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Global outer connected domination number of a graph

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Abstract. For a given graph $G = (V, E)$, a dominating set $D \subseteq V(G)$ is said to be an outer connected dominating set if $D = V(G)$ or $G - D$ is connected. The outer connected domination number of a graph G, denoted by $\widetilde{\gamma}_c(G)$, is the cardinality of a minimum outer connected dominating set of G. A set $S \subseteq V(G)$ is said to be a global outer connected dominating set of a graph G if S is an outer connected dominating set of G and $\overline{G}$. The global outer connected domination number of a graph G, denoted by $\widetilde{\gamma}_{gc}(G)$, is the cardinality of a minimum global outer connected dominating set of G. In this paper we obtain some bounds for outer connected domination numbers and global outer connected domination numbers of graphs. In particular, we show that for connected graph $G \neq K_1$, $\max\{n - \frac{m+1}{2}, \frac{5n+2m-n^2-2}{4}\} \leqslant \widetilde{\gamma}_{gc}(G) \leqslant \min\{m(G), m(\overline{G})\}$. Finally, under the conditions, we show the equality of global outer connected domination numbers and outer connected domination numbers for family of trees.

Introduction

In this paper the number of vertices of graph G denoted by $n(G)$ and the number of edges of graph G denoted by $m(G)$. The neighborhood of

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vertex $u \in V(G)$ is denoted by $N_G(u) = \{v \in V(G) : uv \in E(G)\}$. The degree of the vertex $u \in V(G)$ denoted by $d_G(u)$ and $d_G(u) = |N_G(u)|$. $\delta(G) = \min_{u \in V(G)} d_G(u)$ and $\Delta(G) = \max_{u \in V(G)} d_G(u)$. $u \in V(G)$ is an end-vertex of G, if $d_G(u) = 1$ and the vertex that is not an end-vertex but is adjacent to an end-vertex is named a support vertex. For every $u \in V(G)$, delete all the end-vertices of $N(u)$ except one, the remaining graph is called the pruned of G and denoted by G_p.

The complement of a graph G, denoted by $\overline{G}$, is a graph with $|V(G)|$ vertices and two vertices in $\overline{G}$ are adjacent if and only if they are not adjacent in G. Every maximal connected subgraph of a graph G named a component of G and the number of components of graph G denoted by $c(G)$. For connected graphs we have $c(G) = 1$. For every connected graph G, a set $S \subseteq V(G)$ is a vertex-cut if $G - S$ is disconnected. If S is a vertex-cut of connected graph G and G_1 is a component of $G - S$, then the subgraph $\langle S \cup V(G_1) \rangle$ named a S-lob of graph G. A subdivision of an edge uv is obtained by inserting a new vertex w and replacing the edge uv with the edges uw and wv. A Spider is a tree obtained from a star by subdividing all of its edges. A wounded spider is a tree obtained from a spider by removing at least one end-vertex. For more notation and graph theory terminology not defined herein, we refer the reader to [10]. A set $S \subseteq V(G)$ is a dominating set of G if every vertex of $V(G) - S$ is adjacent to at least one vertex of S. The cardinality of a smallest dominating set of G, denoted by $\gamma(G)$, is called the domination number of G. A dominating set of cardinality $\gamma(G)$ is called a γ-set of G. A set $S \subseteq V(G)$ is a global dominating set of G if S is a dominating set of G and $\overline{G}$. The cardinality of a smallest global dominating set of G, denoted by $\gamma_g(G)$, is called the global domination number of G, for more see [1, 4, 5, 8, 9].

One of many applications of global domination have been given in [1], which relates to a communication network modeled by a graph G, where subnetworks are defined by some matching M_i of cardinality k. The necessity of these subnetworks could be due for reason of security, redundancy or limitation of recipients for different classes of messages. For this practical case, the global domination number represents the minimum number of master stations needed such that a message issued simultaneously from all masters reaches all desired recipients after traveling over only one communication link. We note that Carrington [2] gave two other applications of global dominating sets for graph partitioning commonly used in the implementation of parallel algorithms. With these applications we understand the value of global dominating set.

