induction processes induced by fast variable magnetic fields. Their interaction with plasma is more conventionally described in terms of the magnetic hydrodynamics where the magnetic field intensity is assumed to be the primary value and the electric current and electric intensity are considered to be the secondary ones.

Due to the magnetic field fluctuation the complete MHD field can have a strong non-uniformity distribution, in particular, this is supported by the observations performed on the space plasma. i.e., the regions of the relatively weak intensity alternate with the regions characterized with strong field concentrations (non-uniformities) in the form of layers, bunches, ellipsoids etc. This is well simulated in the approximation of the ideal magnetic hydrodynamics. High-frequency oscillations in the considered non-uniformities can form resonator structures similar to dielectric resonators in electrodynamics and this phenomenon is of special interest as a great deal of energy can be stored up in the resonance structures.

The need for "establishing a proper contact between the theory on the one hand, and an experiment or an observation on the other hand" (G. Alfven [5]) remains one of the most important and still the most difficult problems of investigation into the MHD outer space. Pointing out the experimental investigation limitations and great theoretical difficulties it can be assumed that further progress in this field will depend on the correctly constructed MHD models. These models must describe experimental investigations the most closely. Here again the importance of mathematical simulation for our object of investigation should be stressed. At first it is meaningful to consider a simple model of the MHD non-uniformity allowing to obtain a rigorous analytical solution and only then to make the model more realistic adding to it successively newer effects. Thus, a simple MHD model and complicated numerical calculations will be able to complement each other when developing a realistic MHD model [6 - 21].

A sphere can be one of such simplest non-uniformities, this model admits the detailed theoretical investigation and the possibility of the experimental checking under laboratory conditions.

## 1. GENERAL INTEGRAL STATEMENT OF THE MHD BOUNDARY-VALUE PROBLEM

Let us consider the general case of the boundary-value MHD problem when small perturbations in the plasma medium interpreted as a magneto hydrodynamic one is described by the state vector. The state vector $\vec{\Psi}(\mathbf{r},t)=\{\mathbf{u}(\mathbf{r},t),\mathbf{b}(\mathbf{r},t),\rho(\mathbf{r},t)\}$ – represents the totality of velocity $\mathbf{u}(\mathbf{r},t)$, magnetic $\mathbf{b}(\mathbf{r},t)$ field and density $\rho(\mathbf{r},t)$ deviation from their no perturbed values $\mathbf{U}_i,\mathbf{B}_i,\rho_i,_{i=1,2}$, assigning the MHD media (internal and external ones).

Let us assume that some non-uniformity (geometrically uniform domain) assigned by the parameters $\mathbf{U}_2,\mathbf{B}_2,\rho_2,V_{A2},V_{S2}$, has the volume $V(t)$, depending on time in the general case. Let the considered non-uniformity be placed in the unlimited MHD medium characterized by the parameters $\mathbf{U}_1,\mathbf{B}_1,\rho_1,V_{A1},V_{S1}$, respectively, till its perturbation with the incident field which is assigned by the corresponding state vector

$$\vec{\Psi}_0(\mathbf{r},t)=\{\mathbf{u}_0(\mathbf{r},t),\mathbf{b}_0(\mathbf{r},t),\rho_0(\mathbf{r},t)\}.$$

Here $V_{Ai}=\dfrac{B_i}{\sqrt{4\pi\rho_i}}$ is the Alfven and $V_{Si}=\dfrac{dp}{d\rho}$ ($p$ – pressure) is the sound velocity of the internal $(i=2)$ and external $(i=1)$ media.

