METHOD OF MAGNETIC SEPARATION IN FLIGHT

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(Received 23 July, 2018)

Mathematical models: the source of magnetic field, the lump of ore (paramagnetic body) have been created. The ore dynamics in magnetic field is considered taking into account gravity and aerodynamic drag force. The results of modeling indicate the possibility of magnetic separation of ore in flight. The parameters for the plant optimization are determined.

PACS: 45.40.Gj

1. INTRODUCTION

A lot of works has been devoted to the study of magnetic separation of minerals based on the difference in their magnetic characteristics. Basic approaches are quite fully represented in the works of [1, 2].

Our task was as far as feasible to offer a new original method of separation; therefore the work has a research nature, i.e. the task is to substantiate the principles, to create and study new mathematical models that are ideal by necessity.

As it is well known from the classical courses of macroscopic electrodynamics a rigid body placed in an inhomogeneous magnetic field is under the influence of a force that depends on magnetic permeability of the body and on the degree of a field inhomogeneity. For paramagnets the volume density of force is directed towards aside on increase of the field induction, while in diamagnets the volume density of force is directed toward aside on decrease in the induction of the field.

Thus, the behavior of magnetic and nonmagnetic bodies in a magnetic field is different. In particular, magnetic and nonmagnetic bodies that start moving with the same speed from the same field region will get different accelerations and will move along the different trajectories. This is the idea of our method.

The physical processes occurring in ferromagnetic materials under the influence of a magnetic field are very complex. So, it is both impossible and not feasible to create theoretical models that take into consideration in detail all properties of these substances and bodies.

Therefore simple idealized models have been proposed to advocate operating capability and to evaluate efficiency of the method. Nevertheless these models reflect essential properties of the behavior of bodies in a magnetic field.

The motion equations of a magnetized rigid body of arbitrary shape in an external magnetic field were obtained in [3] — the orientation of the body is given by the rotation matrix and in [4] the unit quaternions were used.

The lump of ore (i.e. rigid body sample) can be of arbitrary form but for our estimates it will be easier to consider the symmetric top model.

The motion equations of a magnetized symmetrical top in an external magnetic field were given in the papers [5, 6].

2. ONE-DIMENSIONAL MODEL OF MOTION OF A MAGNET

We consider the one-dimensional motion of a magnet from the region where the magnetic field has a maximum to a region with zero magnetic field. If we neglect the kinetic energy of magnet rotation the balance of kinetic and potential energy has the form

$$\frac{mv_0^2}{2} - \vec{\mu} \cdot \vec{B}_0 = \frac{mv_\infty^2}{2} - \vec{\mu} \cdot \vec{B}_\infty = \frac{mv_\infty^2}{2}. \quad (1)$$

Hence we have

$$v_\infty = \sqrt{v_0^2 - \frac{2\mu}{m} B_0}. \quad (2)$$

Deceleration between the start and end time

$$\triangle v = v_\infty - v_0 = \sqrt{v_0^2 - \frac{2\mu}{m} B_0} - v_0. \quad (3)$$

We can also write that

$$\triangle v = v_0 \left(\sqrt{1 - \frac{2\mu}{mv_0^2} B_0} - 1 \right) \approx -\frac{\mu B_0}{mv_0}. \quad (4)$$

or

$$\frac{\triangle v}{v_0} \approx -\frac{\mu B_0}{mv_0^2}. \quad (5)$$

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3. MOTION EQUATIONS IN A MAGNETIC FIELD

If there are no other fields, the Hamiltonian of two magnetically interacting tops can be written as the sum of kinetic and potential energies of magnetic dipole

$$T = \frac{\vec{p}^{\,2}}{2m} + \frac{\alpha \vec{m}^2}{2};$$
$$U = -\mu(\vec{\nu}, \vec{B}(\vec{x}));$$

$$H = \frac{\vec{p}^{\,2}}{2m} + \frac{\alpha \vec{m}^2}{2} - \mu(\vec{\nu}, \vec{B}(\vec{x})). \tag{6}$$

