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Modules with minimax Cousin cohomologies*

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Abstract. Let R be a commutative Noetherian ring with non-zero identity and let X be an arbitrary R-module. In this paper, we show that if all the cohomology modules of the Cousin complex for X are minimax, then the following hold for any prime ideal $\mathfrak{p}$ of R and for every integer n less than X—the height of $\mathfrak{p}$:

(i) the nth Bass number of X with respect to $\mathfrak{p}$ is finite;

(ii) the nth local cohomology module of $X_{\mathfrak{p}}$ with respect to $\mathfrak{p}R_{\mathfrak{p}}$ is Artinian.

Introduction

Throughout R will denote a commutative Noetherian ring with non-zero identity, X an arbitrary R-module which is not necessarily finite (i.e., finitely generated), and M a non-zero finite R-module. For basic results, notations and terminology not given in this paper, the reader is referred to [2], [3], and [12].

The notion of the Cousin complex for an R-module X was introduced by Sharp [13] as an analogue of Hartshorne [8]. The Cousin cohomologies (i.e., the cohomology modules of the Cousin complex) have been studied by several authors. Sharp used the vanishing of Cousin cohomologies for investigating the Cohen-Macaulay property, Serre's S_n-condition, and the vanishing of Bass numbers of X in [13], [14], and [15]. Dibaei, Tousi, Jafari, and Kawasaki, in [4], [5], [6], [7], and [10], worked on the finiteness of

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Cousin cohomologies and, in [11, Proposition 9.3.5], Lipman, Nayak, and Sastry generalized their results to complexes on formal schemes.

Sharp, in [14, Theorem 2.4], showed that M is Cohen-Macaulay if and only if the Cousin complex for M is exact. Thus we get the following theorem.

Theorem 1. *Let M be a non-zero finite R-module such that all the cohomology modules of the Cousin complex for M are zero. Then the followings hold for any prime ideal $\mathfrak{p}$ of R and for every integer n less than X—the height of $\mathfrak{p}$.*

(i) The nth Bass number of M with respect to $\mathfrak{p}$ is zero;

(ii) The nth local cohomology module of $M_{\mathfrak{p}}$ with respect to $\mathfrak{p}R_{\mathfrak{p}}$ is zero.

Now, it is natural to ask whether a similar statement is valid if 'zero' is replaced by 'finite'.

Question 1. *Let X be an arbitrary R-module such that all the cohomology modules of the Cousin complex for X are finite. Do the followings hold for any prime ideal $\mathfrak{p}$ of R and for every integer n less than X–height of $\mathfrak{p}$?*

(i) *The nth Bass number of X with respect to $\mathfrak{p}$ is finite;*

(ii) *The nth local cohomology module of $X_{\mathfrak{p}}$ with respect to $\mathfrak{p}R_{\mathfrak{p}}$ is finite.*

In this paper, we answer the above question. We show that the first part of Question 1 is true. In fact, in Theorem 2, we prove that the nth Bass number of X with respect to $\mathfrak{p}$ is finite for any prime ideal $\mathfrak{p}$ of R and for every integer n less than X–height of $\mathfrak{p}$, when all the cohomology modules of the Cousin complex for X are minimax. Even though the second part of Question 1 is false in general, we show in Theorem 3 that if all the cohomology modules of the Cousin complex for X are minimax, then the nth local cohomology module of $X_{\mathfrak{p}}$ with respect to $\mathfrak{p}R_{\mathfrak{p}}$ is Artinian for any prime ideal $\mathfrak{p}$ of R and for every integer n less than X–height of $\mathfrak{p}$.

1. Main results

Suppose that X is an arbitrary R-module. Recall that, for a prime ideal $\mathfrak{p}$ of $\mathrm{Supp}_R(X)$, the X–height of $\mathfrak{p}$ is defined to be $\mathrm{ht}_X(\mathfrak{p}) = \dim_{R_{\mathfrak{p}}}(X_{\mathfrak{p}})$. Let i be a non-negative integer and set $\mathrm{U}^i(X) = \{\mathfrak{p} \in \mathrm{Supp}_R(X) : \mathrm{ht}_X(\mathfrak{p}) \geqslant i\}$. Then $\mathrm{Supp}_R(X) = \mathrm{U}^0(X)$, $\mathrm{U}^i(X) \supseteq \mathrm{U}^{i+1}(X)$, and $\mathrm{U}^i(X) - \mathrm{U}^{i+1}(X)$ ($= \{\mathfrak{p} \in \mathrm{Supp}_R(X) : \mathrm{ht}_X(\mathfrak{p}) = i\}$) is low with respect to $\mathrm{U}^i(X)$ (i.e., each member of $\mathrm{U}^i(X) - \mathrm{U}^{i+1}(X)$ is a minimal member of $\mathrm{U}^i(X)$ with respect to inclusion). The Cousin complex $\mathrm{C}_R(X)$ for X is of the form