A set $S \subseteq V(G)$ is called an outer connected dominating set of G if S is a dominating set of G and $S = V(G)$ or $\langle V(G) - S\rangle$ is connected. The cardinality of a smallest outer connected dominating set of G, denoted by $\widetilde{\gamma}_c(G)$, is called the outer connected domination number of G. An outer connected dominating set of cardinality $\widetilde{\gamma}_c(G)$ is called a $\widetilde{\gamma}_c(G)$-set of G. For more details on outer domination see [3, 6]. We give a new definition as follows:

Definition 1. A set $S \subseteq V(G)$ is a *global outer connected dominating set* of G if S is an outer connected dominating set of G and $\overline{G}$. The cardinality of a smallest global outer connected dominating set of G, denoted by $\widetilde{\gamma}_{gc}(G)$, is called the *global outer connected domination number* of G. A global outer connected dominating set of cardinality $\widetilde{\gamma}_{gc}(G)$ is called a $\widetilde{\gamma}_{gc}$-set of G.

1. Bounds on (global) outer connected domination

In this section we find upper and lower bounds for global outer domination number of graphs. First, we state some results from [3].

Theorem 2. ([3], Theorem 5) *If G is a connected graph, then $\widetilde{\gamma}_c(G) \leqslant n(G) - \delta(G)$.*

The following observation has a straightforward proof.

Observation 3. The only connected graphs without any P_4 as a subgraph, are P_1, C_3 and $K_{1,t}$, $t \geqslant 1$.

Theorem 4. *Let G be a connected graph. If $\widetilde{\gamma}_c(G) = n(G) - 1$, then $G = K_{1,t}$, $t \geqslant 1$.*

Proof. For $G = K_{1,t}, t \geqslant 1$, we have $\widetilde{\gamma}_c(G) = n(G) - 1$. We show that for every graph $G \neq K_{1,t}$, $\widetilde{\gamma}_c(G) \neq n(G) - 1$. If G has a P_4 as a subgraph like $u_0u_1u_2u_3$, then the set $V(G) - \{u_1, u_2\}$ is an outer connected dominating set of G, so $\widetilde{\gamma}_c(G) \leqslant n(G) - 2$. For $G = P_1$ and $G = C_3$ we have $\widetilde{\gamma}_c(G) \neq n(G) - 1$. By Observation 3 the desired result holds. □

Theorem 5. *If $G \neq K_{1,t}, t \geqslant 1$ is a connected graph and S is a $\widetilde{\gamma}_c(G)$-set, then S contains all of the pendant vertices.*

Proof. Let u be an end-vertex of G and let v be its support vertex. If $u \notin S$, then $v \in S$ and because of connectivity of $V - S$ we have $S = V(G) - \{u\}$, so $\widetilde{\gamma}_c(G) = n(G) - 1$. By Theorem 4, $G = K_{1,t}$, that is contradiction. □

Theorem 6. *If G is a connected graph and S is a $\widetilde{\gamma}_c$-set of G and S_p is a $\widetilde{\gamma}_c$-set of G_p, then $|V(G) - S| = |V(G_p) - S_p|$.*

Proof. Let $P(G)$ and $P(G_p)$ be the set of end-vertices (pendant vertices) of G and G_p respectively. If $G = K_{1,t},\ t \geqslant 1$, then $G_p = P_2$, so $|V(G) - S| = |V(G_p) - S_p| = 1$. If $G \neq K_{1,t},\ t \geqslant 1$, then by Theorem 5 we have $P(G) \subseteq S$ and $P(G_p) \subseteq S_p$ and $\langle G - N_G[P(G)]\rangle = \langle G_p - N_{G_p}[P(G_p)]\rangle$, hence $|V(G) - S| = |V(G_p) - S_p|$. $\square$

Theorem 7. *If S is an outer connected dominating set of G and $A \subseteq S$ is a vertex-cut of G and $c(G - A) = t$, then S contains all the vertices of $t - 1$ numbers of S-lobs.*

Proof. Let B_1, B_2 be two S-lobs of G, $u \in V(B_1)$, $v \in V(B_2)$ and $u, v \in V(G) - S$. Then there does not exist any path between u and v in $V(G) - S$. $\square$

Since global outer connected dominating set is an outer connected dominating set, we have the following.