Then the equation can be represented in the form of the convolution of $\hat{\mathbf{G}}$ and $\mathbf{W}$ functional relative to the state vector $\vec{\Psi}(\mathbf{r},t)$ [6]:

$$\vec{\Psi}(\mathbf{r},t)=\vec{\Psi}_0(\mathbf{r},t)+\hat{\mathbf{G}}(\mathbf{r}-\mathbf{r}',t-t')*\mathbf{W}(\mathbf{r},t), \qquad (1)$$

i.e. the integral operation of the form:

$$\hat{\mathbf{G}}*\mathbf{W}=\int_{-\infty}^{\infty}dt'\int_{\infty}\hat{\mathbf{G}}(\mathbf{r}-\mathbf{r}',t-t')\mathbf{W}(\mathbf{r},t)d\mathbf{r}'.$$

Here in terms of the diffraction theory $\vec{\Psi}_0(\mathbf{r},t)$ is the state vector of the incident (nonperturbed) field;

$\mathbf{W}(\mathbf{r},t)$ is the discontinuous function written in the generalized functions' class. This function describes equally the MHD medium inside and outside the non-uniformity taking into account boundary and initial conditions;

$\hat{\mathbf{G}}(\mathbf{r}-\mathbf{r}',t-t')$ is Green's function of the MHD equations of the free space assigned by the parameters $\mathbf{U}_1,\mathbf{B}_1,\rho_1,V_{A1},V_{S1}$, or the fundamental solution of the following system of differential equations:

$$\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{U},\nabla)\mathbf{u}+\frac{V_{S1}}{\rho_1}\nabla\rho+\frac{1}{4\pi\rho_1}[\mathbf{B}_1,rot\mathbf{b}]=\mathbf{S}'_u\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'),$$

$$\frac{\partial\mathbf{b}}{\partial t}-rot[\mathbf{U}_1,\mathbf{b}]+rot[\mathbf{B}_1,\mathbf{u}]=\mathbf{S}'_b\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'),$$

$$\frac{\partial\rho}{\partial t}+\rho_1 div\mathbf{u}+div(\rho\mathbf{U}_1)=S'_\rho\delta(\mathbf{r}-\mathbf{r}')\delta(t-t').$$

In [6] the fundamental solution is obtained and completely described in the dyadic representation. This representation follows naturally from the known fact that any tensor may be written as the sum of three dyads. Whereas Green function for the given class of problems is the tensor function of two points' position: the observation point $(\mathbf{r},t)$ position and the source point $(\mathbf{r}',t')$ position.

In the general case (1) represents the integral-differential equation, its type is defined just by the properties of Green functions. It is easy to derive different special cases of the Green function representation from the fundamental solution of the general form. Each of these functions may turn out to be more preferential when solving a concrete problem in practice. The considered method of boundary problems solution is convenient for the problems of volume scattering and MHD flow. With this method a modern theoretical model for solving self-consistent MHD boundary problems has been developed [2]. Naturally the first step in this problem solution is the analysis of geometric non-uniformities flow with MHD flux in the steady-state case.

## 2. STEADY-STATE INTEGRAL EQUATIONS OF HYDROMAGNETICS

In the stationary hydromagnetics the Green function assumes rather simple form but peculiar to the hydromagnetics in the absence of unperturbed medium movement ($\mathbf{U}_1=0$) [2]:

$$\hat{\mathbf{G}}(\mathbf{R})=\frac{\hat{\Theta}(\theta,\varphi)}{|\mathbf{R}|}. \qquad (2)$$

Here $\hat{\Theta}(\theta,\varphi)$ is the matrix written in the basis $\langle\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\rangle$ connected with the chosen direction of the external magnetic field $\mathbf{e}_2=\mathbf{s}_1=\mathbf{B}_1/B_1$:

$$\hat{\Theta}(\theta,\varphi)=\frac{1}{2\pi V_{A1}^2}\begin{pmatrix}\dfrac{\cos^2\theta\cos^2\varphi-\sin^2\varphi}{\sin^2\theta} & -ctg\theta\cos\varphi & -\dfrac{\sin2\varphi\left(1+\cos^2\theta\right)}{2\sin^2\theta}\\ -ctg\theta\cos\varphi & 1-\dfrac{V_{A1}^2}{V_{S1}^2} & -ctg\theta\sin\varphi\\ -\dfrac{\sin2\varphi\left(1+\cos^2\theta\right)}{2\sin^2\theta} & -ctg\theta\sin\varphi & \dfrac{\cos^2\theta\sin^2\varphi-\cos^2\varphi}{\sin^2\theta}\end{pmatrix}. \qquad (3)$$