$$\mathrm{C}_R(X) : 0 \xrightarrow{d^{-2}} X \xrightarrow{d^{-1}} X^0 \xrightarrow{d^0} X^1 \xrightarrow{d^1} \cdots \xrightarrow{d^{i-2}} X^{i-1} \xrightarrow{d^{i-1}} X^i \xrightarrow{d^i} \cdots$$

where, for all $i \geqslant 0$,

- $X^i = \bigoplus_{\mathfrak{p}\in \mathrm{U}^i(X)-\mathrm{U}^{i+1}(X)}(\operatorname{Coker} d^{i-2})_{\mathfrak{p}}$ and
- $d^{i-1}(x) = \{\frac{x+\operatorname{Im} d^{i-2}}{1}\}_{\mathfrak{p}\in \mathrm{U}^i(X)-\mathrm{U}^{i+1}(X)}$ for every element x of X^{i-1};

and satisfies

- $\operatorname{Supp}_R(X^i) \subseteq \mathrm{U}^i(X)$,
- $\operatorname{Supp}_R(\operatorname{Coker} d^{i-2}) \subseteq \mathrm{U}^i(X)$, and
- $\operatorname{Supp}_R(\mathrm{H}^{i-1}(\mathrm{C}_R(X))) \subseteq \mathrm{U}^{i+1}(X)$

(see [13] for details). Here, we use the notations $\mathrm{C}^{i-2} := \operatorname{Coker} d^{i-2}$ and $\mathrm{H}^{i-1} := \mathrm{H}^{i-1}(\mathrm{C}_R(X))$ for all $i \geqslant 0$.

Recall that an R-module X is said to be minimax, if there is a finite submodule X' of X such that $\frac{X}{X'}$ is Artinian [3]. Thus the class of minimax modules includes all finite and all Artinian modules. Note that, for any short exact sequence

$$0 \longrightarrow X' \longrightarrow X \longrightarrow X'' \longrightarrow 0$$

of R-modules, X is minimax if and only if X' and X'' are both minimax [1, Lemma 2.1].

In the following, we state our first main result. Note that, for an R-module X and a prime ideal $\mathfrak{p}$ of R, the number

$$\mu^n(\mathfrak{p}, X) = \dim_{\frac{R_{\mathfrak{p}}}{\mathfrak{p}R_{\mathfrak{p}}}}(\operatorname{Ext}^n_{R_{\mathfrak{p}}}(\frac{R_{\mathfrak{p}}}{\mathfrak{p}R_{\mathfrak{p}}}, X_{\mathfrak{p}}))$$

is the nth Bass number of X with respect to $\mathfrak{p}$.

Theorem 2. *Let X be an arbitrary R-module such that H^i is minimax for all i. Then $\mu^n(\mathfrak{p}, X)$ is finite for all prime ideals $\mathfrak{p}$ of R and all $n < \operatorname{ht}_X(\mathfrak{p})$.*

Proof. Let $\mathfrak{p}$ be a prime ideal of R and let $n < \operatorname{ht}_X(\mathfrak{p})$. Let i be an integer such that $0 \leqslant i \leqslant n$. By considering the short exact sequences

$$0 \longrightarrow \frac{\mathrm{C}^{i-2}}{\mathrm{H}^{i-1}} \longrightarrow X^i \longrightarrow \mathrm{C}^{i-1} \longrightarrow 0 \tag{1}$$

and

$$0 \longrightarrow \mathrm{H}^{i-1} \longrightarrow \mathrm{C}^{i-2} \longrightarrow \frac{\mathrm{C}^{i-2}}{\mathrm{H}^{i-1}} \longrightarrow 0, \tag{2}$$