Observation 8. For any graph G, $\widetilde{\gamma}_{gc}(G) \geqslant \widetilde{\gamma}_c(G)$ and $\widetilde{\gamma}_{gc}(G) \geqslant \frac{n(G)}{\Delta(G)+1}$

Observation 9. Let $G \neq K_1$ be a graph. If $G, \overline{G}$ are connected, then $\widetilde{\gamma}_{gc}(G) \leqslant \min\{m(G), m(\overline{G})\}$.

Proof. For every vertex $u \in V(G)$, the set $V(G) - \{u\}$ is an outer connected dominating set of G. Connectivity of G implies that $|V(G) - \{u\}| = n(G) - 1 \leqslant m(G)$. Using this reasons for the complement graph $\overline{G}$, the desired result holds. $\square$

Theorem 10. *Let G be a graph with n vertices and m edges. Then* $\widetilde{\gamma}_{gc}(G) \geqslant \max\{n - \frac{m+1}{2}, \frac{5n+2m-n^2-2}{4}\}$

Proof. Let S be a $\widetilde{\gamma}_{gc}(G)$-set. Since S is a dominating set of G and $\langle V(G) - S\rangle$ is connected we count the minimum edges. Since $m \geqslant (n - |S| - 1) + (n - |S|) = 2n - 2|S| - 1$, hence we have $|S| \geqslant n - \frac{m+1}{2}$. Furthermore S is a dominating set of $\overline{G}$, and $\langle \overline{V(G) - S}\rangle$ is connected, so we have $|S| \geqslant n - \frac{m(\overline{G})+1}{2} = n - \frac{(\frac{n(n-1)}{2} - m)+1}{2}$, therefore $|S| \geqslant \frac{5n+2m-n^2-2}{4}$. $\square$

Theorem 11. *For every graph G, $\widetilde{\gamma}_{gc}(G) \notin \{n(G) - 2,\ n(G) - 3\}$*

Proof. Let S be a $\widetilde{\gamma}_{gc}(G)$-set. If $|S| = n-2$, then $V(G) - S = \{x, y\}$, where x and y are adjacent in G, so they are not adjacent in $\overline{G}$, that is a contradiction. If $|S| = n-3$, then $V(G) - S = \{x, y, z\}$, where $\langle V(G) - S\rangle$ is P_3 or C_3 in G. In both of cases $\overline{\langle V(G) - S\rangle}$ is disconnected in $\overline{G}$, that is also a contradiction. □

Theorem 12. *For every graph G, $\widetilde{\gamma}_{gc}(G) = n(G) - 4$ if and only if $\langle V(G) - S\rangle = P_4$.*

Proof. Let S be a $\widetilde{\gamma}_{gc}(G)$-set with cardinality $|S| = n-4$. Let $V(G) - S = \{x, y, z, t\}$. If $\langle V(G) - S\rangle$ has less than tree edges then $\langle V(G) - S\rangle$ is disconnected. If $\langle V(G) - S\rangle$ has more than tree edges then $\overline{\langle V(G) - S\rangle}$ is disconnected. Therefore $\langle V(G) - S\rangle$ has tree edges, and $\langle V(G) - S\rangle$ is P_4 or $K_{1,3}$ or $K_3 + P_1$. If $\langle V(G) - S\rangle = K_{1,3}$ or $K_3 + P_1$, then $\langle V(G) - S\rangle$ or $\overline{\langle V(G) - S\rangle}$ is disconnected, so $\langle V(G) - S\rangle = P_4$. □