$\mathbf{R}=\mathbf{r}-\mathbf{r}'$ is the radius-vector specified by the polar coordinates: $\varphi,\theta:\ 0\le\varphi<2\pi,\quad 0\le\theta\le\pi$.

I.e., the stationary Green function has a special feature of the type $|\mathbf{r}-\mathbf{r}'|^{-1}$, and the problem anisotropy is emphasized by the basis $\langle\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\rangle$ connected with the unperturbed magnetic field.

In this case having applied convolution properties to the integral-differential equation (1) and taking into account the type of the Green function (2) we will obtain the integral equation relative to MHD speed $\mathbf{u}$:

$$\mathbf{u}(\mathbf{r})=\mathbf{u}_0(\mathbf{r})+\left(V_{S1}^2-V_{S2}^2\right)\mathbf{graddiv}\int_{(V)}\mathbf{u}(\mathbf{r}')\hat{\mathbf{G}}(\mathbf{r},\mathbf{r}')d\mathbf{r}'-$$
$$-V_{A1}^2\left[\mathbf{s}_1,\mathbf{rotrot}\left[\mathbf{s}_1-\frac{B_2}{B_1}\mathbf{s}_2,\int_{(V)}\mathbf{u}(\mathbf{r}')\hat{\mathbf{G}}(\mathbf{r},\mathbf{r}')d\mathbf{r}'\right]\right] \qquad (4)$$

Similarly the equation for the magnetic field deviation $\mathbf{b}(\mathbf{r})$ is written.

One of the integral equations' (4) peculiarity is that they represent a mathematical description of the phenomena with the retarded potentials which describe interaction at the finite distances. In this instance this potential is the velocity potential $\vec{\Pi}_{\mathbf{u}}(\mathbf{r}) = \int_{(V)} \mathbf{u}(\mathbf{r}')\hat{\mathbf{G}}(\mathbf{r},\mathbf{r}')d\mathbf{r}'$, and the magnetic field potential $\vec{\Pi}_b(\mathbf{r}) = \int_{(V)} \mathbf{b}(\mathbf{r}')\hat{\mathbf{G}}(\mathbf{r},\mathbf{r}')d\mathbf{r}'$. According to the structure these potentials are similar to the Hertz electrodynamics potential.

Whereas another peculiarity consists in that the considered method for boundary problems solution assumes essentially not merely reduction of the initial differential equations to the integral form (4) or, in the general case, to the integral-differential equation (1) (it is always possible to realize having built the corresponding Green function), but to the application of the additional statement, namely, the extinction principle. It is precisely the latter that results in a clear simple algorithm of the boundary problem solution. And in accordance with this algorithm the relations (4) are properly integral equations only for the internal points of non-uniformity. For the external points they represent quadrature formulas making it possible to find the external field using the internal field found by this time. Hence, it follows that it is just the internal problem that represents the greatest severity in terms of mathematics. Let us dwell upon its analysis.

## 3. SPHERICAL MHD NON-UNIFORMITY

Thus, let us consider the simplest model of the non-uniformity, namely, a sphere with a radius $a$. This model really admits the detailed theoretical investigation (a rigorous analytical solution is built for it) and gives the possibility to compare theoretical results with the observations performed under laboratory conditions. Let us investigate the integral characteristics of the internal field, namely, the potential of velocity:

$$\vec{\Pi}_u(\mathbf{r}) = \int_{(V)} \frac{\mathbf{u}(\mathbf{r}')\hat{\Theta}(\theta,\varphi)}{R} d\mathbf{r}', \tag{5}$$

where $\theta, \varphi$ are polar angles of the radius-vector $\mathbf{R} = \mathbf{r} - \mathbf{r}'$, and R is the distance between the element whose volume is $d\mathbf{r}'$ to the observation point $\mathbf{r}$.