we have the long exact sequences

$$0 \longrightarrow \operatorname{Hom}_R(\frac{R}{\mathfrak{p}}, \frac{\mathrm{C}^{i-2}}{\mathrm{H}^{i-1}}) \longrightarrow \operatorname{Hom}_R(\frac{R}{\mathfrak{p}}, X^i) \longrightarrow \operatorname{Hom}_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{i-1})$$

$$\longrightarrow \mathrm{Ext}^1_R(\frac{R}{\mathfrak{p}}, \frac{\mathrm{C}^{i-2}}{\mathrm{H}^{i-1}}) \longrightarrow \mathrm{Ext}^1_R(\frac{R}{\mathfrak{p}}, X^i) \longrightarrow \mathrm{Ext}^1_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{i-1})$$
$$\longrightarrow \cdots$$
$$\longrightarrow \mathrm{Ext}^{n-i-1}_R(\frac{R}{\mathfrak{p}}, \frac{\mathrm{C}^{i-2}}{\mathrm{H}^{i-1}}) \longrightarrow \mathrm{Ext}^{n-i-1}_R(\frac{R}{\mathfrak{p}}, X^i) \longrightarrow \mathrm{Ext}^{n-i-1}_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{i-1})$$
$$\longrightarrow \mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}}, \frac{\mathrm{C}^{i-2}}{\mathrm{H}^{i-1}}) \longrightarrow \mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}}, X^i) \longrightarrow \mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{i-1})$$
$$\longrightarrow \cdots$$

and

$$0 \longrightarrow \mathrm{Hom}_R(\frac{R}{\mathfrak{p}}, \mathrm{H}^{i-1}) \longrightarrow \mathrm{Hom}_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{i-2}) \longrightarrow \mathrm{Hom}_R(\frac{R}{\mathfrak{p}}, \frac{\mathrm{C}^{i-2}}{\mathrm{H}^{i-1}})$$
$$\longrightarrow \mathrm{Ext}^1_R(\frac{R}{\mathfrak{p}}, \mathrm{H}^{i-1}) \longrightarrow \mathrm{Ext}^1_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{i-2}) \longrightarrow \mathrm{Ext}^1_R(\frac{R}{\mathfrak{p}}, \frac{\mathrm{C}^{i-2}}{\mathrm{H}^{i-1}})$$
$$\longrightarrow \cdots$$
$$\longrightarrow \mathrm{Ext}^{n-i-1}_R(\frac{R}{\mathfrak{p}}, \mathrm{H}^{i-1}) \longrightarrow \mathrm{Ext}^{n-i-1}_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{i-2}) \longrightarrow \mathrm{Ext}^{n-i-1}_R(\frac{R}{\mathfrak{p}}, \frac{\mathrm{C}^{i-2}}{\mathrm{H}^{i-1}})$$
$$\longrightarrow \mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}}, \mathrm{H}^{i-1}) \longrightarrow \mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{i-2}) \longrightarrow \mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}}, \frac{\mathrm{C}^{i-2}}{\mathrm{H}^{i-1}})$$
$$\longrightarrow \cdots.$$

Since H^i is minimax for all i, $\mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}}, \mathrm{H}^{i-1})$ is minimax for all $0 \leqslant i \leqslant n$. On the other hand, by [13, Lemma 4.5], $\mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}}, X^i) = 0$ for all $0 \leqslant i \leqslant n$. Thus, from the above long exact sequences, $\mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{i-2})$ is minimax whenever $\mathrm{Ext}^{n-i-1}_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{i-1})$ is minimax. Hence $\mathrm{Ext}^n_R(\frac{R}{\mathfrak{p}}, \mathrm{C}^{-2})$ is minimax. Therefore $\mathrm{Ext}^n_R(\frac{R}{\mathfrak{p}}, X)$ is minimax. Thus there is a finite submodule E' of $\mathrm{Ext}^n_R(\frac{R}{\mathfrak{p}}, X)$ such that $\frac{\mathrm{Ext}^n_R(\frac{R}{\mathfrak{p}}, X)}{E'}$ is Artinian. Since $\mathfrak{p}R_\mathfrak{p}(\frac{\mathrm{Ext}^n_{R_\mathfrak{p}}(\frac{R_\mathfrak{p}}{\mathfrak{p}R_\mathfrak{p}}, X_\mathfrak{p})}{E'_\mathfrak{p}}) = 0$, $\frac{\mathrm{Ext}^n_{R_\mathfrak{p}}(\frac{R_\mathfrak{p}}{\mathfrak{p}R_\mathfrak{p}}, X_\mathfrak{p})}{E'_\mathfrak{p}}$ is finite. Thus $\mathrm{Ext}^n_{R_\mathfrak{p}}(\frac{R_\mathfrak{p}}{\mathfrak{p}R_\mathfrak{p}}, X_\mathfrak{p})$ is finite. Hence $\mu^n(\mathfrak{p}, X)$ is finite as we desired. □