The system of equations under study has the form

$$\begin{cases} \dot{\vec{x}} = \frac{1}{m}\vec{p}; \\ \dot{\vec{p}} = \mu \nu_s B_{s,r} \vec{e}_r; \\ \dot{\vec{\nu}} = \alpha(\vec{m} \times \vec{\nu}); \\ \dot{\vec{m}} = \mu(\vec{\nu} \times \vec{B}). \end{cases} \tag{7}$$

Our system of equations must be supplemented by relations of the form

$$\begin{cases} \vec{\nu}^{\,2} = 1; \\ (\vec{\nu}, \vec{m}) = M_3 = const. \end{cases} \tag{8}$$

From essence of the problem the translational velocity of the lump is more suitable than the momentum. Moreover, when using the speed value, it is possible to exclude the mass as a parameter from our calculations. Therefore, we make the following changes of variables in the system of equations (7).

$$\vec{v} = \frac{1}{m}\vec{p}; \quad \vec{n} = \frac{1}{m}\vec{m} \tag{9}$$

and the corresponding parameter substitutions

$$\bar{\alpha} = m\alpha = \frac{1}{I/m}; \quad \bar{\mu} = \frac{\mu}{m}. \tag{10}$$

Then equations of a motion take the form

$$\begin{cases} \dot{\vec{x}} = \vec{v}; \\ \dot{\vec{v}} = \bar{\mu} \nu_s B_{s,r} \vec{e}_r - g\vec{e}_3; \\ \dot{\vec{\nu}} = \bar{\alpha}(\vec{n} \times \vec{\nu}); \\ \dot{\vec{n}} = \bar{\mu}(\vec{\nu} \times \vec{B}). \end{cases} \tag{11}$$

The deceleration (5) between the initial and final moment of time that was previously calculated can now be written in the form

$$\frac{\triangle v}{v_0} \approx -\frac{\bar{\mu} B_0}{v_0^2}. \tag{12}$$

4. MAGNETIC FIELD SOURCE MODEL — BRICK

We need to have a realistic model of a power (electro)magnet.

It is suggested that acceptable model is a plate z, magnetized along its axis. The wide side of the plate is perpendicular to the direction of motion of the lump, and the narrow side is parallel to the direction of lump motion.

We cannot neglect the influence of lower edge because a height of the plate is essentially large.

Then we can combine two plates to create a quasi-homogeneous field in the region of injection the lump.

The derivation of formulas for such a magnet is a separate task. The main results of solving this task are as follows. Let introduce the quantities ψ_x, ψ_y, ψ_z

$$\psi_x(x, y, z; a, b, c) = -\ln\left(b - y + [(x-a)^2 + (y-b)^2 + (z-c)^2]^{\frac{1}{2}}\right); \tag{13}$$

$$\psi_y(x, y, z; a, b, c) = -\ln\left(a - x + [(x-a)^2 + (y-b)^2 + (z-c)^2]^{\frac{1}{2}}\right); \tag{14}$$

$$\psi_z(x, y, z; a, b, c) = -\arctan\frac{(x-a)(y-b)}{(z-c)[(x-a)^2 + (y-b)^2 + (z-c)^2]^{\frac{1}{2}}}. \tag{15}$$

The obtained quantities describe the field created by a layer of rectangular shape with sides u and w of magnetic poles located at an altitude of c.

To describe the field of the brick it is necessary to subtract the field of a similar layer located at an altitude of $c - l$

$$\begin{aligned} \beta_i(x, y, z; z_0, u, w, l) = {} & \psi_i(x, y, z; \frac{u}{2}, \frac{w}{2}, z_0) - \psi_i(x, y, z; -\frac{u}{2}, \frac{w}{2}, z_0) \\ & - \psi_i(x, y, z; \frac{u}{2}, -\frac{w}{2}, z_0) + \psi_i(x, y, z; -\frac{u}{2}, -\frac{w}{2}, z_0) \\ & - \psi_i(x, y, z; \frac{u}{2}, \frac{w}{2}, z_0 - l) + \psi_i(x, y, z; -\frac{u}{2}, \frac{w}{2}, z_0 - l) \\ & + \psi_i(x, y, z; \frac{u}{2}, -\frac{w}{2}, z_0 - l) - \psi_i(x, y, z; -\frac{u}{2}, -\frac{w}{2}, z_0 - l), \end{aligned} \tag{16}$$

where u, w, l are the geometric dimensions of the parallelepiped, z_0 is the coordinate of its upper bound, and the index i takes the value x, y, z.