Having assumed to start with that $\mathbf{u}(\mathbf{r})$ is constant and the observation point is inside the sphere with the volume of (V), let us introduce the spherical coordinate system with the center in the observation point. Then the integral (5) is easily reduced to the form:

$$\vec{\Pi}(\mathrm{r}) = \mathrm{uI}(\mathrm{r}), \quad \mathrm{I}(\mathrm{r}) = \iint_{(\Omega)} \hat{\Theta}(\theta,\varphi)\, d\omega \int_0^R \rho\, d\rho, \tag{6}$$

where $d\omega$ is an element of the sphere area $(\Omega)$, $\mathbf{R}$ is a radius-vector of the points on the surface limiting the volume (V). In this case the Cartesian coordinates of the sphere surface points may be expressed using the direction cosines $\alpha, \beta, \gamma$ of angles of the radius-vector $\mathbf{R}$ with its main axes in the following way: $\varsigma = x + R\alpha, \xi = y + R\beta, \zeta = z + R\gamma$.

Having performed integration over the whole surface $(\Omega)$ of the unit radius, we shall obtain as a result:

$$\mathbf{I}(\mathbf{r}) = \frac{1}{a^2 V_{A1}^2} \begin{pmatrix} \frac{x^2 - z^2}{3} + y^2 & -\frac{4}{3}xy & -\frac{2}{3}yz \\ -\frac{4}{3}xy & \frac{8}{3}\left(\frac{V_{A1}^2}{V_{S1}^2} - 1\right)\left(x^2 + y^2 + z^2\right) & -\frac{4}{3}xz \\ -\frac{2}{3}yz & -\frac{4}{3}xz & \frac{x^2 - y^2}{3} + z^2 \end{pmatrix}. \tag{7}$$

From (7) it is seen that the potential of velocity $\vec{\Pi}_u(\mathbf{r})$ analogously true for the potential of the magnetic field $\vec{\Pi}_b(\mathbf{r})$ is the second power polynomial of the Cartesian coordinates. It immediately follows that if the unperturbed field is uniform then the internal field of the MHD sphere is also uniform. That is to say, if the unperturbed field of velocities is of the form:

$$\mathbf{u}_0 = \{u_{ox}, u_{0y}, u_{oz}\}, \tag{8}$$

then the internal field should also be sought for in the form of the constant vector:

$$\mathbf{u} = \{u_x, u_y, u_z\}. \tag{9}$$

We emphasize once again that the considered property of the field uniformity is well known in electrodynamics. But the essential dissimilarity of the MHD potentials from the Newton potential for the problems of electrodynamics is that these potentials are written in the form of a matrix. The matrix is set in the basis $\langle \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \rangle$ connected with the direction of the unperturbed magnetic field $\mathbf{s}_1$. This involves a strong dependence on the direction in relation to the magnetic field not only of the internal field but also as a result, of the external one. Having written the vector differential operations in (4) let us reduce the internal problem of finding the field of velocities in the sphere with radius a to the linear algebraic equations system:

$$\begin{gathered} u_x\left[3a^2 - 2\mathrm{E} - 8\Gamma + 2\eta s_{2x}\right] + 4u_y\left[\mathrm{E} + \eta s_{2x}\mathrm{H} - \Gamma\right] - u_z\eta\left(8s_{2z} + 2s_{2x}\right) = 3a^2 u_{0x}, \\ 2u_x\mathrm{E} + u_y\left[3a^2 - 16\mathrm{E}(\mathrm{H} - 4)\right] = 3a^2 u_{0y}, \\ 2u_x\eta s_{2x} + 32u_y\eta s_{2z}\mathrm{E} + u_z\left[3a^2 - 6\mathrm{E} + 4\Gamma - 2\eta\left(s_{2z} + 2s_{2x}\right)\right] = 3a^2 u_{0z}, \end{gathered} \tag{10}$$

