PARTICLE SUBENSEMBLES IN RANDOM FIELD WITH FINITE CORRELATION TIME

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Particle transport in a random electric field across constant magnetic field is studied by numerical simulation and analytical approach. We consider the effects of finite Larmor radius and finite correlation time on evolution of a particle subensemble, i.e. a group of particles which are initially in vicinity of the chosen equipotential line. The account for difference in evolution of subensembles improves agreement with a direct numerical simulation.

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INTRODUCTION

It is commonly known that in non-equilibrium plasma an anomalous transport can considerably exceed collisional one because of particle interaction with intense fields generated by instabilities in such plasma. Here the effect of particle trapping on anomalous two-dimensional transport in constant magnetic and random electric fields is studied.

This effect is most clearly manifested for two-dimensional drift particle motion undergoing static random electric field. The trapped particles move along closed streamlines, and in static random field all particles are trapped. Motion of these particles is correlated; therefore Gaussian displacements approximation [1] isn't valid [2]. Our approach accounts for particle trapping [3], finite Larmor radius [4, 5], and can be applied in a wide range of correlation times [6].

The idea of particle subensembles [7] applied to our approach [8] for drift particle motion in static random field improves consistency with results of direct numerical simulations. However, Larmor gyration and temporal variation of random field cause deviation of a particle orbit from a streamline; consequently particles are no more trapped for an infinite time. The effect of partial particle trapping on subensemble evolution is a subject of our study.

1. MODEL

We consider particle motion across a constant magnetic field $\boldsymbol{B} = B\,\boldsymbol{e}_Z$ exactly $\boldsymbol{x} = \boldsymbol{x}_d + \boldsymbol{r}_L$, and in $\boldsymbol{E}\times\boldsymbol{B}$ drift approximation

$$\frac{d}{dt}\boldsymbol{r}_L = -\frac{e}{m\Omega_B}\left[\boldsymbol{E}(\boldsymbol{x}_d + \boldsymbol{r}_L, t)\times\boldsymbol{e}_B\right] + \Omega_B\left[\boldsymbol{r}_L\times\boldsymbol{e}_B\right],$$

$$\frac{d}{dt}\boldsymbol{x}_d = \frac{e}{m\Omega_B}\left[\boldsymbol{E}(\boldsymbol{x}_d + \boldsymbol{r}_L, t)\times\boldsymbol{e}_B\right],\ \Omega_B = \frac{eB}{mc}.$$

A random electric field $\boldsymbol{E}(\boldsymbol{x}, t) = -\partial/\partial\boldsymbol{x}\ \phi(\boldsymbol{x}, t)$ is given through a potential as superposition of harmonics with a common amplitude ϕ_0

$$\phi(\boldsymbol{x},t) \propto \sum\nolimits_{s=1}^{N}\phi_0 \exp\left(-\frac{1}{2}\frac{\boldsymbol{k}_s^2}{\Delta k^2}\right)\cos(\alpha_s(t) - \boldsymbol{k}_s\boldsymbol{x}).$$

Each realization is determined by the set of random phases $\{\alpha_S\}$. If $\{\alpha_S\}$ is constant then a random field is static, so the field correlation time and the Kubo number tend to infinity, $K\to\infty$. When $\{\alpha_S\}$ depends on time, particularly being reseeded during simulation with frequency ν and probability p, the correlation time $t_c = 1/(\nu/\ln(1-p))$, as well as the Kubo number, are finite $0 < K < \infty$.

Numerical calculation of particles trajectories is done by Runge-Kutta method of the 5-th order. Particle trajectories are used to calculate mean square displacement, diffusion coefficient and correlation function of drift velocity components along particle trajectories. The correlation function of drift velocity components in fixed points of reference frame is obtained using equation for random field. For drift motion, the Larmor radius is set to be $\boldsymbol{r}_L = 0$.

2. STATISTICAL APPROACH

Our analytical approach is based on the Taylor relation [9], that gives a diffusion coefficient $D_i(t)$ and a mean square displacement $\Delta_i(t)$

$$D_i(t) = \frac{1}{2}\frac{d}{dt}\Delta_i(t) = \int d\tilde{t}\, C^L_{v_i v_i}(\tilde{t}),$$

$$C^L_{v_i v_i}(\tilde{t}) = \langle \boldsymbol{v}_i(\boldsymbol{x}(t) + \boldsymbol{x}(t_0), t + t_0)\boldsymbol{v}_i(\boldsymbol{x}(t_0), t_0)\rangle,$$

through an unknonwn correlation function along particles trajectories, Lagrangian one. The Eulerian correlation function in fixed points of laboratory frame is known

$$C^E_{v_i v_i}(t) = \langle \boldsymbol{v}_i(\boldsymbol{x} + \boldsymbol{x}_0, t + t_0)\boldsymbol{v}_i(\boldsymbol{x}_0, t_0)\rangle.$$