Taking into account the relationship between the magnetic field and magnetic poles we can finally obtain the expression for the magnetic induction created by our magnet in form of brick

$$B_i(x, y, z; z_0, u, w, l) = -\frac{\mu_0 J}{4\pi}\beta_i(Ibid), \tag{17}$$

where J $[A/m]$ is the magnetization of the brick material.
Sign Minus appears because of $\vec{B} = -\nabla\psi$, where ψ is the scalar potential of the magnetic field.

To find the gradient of magnetic induction, we proceed similarly.

It makes sense for the compactness of the notation to introduce vector notation

$$\vec{x} = (x, y, z);$$
$$\vec{a} = (a, b, c);$$
$$|\vec{x} - \vec{a}| = [(a-x)^2 + (b-y)^2 + (z-c)^2]^{1/2}. \quad (18)$$

Then the necessary formulas will have the following form

$$\psi_{xx}(x, y, z; a, b, c) = -\frac{(\vec{x}-\vec{a})_1}{|\vec{x}-\vec{a}| \cdot (|\vec{x}-\vec{a}| - (\vec{x}-\vec{a})_2)}; \quad (19)$$

$$\psi_{xy}(x, y, z; a, b, c) = \frac{1}{|\vec{x}-\vec{a}|}; \quad (20)$$

$$\psi_{xz}(x, y, z; a, b, c) = -\frac{(\vec{x}-\vec{a})_3}{|\vec{x}-\vec{a}| \cdot (|\vec{x}-\vec{a}| - (\vec{x}-\vec{a})_2)}; \quad (21)$$

$$\psi_{yx}(x, y, z; a, b, c) = \frac{1}{|\vec{x}-\vec{a}|}; \quad (22)$$

$$\psi_{yy}(x, y, z; a, b, c) = -\frac{(\vec{x}-\vec{a})_2}{|\vec{x}-\vec{a}| \cdot (|\vec{x}-\vec{a}| - (\vec{x}-\vec{a})_1)}; \quad (23)$$

$$\psi_{yz}(x, y, z; a, b, c) = -\frac{(\vec{x}-\vec{a})_3}{|\vec{x}-\vec{a}| \cdot (|\vec{x}-\vec{a}| - (\vec{x}-\vec{a})_1)}; \quad (24)$$

$$\psi_{zx}(x, y, z; a, b, c) = -\frac{(\vec{x}-\vec{a})_2(\vec{x}-\vec{a})_3}{|\vec{x}-\vec{a}| \cdot (|\vec{x}-\vec{a}|^2 - (\vec{x}-\vec{a})_2^2)}; \quad (25)$$

$$\psi_{zy}(x, y, z; a, b, c) = -\frac{(\vec{x}-\vec{a})_1(\vec{x}-\vec{a})_3}{|\vec{x}-\vec{a}| \cdot (|\vec{x}-\vec{a}|^2 - (\vec{x}-\vec{a})_1^2)}; \quad (26)$$

$$\psi_{zz}(x, y, z; a, b, c) = \frac{(\vec{x}-\vec{a})_1(\vec{x}-\vec{a})_2 \cdot (|\vec{x}-\vec{a}|^2 + (\vec{x}-\vec{a})_3^2)}{|\vec{x}-\vec{a}| \cdot (|\vec{x}-\vec{a}|^2(\vec{x}-\vec{a})_3^2 + (\vec{x}-\vec{a})_1^2(\vec{x}-\vec{a})_2^2)}. \quad (27)$$

To obtain the final answer we have to act like in equation (17).

5. LUMP MODEL

Additional simplifications of the lump model are connected with a specific choice of body shape. We shall assume that the body has the shape of a solid sphere with the moment of inertia I_i of a solid sphere with mass m and radius R, where $I_1 = I_2 = I_3 = \frac{2}{5}mR^2$.

Usually in the courses on electromagnetism, first consider the auxiliary task of a dielectric solid sphere in a constant electric field, and then, by analogy, the task of a solid sphere from magnetic material in a constant magnetic field.