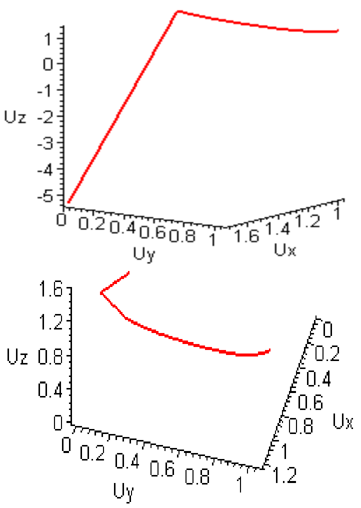
here the following designations are introduced to reduce the notation: $E = \frac{V_{S1}^2 - V_{S2}^2}{V_{A1}^2}$, $H = \frac{V_{A1}^2}{V_{S1}^2} - 1$, $\Gamma = 1 - \eta s_{2y}$, $\eta = \frac{B_2}{B_1}$.

Let us perform the numerical analysis of the system (10) solution, which gives the complete picture of the MHD sphere internal field development. In this case just the resonant structures are of evident interest. Let us analyze, in particular, how the sphere dimensions and other medium characteristics act on the resonance or the MHD instability rise.

As mentioned above, plasma is non-uniform, as a rule. And plasma parameters' fluctuation is one of the reasons for the non-uniformities rise. Let us consider the MHD sphere as a multiparameter system. We will consider the internal MHD flux velocity as a function of the following parameters $\mathbf{u} = \mathbf{u}(V_{Ai}, V_{Si}, B_i, \mathbf{u}_0, \mathbf{s}_2, a)_{i=1,2}$, where $\mathbf{u} = \{u_x, u_y, u_z\}$. Each of the above-listed parameters describes a particular type of non-uniformity.

Topographical picture of the internal MHD velocity variation in two- and one-parametric cases appears as follows.

Fig. 1 shows one-parametric hodograph of the variable vector $\mathbf{u} = \mathbf{u}(\varsigma)$ of the real parameter $\varsigma = V_{A1}^2$ with the fixed values of the remaining parameters; it gives a pictorial view of variations of the absolute value $\mathbf{u}$ depicted by the variable vector itself and of this variation velocity with the tangent direction to the hodograph curve.



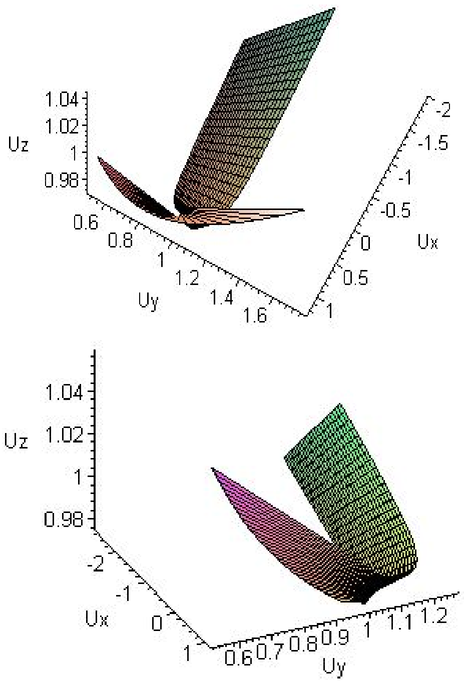

$a - \mathbf{s}_2 = \left\{\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3\right\}$; $b - \mathbf{s}_2 = \left\{\sqrt{2}/2, \sqrt{2}/2, 0\right\}$

*Fig. 1. Topographic curve of the velocity component variation depending on the parameter* $V_{A1}^2$

Fig. 2 shows two-parametric hodograph of the variable vector $\mathbf{u} = \mathbf{u}(\varsigma, \xi)$ of two real parameters $\varsigma = V_{A1}^2$ and $\xi = V_{S1}^2 - V_{S2}^2\big|_{V_{S1}^2=const}$, the latter is considered relative to the level $V_{S1}^2$. In the given case we have the surface representing the continuous set of the variable radius-vector $\mathbf{u} = \mathbf{u}(\varsigma, \xi)$ endpoints.