For an R-module X and an ideal $\mathfrak{a}$ of R, we write $\mathrm{H}^n_\mathfrak{a}(X)$ as the nth local cohomology module of X with respect to $\mathfrak{a}$. An important problem in commutative algebra is to determine when $\mathrm{H}^n_\mathfrak{a}(X)$ is Artinian. In the second main result of this paper, we show that for an arbitrary R-module X (not necessarily finite) with minimax Cousin cohomologies, $\mathrm{H}^n_{\mathfrak{p}R_\mathfrak{p}}(X_\mathfrak{p})$ is Artinian for all prime ideals $\mathfrak{p}$ of R and all $n < \mathrm{ht}_X(\mathfrak{p})$, which is related to the third of Huneke's four problems in local cohomology modules [9].

Theorem 3. *Let X be an arbitrary R-module such that H^i is minimax for all i. Then $\mathrm{H}^n_{\mathfrak{p}R_\mathfrak{p}}(X_\mathfrak{p})$ is Artinian for all prime ideals $\mathfrak{p}$ of R and all $n < \mathrm{ht}_X(\mathfrak{p})$.*

Proof. The proof is similar to that of Theorem 2. We bring it here for the sake of completeness. Let $\mathfrak{p}$ be a prime ideal of R and let $n < \mathrm{ht}_X(\mathfrak{p})$. Let i be an integer such that $0 \leqslant i \leqslant n$. By considering the short exact sequences (1) and (2), we have the long exact sequences

$$\begin{aligned}
0 &\longrightarrow \Gamma_{\mathfrak{p}R_\mathfrak{p}}(\frac{\mathrm{C}^{i-2}_\mathfrak{p}}{\mathrm{H}^{i-1}_\mathfrak{p}}) \longrightarrow \Gamma_{\mathfrak{p}R_\mathfrak{p}}(X^i_\mathfrak{p}) \longrightarrow \Gamma_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{i-1}_\mathfrak{p}) \\
&\longrightarrow \mathrm{H}^1_{\mathfrak{p}R_\mathfrak{p}}(\frac{\mathrm{C}^{i-2}_\mathfrak{p}}{\mathrm{H}^{i-1}_\mathfrak{p}}) \longrightarrow \mathrm{H}^1_{\mathfrak{p}R_\mathfrak{p}}(X^i_\mathfrak{p}) \longrightarrow \mathrm{H}^1_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{i-1}_\mathfrak{p}) \\
&\longrightarrow \cdots \\
&\longrightarrow \mathrm{H}^{n-i-1}_{\mathfrak{p}R_\mathfrak{p}}(\frac{\mathrm{C}^{i-2}_\mathfrak{p}}{\mathrm{H}^{i-1}_\mathfrak{p}}) \longrightarrow \mathrm{H}^{n-i-1}_{\mathfrak{p}R_\mathfrak{p}}(X^i_\mathfrak{p}) \longrightarrow \mathrm{H}^{n-i-1}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{i-1}_\mathfrak{p}) \\
&\longrightarrow \mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(\frac{\mathrm{C}^{i-2}_\mathfrak{p}}{\mathrm{H}^{i-1}_\mathfrak{p}}) \longrightarrow \mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(X^i_\mathfrak{p}) \longrightarrow \mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{i-1}_\mathfrak{p}) \\
&\longrightarrow \cdots
\end{aligned}$$