To compile a mathematical model, we need the expressions for the scalar potential φ_e – the electric dipole ([7, (16.84),p.128]; [8, (26),p.161]), the scalar potential φ_1 inside and φ_2 outside the dielectric solid sphere ([7, p.150-151]; [8, p.187-188]), the energy of the solid sphere for given external charges ([7, (18.30), (18.35), p.158-159]; [8, p.188])

$$\begin{cases} \varphi_e(\vec{r}) = \frac{1}{4\pi\varepsilon}\frac{\vec{p}_e\cdot\vec{r}}{r^3}; \\ \varphi_1(\vec{r}) = -\frac{3\varepsilon_2}{\varepsilon_1+2\varepsilon_2}E_0 r\cos\theta = -\frac{3\varepsilon_2}{\varepsilon_1+2\varepsilon_2}E_0 z; \\ \varphi_2(\vec{r}) = -E_0 z + \frac{R^3}{r^3}\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+2\varepsilon_2}E_0 z = -E_0 z + \frac{1}{4\pi\varepsilon_2}\frac{\vec{P}_e\cdot\vec{r}}{r^3}; \\ W_e = -\frac{1}{2}\int(\varepsilon_1-\varepsilon_2)\vec{E}_1\cdot\vec{E}_2 = -\frac{1}{2}(\vec{P}_e\cdot\vec{E}_0), \end{cases} \quad (28)$$

where $\vec{P}_e = V(\varepsilon_1-\varepsilon_2)\frac{3\varepsilon_2}{\varepsilon_1+2\varepsilon_2}\vec{E}_0$, $V = \frac{4}{3}\pi R^3$ is the volume of the solid sphere, θ is the angle between the vectors $\vec{E}_0$ and $\vec{r}$, $\vec{E}_0$ is the external electric field strength, $\vec{E}_1, \vec{E}_2$ is the electric field strength inside and outside the solid sphere, similarly $\varepsilon_1, \varepsilon_2$ – dielectric permeabilities inside and out solid sphere.

Thus, the ball field caused by the displacement of bound charges is a dipole field with the dipole moment $\vec{P}_e$. Accordingly, we have the scalar potential φ_m of the magnetic dipole ([8, (18), p.208]) in the form

$$\varphi_m(\vec{r}) = \frac{1}{4\pi}\frac{\vec{p}_m\cdot\vec{r}}{r^3}. \quad (29)$$

Note 1. There is no μ_0 in (29), since at given currents the strength of the magnetic field does not depend on the magnetic permeability of the homogeneous medium (see the note after [7, (38.27), p.271]), and φ_m is the potential precisely for the strength of the magnetic field.

Similarly, for a solid sphere of magnetic material in a magnetic field, we get

$$H_1 = \frac{3\mu_2}{\mu_1+2\mu_2}H_0, \quad (30)$$

where $\vec{H}_0$ — the strength of the external magnetic field, $\vec{H}_1$ — the strength of the field inside the solid sphere, similarly to μ_1, μ_2 — magnetic permeabilities inside and outside the solid sphere.

Then, by analogy with electrostatics, outside the solid sphere the scalar potential of the magnetic field strength will be

$$\varphi_{m2}(\vec{r}) = -H_0 z + \frac{R^3}{r^3}\frac{\mu_1 - \mu_2}{\mu_1 + 2\mu_2} H_0 z \qquad (31)$$

or

$$\varphi_{m2}(\vec{r}) = -H_0 z + \frac{1}{4\pi}\frac{\vec{P}_m \cdot \vec{r}}{r^3}, \qquad (32)$$

where

$$\vec{P}_m = V\frac{(\mu_1 - \mu_2)}{\mu_2}\frac{3\mu_2}{\mu_1 + 2\mu_2}\vec{H}_0. \qquad (33)$$

The energy of the solid sphere for given external currents is given by the following expression ([7, (47.42), (47.43), p.329])

$$W_m = \frac{1}{2}\int (\mu_1 - \mu_2)\vec{H}_1 \cdot \vec{H}_2 = \frac{1}{2}(\vec{P}_m \cdot \vec{B}_0). \qquad (34)$$