In the context of our concrete model it is not difficult to consider development of the module of velocity field $u = \sqrt{u_x^2 + u_y^2 + u_z^2}$ depending on the given above parameter $\varsigma$ (abscissa axis), i.e. to trace the action of different non-uniformity types on the internal MHD velocity module.

Plots of Fig. 3 demonstrate extremums. They can be interpreted as a sort of resonances and antiresonances. As it is common knowledge the presence of no less than two independently varying parameters in the system may cause the rise in the geometric resonance.

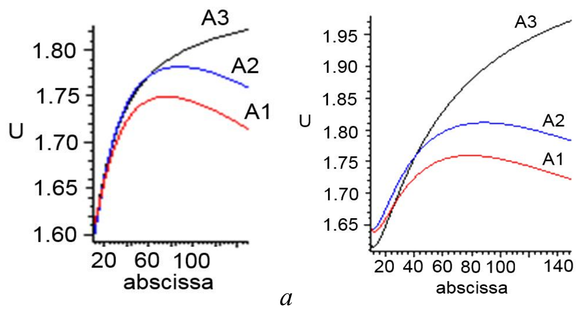



$a - \mathbf{s}_2 = \left\{\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3\right\}$; $b - s_2 = \left\{\sqrt{2}/2, \sqrt{2}/2, 0\right\}$.

*Fig. 2. Topographic surface of the velocity component variation depending on the parameters* $\varsigma = V_{A1}^2$, $\xi = V_{S1}^2 - V_{S2}^2\big|_{V_{S1}^2=const}$

As this takes place, the latter become less expressed with the increase in the sphere radius (see Fig. 3,a).

Let us consider the field quadratic characteristics (Fig. 4).




*Curve A1* – $\mathbf{s}_2 = \left\{\sqrt{2}/2, \sqrt{2}/2, 0\right\}$,

*curve A2* – $s_2 = \left\{\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3\right\}$,

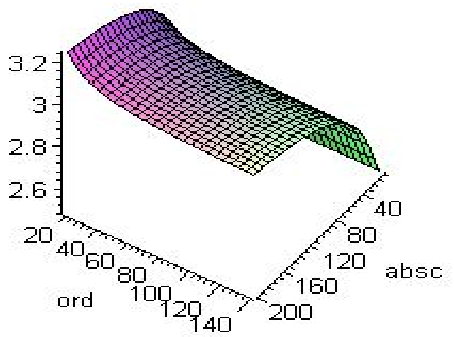
*curve A3* – $\mathbf{s}_2 = \{0, 1, 0\}$

*Fig. 3. Dependence of the velocity module on the parameter* $\varsigma = V_{A1}^2$

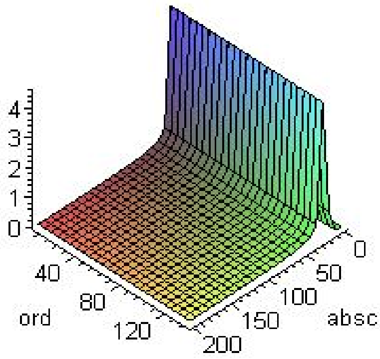
The radius of the sphere in Fig. 4,a is by an order of magnitude greater than the radius of the sphere in Fig. 4,b.

The considered dependence causes new interesting effects. On the one hand, in particular, the square of velocity can give an idea of the MHD field power characteristics and on the other hand, it can give the possibility to trace the plasma formations instability.