and

$$\begin{aligned}
0 &\longrightarrow \Gamma_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{H}^{i-1}_\mathfrak{p}) \longrightarrow \Gamma_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{i-2}_\mathfrak{p}) \longrightarrow \Gamma_{\mathfrak{p}R_\mathfrak{p}}(\frac{\mathrm{C}^{i-2}_\mathfrak{p}}{\mathrm{H}^{i-1}_\mathfrak{p}}) \\
&\longrightarrow \mathrm{H}^1_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{H}^{i-1}_\mathfrak{p}) \longrightarrow \mathrm{H}^1_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{i-2}_\mathfrak{p}) \longrightarrow \mathrm{H}^1_{\mathfrak{p}R_\mathfrak{p}}(\frac{\mathrm{C}^{i-2}_\mathfrak{p}}{\mathrm{H}^{i-1}_\mathfrak{p}}) \\
&\longrightarrow \cdots \\
&\longrightarrow \mathrm{H}^{n-i-1}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{H}^{i-1}_\mathfrak{p}) \longrightarrow \mathrm{H}^{n-i-1}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{i-2}_\mathfrak{p}) \longrightarrow \mathrm{H}^{n-i-1}_{\mathfrak{p}R_\mathfrak{p}}(\frac{\mathrm{C}^{i-2}_\mathfrak{p}}{\mathrm{H}^{i-1}_\mathfrak{p}}) \\
&\longrightarrow \mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{H}^{i-1}_\mathfrak{p}) \longrightarrow \mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{i-2}_\mathfrak{p}) \longrightarrow \mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(\frac{\mathrm{C}^{i-2}_\mathfrak{p}}{\mathrm{H}^{i-1}_\mathfrak{p}}) \\
&\longrightarrow \cdots .
\end{aligned}$$

Since H^i is minimax for all i, there is a finite submodule $\mathrm{H}^{i'}$ of H^i such that $\frac{\mathrm{H}^i}{\mathrm{H}^{i'}}$ is Artinian. Therefore, from the exact sequence

$$\mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{H}^{i-1'}_\mathfrak{p}) \longrightarrow \mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{H}^{i-1}_\mathfrak{p}) \longrightarrow \mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(\frac{\mathrm{H}^{i-1}_\mathfrak{p}}{\mathrm{H}^{i-1'}_\mathfrak{p}}),$$

$\mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{H}^{i-1}_\mathfrak{p})$ is Artinian for all $0 \leqslant i \leqslant n$. On the other hand, by [13, Lemma 4.5], for all $0 \leqslant i \leqslant n$ and all $j \geqslant 0$, $\mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}^j}, X^i) = 0$ and so $\mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(X^i_\mathfrak{p}) \cong (\mathrm{H}^{n-i}_\mathfrak{p}(X^i))_\mathfrak{p} = 0$ because

$$\mathrm{H}^{n-i}_\mathfrak{p}(X^i) \cong \varinjlim_{j\geqslant 0} \mathrm{Ext}^{n-i}_R(\frac{R}{\mathfrak{p}^j}, X^i).$$

Thus, from the above long exact sequences, $\mathrm{H}^{n-i}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{i-2}_\mathfrak{p})$ is Artinian whenever $\mathrm{H}^{n-i-1}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{i-1}_\mathfrak{p})$ is Artinian. Hence $\mathrm{H}^{n}_{\mathfrak{p}R_\mathfrak{p}}(\mathrm{C}^{-2}_\mathfrak{p})$ is Artinian. Therefore $\mathrm{H}^{n}_{\mathfrak{p}R_\mathfrak{p}}(X_\mathfrak{p})$ is Artinian. □

The following corollaries are immediate applications of the above theorems.

Corollary 1. *Let X be an arbitrary R-module such that H^i is finite for all i. Then*
(i) $\mu^n(\mathfrak{p}, X)$ is finite and
(ii) $\mathrm{H}^n_{\mathfrak{p}R_\mathfrak{p}}(X_\mathfrak{p})$ is Artinian
for all prime ideals $\mathfrak{p}$ of R and all $n < \mathrm{ht}_X(\mathfrak{p})$.

Corollary 2. *Let X be an arbitrary R-module such that H^i is Artinian for all i. Then*
(i) $\mu^n(\mathfrak{p}, X)$ is finite and
(ii) $\mathrm{H}^n_{\mathfrak{p}R_\mathfrak{p}}(X_\mathfrak{p})$ is Artinian
for all prime ideals $\mathfrak{p}$ of R and all $n < \mathrm{ht}_X(\mathfrak{p})$.

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