Substituting (33) in (34), we obtain

$$\begin{aligned} W_m = &\ \tfrac{1}{2}V\tfrac{(\mu_1-\mu_2)}{\mu_2}\tfrac{3\mu_2}{\mu_1+2\mu_2}\vec{H}_0 \cdot \vec{B}_0 = \\ = &\ \tfrac{1}{2}V\tfrac{3}{\mu_1+2\mu_2}\tfrac{\mu_1-\mu_2}{\mu_2}\vec{B}_0^2. \end{aligned} \qquad (35)$$

If $\mu_2 = \mu_0$ and $\mu_1 = \mu$ then

$$\begin{aligned} W_m = &\ \tfrac{1}{2}V\tfrac{3}{\mu+2\mu_0}\tfrac{\mu-\mu_0}{\mu_0}\vec{B}_0^2 = \\ = &\ \tfrac{1}{2}V\tfrac{3\chi}{3+\chi}\tfrac{1}{\mu_0}\vec{B}_0^2. \end{aligned} \qquad (36)$$

We find the force acting on the solid sphere of magnetic [7, (47.53), p.331]

$$\vec{F} = \nabla W_m. \qquad (37)$$

We assume that within the limits of the solid sphere the field varies slightly, so that for the magnetic moment of the solid sphere and its energy in an external field, one can use the formulas (29)-(37).

6. MOTION OF THE LUMP IN THE EXTERNAL FIELDS

Since the magnetic moment of the solid sphere is parallel to the external field (33), the magnetic field does not create a moment of force. The force of gravity also does not create a moment of force. Then we can not consider the rotational degrees of freedom.

Thus, the complete system of motion equations has the form

$$\begin{cases} \dot{\vec{x}} = \vec{v}; \\ \dot{\vec{v}} = \kappa B_s B_{s,r}\vec{e}_r - g\vec{e}_3, \end{cases} \qquad (38)$$

where

$$\kappa = \frac{3\chi}{3+\chi}\frac{1}{\varrho\mu_0}. \qquad (39)$$

For the system (38) we can introduce the energy integral. We introduce the energies: kinetic T, potential U and total E

$$\begin{aligned} T &= \frac{1}{2}v^2; \\ U &= -\frac{1}{2}\kappa B^2(\vec{x}) + gz; \\ E &= T + U. \end{aligned} \qquad (40)$$

Having neglected for a time the influence of the gravity field, we consider a one-dimensional motion along the x axis. In this case it is sufficient to use the energy conservation law $E = E_0$ from which

$$\begin{aligned} v &= v_0\left[1 + \kappa\frac{B^2 - B_0^2}{v_0^2}\right]^{1/2}; \\ \frac{v - v_0}{v_0} &= \left[1 + \kappa\frac{B^2 - B_0^2}{v_0^2}\right]^{1/2} - 1 \end{aligned} \qquad (41)$$

follows.

If the action of the force is relatively small (the increment of the kinetic energy is much less than the initial one), then

$$\frac{v - v_0}{v_0} = \frac{1}{2}\kappa\frac{B^2 - B_0^2}{v_0^2}. \qquad (42)$$

Note 2. In [7, p.270] we find the following statement "At room temperature, the paramagnetic susceptibility of substances in the solid state is of the order of $\sim 10^{-3}$, i.e. approximately two orders of magnitude greater than the diamagnetic susceptibility".

This means that in the case of weakly ferromagnetic materials the paramagnetic susceptibility is larger than the diamagnetic susceptibility. In addition, this means that the "demagnetizing" field is very small and can be neglected.

We carry out estimates for our case, i.e. substitute $\chi = 3.2 \cdot 10^{-3}$, $\varrho = 4 \cdot 10^3$ kg/m^3 in the formula (39) and then in (42) and have

$$\begin{cases} \kappa \simeq 8; \\ \frac{v-v_0}{v_0} \simeq -4\frac{B_0^2}{v_0^2}. \end{cases} \qquad (43)$$

7. THE WHITE-WOODSON APPROACH TO NONLINEAR MAGNETISM

As shown in the literature, the forces acting on a ferromagnet, are among the most poorly developed questions in macroscopic electrodynamics.