Theorem 13. *Let G be a graph with at most 5 vertices. If $G \neq K_n$ and $G \neq \overline{K_n}$, then $\widetilde{\gamma}_{gc}(G) = n(G) - 1$.*

Proof. Let S be a $\widetilde{\gamma}_{gc}(G)$-set. Every global dominating set of G has at least two vertices, so $|V(G) - S| \leqslant 3$. By Theorem 11 we have $\widetilde{\gamma}_{gc}(G) = n-1$. □

Theorem 14. *Let $G \neq K_6$ and $G \neq \overline{K_6}$. If $n(G) = 6$ and $m(G) \neq 7, 8$, then $\widetilde{\gamma}_{gc}(G) = 5$.*

Proof. Let S be a $\widetilde{\gamma}_{gc}(G)$-set. Since S is a global dominating set of G we have $|S| \geqslant 2$, so $|V(G) - S| \leqslant 4$, by Theorem 11, we have $|V(G) - S| = 1$ or $|V(G) - S| = 4$. Let $|V(G) - S| = 4$ and $S = \{x, y\}$. Then by Theorem 12, $\langle V(G) - S\rangle = P_4$ and every vertex of $V(G) - S$ is adjacent to exact one of the vertices x or y. Hence the number of edges of G equals to 8 if x and y are adjacent, and equals to 7 if not adjacent, therefore $\widetilde{\gamma}_{gc}(G) = 5$. □

2. Global outer connected domination number of trees

In this section we study the global outer connected dominating set of Trees. We start by a theorem from [3].

Theorem 15. ([3], Theorem 6) *If T is a tree and $n(T) \geqslant 3$, then $\widetilde{\gamma}_c(T) \geqslant \Delta(T)$. Furthermore $\widetilde{\gamma}_c(T) = \Delta(T)$ if and only if T is a wounded spider.*

Theorem 16. *If G is a tree, then $\widetilde{\gamma}_c(G_p) \leqslant n(G_p) - \Delta(G_p)$.*

Proof. The inequality holds for graphs without vertices. Let G has at least on edge. Let $d_{G_p}(u) = \Delta(G_p)$. If u is a support vertex and $v \in N_{G_p}(u)$ is an end-vertex of G_p, then consider $S = V(G_p) - N_{G_p}[u] \cup \{v\}$. If u is not a support vertex, then consider $S = V(G_p) - N_{G_p}[u] \cup \{w\}$, that w is an arbitrary vertex of $N_{G_p}(u)$. The set S is an outer connected dominating set of G_p with cardinality $n(G_p) - \Delta(G_p)$. □

Theorem 17. ([3], Theorem 7) *If T is a tree of order $n(T) \geqslant 3$, then $\widetilde{\gamma}_c(T) \geqslant \lceil \frac{n}{2} \rceil$.*

Theorem 18. *Let T be a tree, S be a $\widetilde{\gamma}_c(T)$-set and $|S| \leqslant n(T)-2$. Then S is a dominating set of $\overline{T}$.*

Proof. Since $|S| \leqslant n(T) - 2$, so $V(T) - S$ has at least two vertices. Let u be a vertex of $V(T) - S$ which be adjacent to all vertices of S. Every vertex $v \in V(T) - S - \{u\}$ is adjacent to at least one vertex of S like w. Since $\langle V(G) - S \rangle$ is connected there exists a path between vertices u and v in $\langle V(T) - S \rangle$ like P. The path P together with the edges uw and vw forms a cycle in T, that is a contradiction. □

It is well known that if $\mathrm{diam}(G) \geqslant 3$, then $\mathrm{diam}(\overline{G}) \leqslant 3$, and therefore $\overline{G}$ is connected. Now we have the following.