The crucial problem is to find a Lagrangian correlation function from a given Eulerian one. There is no mathematically strict method in general case, so approximated approaches are used. This is the moment approximation [3] derived from microscopic equations without use of free parameters. The closure is based on the assumption that particle trajectories are characterized by a mean square displacement

$$C^L_{vv}(t) = C^L_{v_x v_x}(t) + C^L_{v_y v_y}(t),$$

$$X_i(t) = \Delta_i^{1/2}(t),\ i = x, y,$$

$$C^L_{vv}(t) = C^E_{vv}(\boldsymbol{X}(t)),$$

$$C^L_{vv}(t) = \int d\sigma_0 \frac{\exp\left(-\sigma_0^2/2C^E_{\sigma\sigma}(0,0)\right)}{\sqrt{2\pi C^E_{\sigma\sigma}(0,0)}} C^E_{vv}(\boldsymbol{X}(t;\sigma_0)).$$

The final equation for a mean square displacement in a subensemble by initial potential reads

$$\frac{\partial^2}{\partial t^2}\Delta = \frac{\sigma_0^2}{2}\exp\left(-\frac{\pi^2\Delta}{2}\right)\times$$
$$\times\left(\mathrm{I}_0\left(\frac{\pi^2\Delta}{2}\right)\left(1-\pi^2\Delta\right)+\mathrm{I}_1\left(\frac{\pi^2\Delta}{2}\right)\pi^2\Delta\right).$$

For a finite Larmor radius effects the gyroavereged potential is used [4], [5] and the Lagrangian correlation function is

$$C_{vv}^L(t)=\frac{1}{2\pi}\int d\boldsymbol{k}\exp\left(\imath\boldsymbol{kX}(t)\right)C_{vv}^E(\boldsymbol{k})J_0^2(kr_L).$$

For a finite correlation time the additional exponential decorrelation factor for the Lagrangian correlation function is introduced [6]

$$C_{vv}^L(t)=\exp(-t/t_c)C_{vv}^L(t;r_L).$$

3. RESULTS OF SIMULATION

The main feature of particle drift motion in static random field is an asymptotically zero diffusion coefficient, since all particles are trapped. In terms of the Lagrangian correlation function this implies appearance of an infinitely long negative tail as it is shown in Fig. 1. Despite all particles are trapped, their dynamics are not the same, it depends on random potential, which is constant along particle drift trajectories. Partial Lagrangian correlation functions for different initial values of potential are presented in Fig. 2. It reflects that a particle with a small absolute value of potential travels for larger distances. The difference between the basic moment approximation (MA) and the moment approximation with subensembles (MAS) is demonstrated in Fig. 1; (NS) means a numerical simulation.

Random potential is not constant along exact particle trajectory and this leads to a less correlated particle motion. Depending on initial velocity or Larmor radius $\boldsymbol{r}_L$, particles wander between equipotential lines with different rates, so while particles with small initial radius $\boldsymbol{r}_L \approx 0.1$ are trapped, the other ones with larger $\boldsymbol{r}_L \approx 1$ are not. Despite a finite Larmor radius cause a decorrelation, the account for subensembles dynamics improves the the consistency of analytical approach with direct numerical simulation. The diffusion coefficient calculated with subensembles (MAS) shows a better agreement with results of numerical simulation (NS) than the basic method (MA) (Fig. 3).

A temporal variation of random potential enhances a decorrelation effect as well. Even particles located in places of high potential are trapped by the field within a limited time interval.

The Lagrangian correlation function for different correlation times is presented in Fig. 4. For a small correlation times $t_c < 1$ there is no particle trapping, but for $t_c \geq 1$ it is noticeable, and account for subensembles becomes important.