A general phenomenological approach was given in the D. C. Whit and H. H. Woodson monograph [9].

The authors introduce the concept of force functions. And, in particular, they consider energy and coenergy.

The stored energy is determined by the expression

$$W = \int_{0,\dots,0}^{\Psi_1,\dots,\Psi_n}\sum_{s=1}^{n} i_s^{'} d\Psi_s^{'}, \qquad (44)$$

and the coenergy by the expression

$$W^{'} = \int_{0,\dots,0}^{i_1,\dots,i_n}\sum_{s=1}^{n}\Psi_s^{'} di_s^{'}. \qquad (45)$$

The relationship between energy and coenergy is

$$W + W' = \sum_{s=1}^{n} i_s \Psi_s. \tag{46}$$

The expression of the mechanical forces acting on the body through the mentioned force functions is represented due to the choice of independent variables.

If the coordinates of the body x_r and currents i_s are chosen as independent variables, then the mechanical force is expressed in terms of magnetic W energy as follows

$$f_r = -\frac{\partial W}{\partial x_r} + \sum_{s=1}^{n} i_s \frac{\partial \Psi_s}{\partial x_r}. \tag{47}$$

And through the energy of W'

$$f_r = \frac{\partial W'}{\partial x_r}. \tag{48}$$

If the coordinates of the body x_r and the flux linkage Ψ_s are chosen as independent variables, then the mechanical force is expressed in terms of the magnetic W energy as follows

$$f_r = -\frac{\partial W}{\partial x_r}. \tag{49}$$

And through the energy of W'

$$f_r = \frac{\partial W'}{\partial x_r} - \sum_{s=1}^{n} \Psi_s \frac{\partial i_s}{\partial x_r}. \tag{50}$$

8. ACCOUNTING EFFECT OF SATURATION

In this section, we attempt to take into account the main effect of nonlinearity in the dependence of magnetization on the strength of magnetic field — this is so called saturation effect.

We shall consider the following dependence of the coenergy on the field strength as the analog of formula (45)

$$W' = \mu_0 \int_0^{H_0} \vec{P}_m(\vec{H}) d\vec{H}, \tag{51}$$

where $\vec{P}_m(\vec{H})$ is the dependence of the magnetic moment of the sample on the strength of the external magnetic field.

To take into account saturation, we can propose a rather crude model with a piecewise continuous dependence of the magnetic moment of the sample on the strength of the magnetic field.

That is, up to some field $H^{(S)}$ we will use the result of the 5th section, namely: calculate the coenergy of the sample as

$$W' = \mu_0 \int_0^{H_0} P_m(H) dH = \frac{1}{2}\mu_0 P_m(H_0) H_0. \tag{52}$$

For field strengths above $H^{(S)}$, we will assume that the magnetic moment of the sample is no longer depends on the applied field and is equal to $P_m(H^{(S)})$.

Thus

$$\begin{aligned} W' &= \mu_0 \int_0^{H_0} P_m(H) dH = \\ &= \frac{1}{2}\mu_0 P_m(H^{(S)}) H^{(S)} + \\ &+ \mu_0 P_m(H^{(S)})(H_0 - H^{(S)}). \end{aligned} \tag{53}$$

$$W' = \mu_0 P_m(H^{(S)}) H_0 - \frac{1}{2}\mu_0 P_m(H^{(S)}) H^{(S)}. \tag{54}$$

Finally, we can write this way

$$W'(H_0) = \begin{cases} \frac{1}{2}\mu_0 P_m(H_0) H_0, & H_0 \le H^{(S)} \\ \mu_0 P_m(H^{(S)}) H_0 - \\ -\frac{1}{2}\mu_0 P_m(H^{(S)}) H^{(S)}, & H_0 > H^{(S)}. \end{cases} \tag{55}$$