In this case the increase in velocity is of certain interest. Here, the dependence on the spherical formation radius is also clearly traced.



*a*



*b*

*Fig. 4. Dependence of velocity square $u^2$ on the parameters $\varsigma = V_{A_1}^2$ (X-axis) and $\zeta = V_{s_1}^2$ (Y-axis)*

## CONCLUSIONS

Having summarized, we can say that though the considered smooth variations of the internal field parameters of the spherical profile non-uniformity are hardly realizable in the real situation but, nevertheless, the performed simulation is important as it is the first rigorously analytical step in studying the MHD sphere in the MHD field.

The possibility of the resonance structures emergence in the non-uniformities of such a type was shown in [18], where the MHD sphere was in the uniform nonmagnetic liquid of the set density and adiabatic compressibility. The solution was obtained in the form of decomposition in terms of vector spherical harmonics; this gave the possibility to reveal the conditions of rise of the magnetic field geometric resonance and velocity field. The further development of the considered model is the MHD waves scattering on the small sphere. To perform this small parameter, $a/\lambda, \lambda$ is a wavelength, is introduced direction. Having presented the fields and Green function as this parameter decomposition we reduce the problem to a sequence of the integral equations of (4) type, their constant term is defined by the corresponding decomposition of the incident field and the integral summands depending on the previous approximations solutions. The considered model solution is taken as a zero approximation.

## REFERENCES

1. A.A. Aleksandrova and Yu.N. Aleksandrov. Model Problem of Wave Refraction by a Periodically Nonuniform Boundary of the Solar Plasma // *Technical Physics*. 2004, v. 49.
2. I. Nevlyudov, V. Palagin, and I. Botsman. The general principles of electromagnetic compatibility improving with microsystem technology using // *3rd International Scientific-Practical Conference Problems of Infocommunications Science and Technology*. 2016, p. 237-238.
3. E. Mathew, T. Pawar, and B.J. Pandian. Control of a Coupled CSTR Process using MRAC-MIT Rule // *Innovations in Power and Advanced Computing Technologies*. 2019, v. 1, p. 1-5.
4. I. Nevliudov, S. Maksymova, A. Funkendorf, O. Chala, and K. Khrustalev. Using MEMS to adapt ultrasonic welding processes control in the implementation of modular robots assembly processes // *XIV-th International Conference Perspective Technologies and Methods in MEMS Design (MEMSTECH)*. 2018, p. 223-226.
5. G.M. Webb, G.P. Zank, E.K. Kaghashvili, R.E. Ratkiewicz. Magnetohydrodynamic waves in non-uniform flows I: a variational approach // *Journal of Plasma Physics*. 2005, v. 71(6), p. 785-809.
6. O. Filipenko, O. Chala, O. Sychova. Some Issues of Dependencies of Loss from Technological Features of Optical Switches for Communication Systems // *International Scientific-Practical Conference Problems of Infocommunications. Science and Technology*. 2018, p. 599-603.
7. N. Sakhnenko, N. Stognii, A. Nerukh. Hybridization of plasmons in coupled nanowires // *Micro- and nano-photonic materials and devices*. Trento, Italy, 2012, p. 69-72.
8. N. Biznyuk, R. Sauleau, A.I. Nosich. Conical-pattern circular dielectric disk antenna analyzed using generalized boundary condition and dual integral equations // *Asia Pacific Microwave Conference, APMC'2018*, Kyoto, Japan, 2018.
9. I. Zharikova, S. Novoselov, V. Nevlyudova, O. Botsman. Electromagnetic Compatibility Analysis of the Infocommunication Systems Components on the Flexible Structures Basis // *Second International Scientific-Practical Conference Problems of Infocommunications Science and Technology*. 2015, p. 101-103.
10. A.A. Aleksandrova, Yu.N. Aleksandrov. Fundamentalnoe reshenie uravneniy lineynoy magnitnoy gidrodinamiki v dvizhuscheysya srede // *ZhTF*. 2001, t. 71, v. 7, p.1-6.
11. M. Andriychuk. Linear antenna synthesis by an amplitude radiation pattern // *21st International Conference on Microwave, Radar and Wireless Communications (MIKON)*, IEEE. 2016, p. 1-4.
12. K.G. Selivanova, O.G. Avrunin, S.M. Zlepko, et al. Virtual training system for tremor prevention // *Information Technologies in Medical Diagnostics II – Editor by Wojcik*, Pavlov, Kalmodaev, ISBN 978-0-367-17769-0, 2019, p. 9-14.
13. A.A. Aleksandrova, N.A. Hizhnyak. *Kraevyie zadachi magnitnoy gidrodinamiki.* Harkov: "TestRadio" LTD. 1993, s. 230 (in Russian).
14. D.M. Natarov and A.O. Natarova. Lattice Lasing Modes of Finite Grating of Silver Nanowires with Active Shells // *IEEE 2nd Ukraine Conference on Electrical and Computer Engineering (UKRCON)*, Lviv, Ukraine, 2019, p. 743-746.