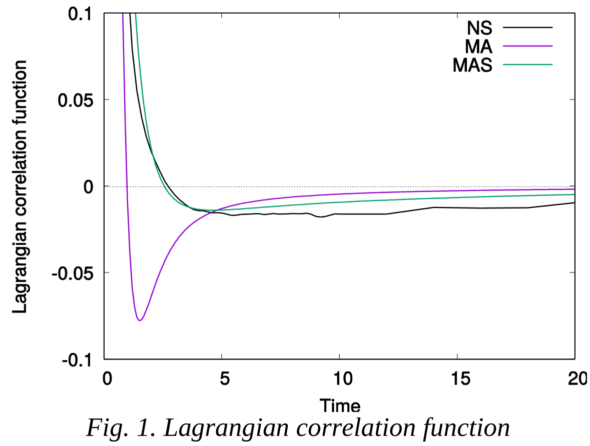

Fig. 1. Lagrangian correlation function

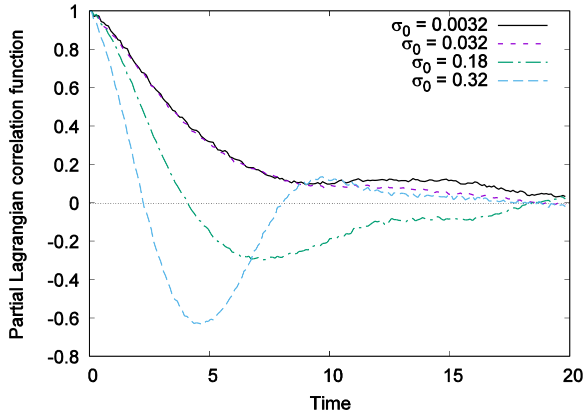

Fig. 2. Partial Lagrangian correlation function

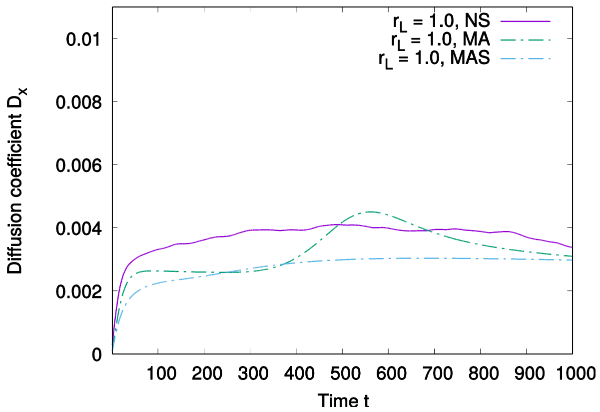

Fig. 3. Evolution of diffusion coefficient

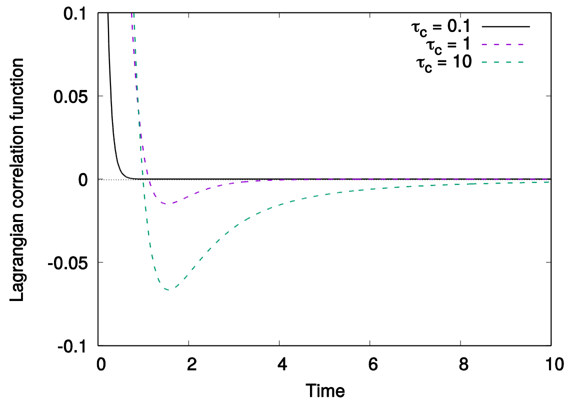

Fig. 4. Lagrangian correlation function

CONCLUSIONS

Transport of particles across constant magnetic field undergoing random electric fields in a wide range of correlation times and initial Larmor radius was considered. Despite the decorelection of particle motion through temporal variation of a random field and Larmor gyration, the trapping effect could be significant. An account for a specific particle dynamics in various subensembles improves consistency with a direct numerical simulation.

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ПОДАНСАМБЛИ ЧАСТИЦ В СЛУЧАЙНОМ ПОЛЕ С КОНЕЧНЫМ ВРЕМЕНЕМ КОРРЕЛЯЦИИ

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Перенос частиц в случайном электрическом поле поперек постоянного магнитного поля рассматривается в численном моделировании с аналитическим приближением. Исследуется влияние конечного радиуса Лармора и конечного времени корреляции на эволюцию подансамблей частиц, то есть группы частиц, которые находятся в окрестности выбранной эквипотенциальной линии в начальный момент. Учет различий в эволюции подансамблей улучшает согласие с прямым численным моделированием.

ПІДАНСАМБЛІ ЧАСТИНОК У ВИПАДКОВОМУ ПОЛІ ЗІ СКІНЧЕННИМ ЧАСОМ КОРЕЛЯЦІЇ

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Перенесення частинок у випадковому електричному полі поперек постійного магнітного поля досліджується в числовому моделюванні з аналітичним наближенням. Розглядається вплив кінцевого радіуса Лармора та кінцевого часу кореляції на еволюцію піддансамблів частинок, тобто групи частинок, що перебувають поблизу вибраної еквіпотенціальної лінії в початковий момент. Врахування відмінностей в еволюції піддансамблів покращує узгодженість із прямим числовим моделюванням.