Then the equations of motion can be written using the potential energy U

$$U(x) = \begin{cases} -\frac{1}{2}\kappa B^2(x) + gz, & B(x) \le B^{(S)} \\ \frac{1}{2}\kappa (B^{(S)})^2 - \\ -\kappa B^{(S)} B(x) + gz, & B(x) > B^{(S)}, \end{cases} \tag{56}$$

namely

$$\begin{cases} \dot{\vec{x}} = \vec{v}; \\ \dot{\vec{v}} = \vec{F}, \end{cases} \tag{57}$$

where $\vec{F} = -\nabla U(x)$,

$$\vec{F} = \begin{cases} \kappa B_i B_{i,k} \vec{e}_k - g\vec{e}_3, & B(x) \le B^{(S)} \\ \kappa \frac{B^{(S)}}{B} B_i B_{i,k} \vec{e}_k - g\vec{e}_3, & B(x) > B^{(S)}. \end{cases} \tag{58}$$

The ratio (42), assuming the field to be vanishingly small at a distance from the magnet, now looks like this

$$\frac{v - v_0}{v_0} = \frac{1}{2}\kappa \frac{B^{(S)}(\frac{1}{2}B^{(S)} - B_0)}{v_0^2}. \tag{59}$$

Note 3. The strength of air resistance

$$\begin{aligned} \vec{F}_{air} &= -C_{air} \varrho_{air} S_{body} v^2 \frac{\vec{v}}{|\vec{v}|} = \\ &= -C_{air} \varrho_{air} S_{body} v \vec{v}. \end{aligned} \tag{60}$$

As is known, for a sphere the volume and area, respectively, are equal

$$\begin{cases} V_{body} = \frac{4}{3}\pi R^3; \\ S_{body} = \pi R^2. \end{cases} \tag{61}$$

Then the corresponding acceleration has the form

$$\vec{a}_{air} = \frac{\vec{F}_{air}}{m_{body}} = -\frac{3}{4} C_{air} \frac{\varrho_{air}}{\varrho_{body}} \frac{v\vec{v}}{R}. \tag{62}$$

9. CONCLUSIONS

Calculations based on the proposed theoretical models show the validity of "magnetic separation in flight". Trajectories of magnetized and nonmagnetic rigid bodies turn out to be spatially separated. Magnitude of separation on the one hand essentially depends on magnetic properties of the substance, and, on the other hand, depends on the magnitude of the field in the injection region. The magnitude of the separation is noticeable even for weakly ferromagnetic ores (hematites), but the conclusion about the technological applicability of the method for a particular production will essentially depend on other (not physical) factors (for example, economic, etc.). For strongly ferromagnetic ores, the effect is unambiguous — the separation can be very significant. Spatial separation also depends on other parameters such as initial velocity, throw angle, geometry of a magnet. For these parameters optimization is possible.

The used models are idealized and take into account only the most essential characteristics of interaction between magnetized bodies and source field. These models are open for giving weight to other factors that affect the movement of bodies (collision effects, etc.). Calculation of a trajectory is performed very fast due to efficient algorithm. It allows setting and solving problems of optimization by the parameters.

But nevertheless, direct experimental confirmation of the separation method efficiency is much more preferable. Clarification of theoretical models cannot be a final purpose by itself, and an experiment alone can give a practical answer to the question of the applying method efficiency to the certain ores.

The proposed theoretical models and calculation programs enable us to put forward the necessary requirements for separators construction that use the proposed method. They also provide a basis for optimal engineering decisions making.

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МЕТОД МАГНИТНОГО РАЗДЕЛЕНИЯ НА ЛЕТУ

С. И. Зуб *,* ***С. С. Зуб, С. И. Ляшко***

Созданы математические модели: источника магнитного поля и штуфа (парамагнитное тело). Рассмотрена динамика руды в магнитном поле с учетом силы тяжести и силы аэродинамического сопротивления. Результаты моделирования указывают на возможность магнитной сепарации руды в полёте. Определены параметры оптимизации установки.

МЕТОД МАГНІТНОГО РОЗДІЛЕННЯ НА ЛЬОТУ

С. І. Зуб *,* ***С. С. Зуб, С. І. Ляшко***

Створено математичні моделі: джерела магнітного поля та штуфа (парамагнітне тіло). Розглянуто динаміку руди в магнітному полі з урахуванням сили тяжіння та сили аеродинамічного опору. Результати моделювання вказують на можливість магнітної сепарації руди в польоті. Визначено параметри оптимізації установки.