15. D.M. Natarov, T.M. Benson, A.I. Nosich. Electromagnetic analysis of the lasing thresholds of hybrid plasmon modes of a silver tube nanolaser with active core and active shell // *Beilstein J. Nanotechnol.* 2019, v. 10, p. 294-304.
16. R. Mittra. A look at some challenging problems in computation electromagnetic // *IEEE Transactions on Antennas and Propagation*. 2004, v. 46, № 5, p. 18-32.
17. E.R. Pries. Solar magnetohydrodynamics. Reidel, Dordrecht, 1982.
18. A.A. Aleksandrova, N.A. Hizhnyak. Rasseyanie na MGD share proizvolnogo radiusa, nahodyaschemsya v gidrodinamicheskoy srede // *Fizika plazmyi*. 1991, t. 17, v. 11, s. 1303-1309 (in Russian).
19. H. Alfven and C.-G. Falthammar. Cosmical electrodynamics / 2nd ed. *Oxford Univ. Press (Clarendon)*, London and New York, 1963.
20. L.N. Sretinskiy. *Teoriya nyutonovskogo potentsiala*. M.-L.: "OGIZ", 1949, s. 317 (in Russian).
21. V. Bazhenov and L. Igumnov. Metodyi granichnyih integralnyih uravneniy i granichnyih elementov. *Litres*, 2018 (in Russian).

*Article received 18.05.2021*

## ИССЛЕДОВАНИЕ КРАЕВОЙ ЗАДАЧИ РЕЗОНАНСНОЙ МГД-НЕОДНОРОДНОСТИ С ПОМОЩЬЮ ИНТЕГРАЛЬНЫХ УРАВНЕНИЙ

***Ю.Н. Александров, И.Ш. Невлюдов, Е.А. Чалая, И.Б. Боцман, В.В. Невлюдова***

Проведено численное моделирование скорости внутреннего поля для МГД-неоднородности на основе строго аналитического решения краевой задачи магнитогидродинамики. В основе аналитического решения лежит метод интегральных уравнений линейной магнитогидродинамики. Проведен анализ полученных результатов.

## ДОСЛІДЖЕННЯ КРАЙОВОЇ ЗАДАЧІ РЕЗОНАНСНОЇ МГД-НЕОДНОРІДНОСТІ З ВИКОРИСТАННЯМ ІНТЕГРАЛЬНИХ РІВНЯНЬ

***Ю.М. Олександров, І.Ш. Невлюдов, О.О. Чала, І.Б. Боцман, В.В. Невлюдова***

Проведено чисельне моделювання швидкості внутрішнього поля для МГД-неоднорідності на основі точно аналітичного рішення крайової задачі магнітогідродинаміки. В основі аналітичного рішення лежить метод інтегральних рівнянь лінійної магнітогідродинаміки. Проведено аналіз отриманих результатів.