Theorem 19. *Let T be a tree and S be an outer connected dominating set of T. If $\mathrm{diam}(\langle V(T) - S \rangle) \geqslant 3$, then S is a global outer connected dominating set of T.*

Proof. By Theorem 18, the result holds. □

Lemma 20. *Let T be a tree. Then $\overline{T}$ is disconnected if and only if $T = K_{1,t}$, $t \geqslant 1$.*

Proof. It is clear that $\overline{K}_{1,t}$ is disconnected. If $T = P_1$ then $\overline{T}$ is connected. If $T \neq P_1, K_{1,t}$, then $\mathrm{diam}(T) \geqslant 3$, so $\overline{T}$ is connected. □

3. Trees T with $\widetilde{\gamma}_{gc}(T) = \widetilde{\gamma}_c(T)$

In this section we verify the conditions that for a family trees tree T, $\widetilde{\gamma}_{gc}(T) = \widetilde{\gamma}_c(T)$.

As an immediate result from Lemma 20 we have.

Corollary 21. *Let T be a tree and S be a $\widetilde{\gamma}_c(T)$-set. If $\langle V(T) - S \rangle \neq K_1,\ K_{1,t}$, then $\widetilde{\gamma}_{gc}(T) = \widetilde{\gamma}_c(T)$.*

Theorem 22. *Let T be a tree. If $\widetilde{\gamma}_c(T) < n-\Delta(T)$, then $\widetilde{\gamma}_{gc}(T) = \widetilde{\gamma}_c(T)$.*

Proof. Let S be a $\widetilde{\gamma}_c(T)$-set and $|S| < n-\Delta(G)$. Then $|V(T)-S| > \Delta(T)$. If $\langle V(T)-S\rangle = K_{1,t}$, and $u \in V(T)-S$, $d(u) = t$, then since $N(u)\cap S \neq \varnothing$, we have $|V(T)-S| \leqslant d(u) \leqslant \Delta(G)$, that is contradiction, therefore $\langle V(T)-S\rangle$ is not K_1 or $K_{1,t}$. Now Corollary 21 implies that $\widetilde{\gamma}_{gc}(T) = \widetilde{\gamma}_c(T)$. □

Theorem 23. *Let T be a tree. If there exist two adjacent vertices $u, v \in V(T_p)$ such that $d_{T_p}(u) \geqslant 3$, $d_{T_p}(v) \geqslant 3$ and $d_{T_p}(u) + d_{T_p}(v) > \Delta(T_p) + 2$, then $\widetilde{\gamma}_{gc}(T) = \widetilde{\gamma}_c(T)$.*

Proof. If u is a support vertex, let $u_1 \in N_{T_p}(u)$ be the end-vertex in T_p and if u is not a support vertex, let u_1 be an arbitrary vertex of $N_{T_p}(u)-\{v\}$. Consider the vertex v_1 with the same definition about u_1. The set $S = V(T_p) - (N_{T_p}(u) \cup N_{T_p}(v)) \cup \{u_1, v_1\}$ is an outer connected dominating set of T_p such that $\operatorname{diam}(\langle V(T_p)-S\rangle) = 3$ and $|V(T_p)-S| \geqslant d_{T_p}(u) + d_{T_p}(v) - 2 > \Delta(T)$, therefore $|S| < n-\Delta(T)$. Now using Theorem 22 we have $\widetilde{\gamma}_{gc}(T) = \widetilde{\gamma}_c(T)$. □

Theorem 24. *Let T be a pruned tree. If T has a subtree as path $P_k = u_1u_2\ldots u_k$, $k \geqslant 4$ such that $d(u_1) \geqslant 2$, $d(u_k) \geqslant 2$, $d(u_i) \geqslant 3$, $i = 2, 3, \ldots, k-1$, and $\Sigma_{i=1}^{k} d(u_i) - 2(k-1) > \Delta(T)$, then $\widetilde{\gamma}_{gc}(T) = \widetilde{\gamma}_c(T)$.*

Proof. Let $U = \{u_1, u_2, \ldots, u_k\}$ and for $i \in \{1, 2, \ldots, k\}$ let T_{u_i} be the maximal subtree of T containing u_i and not containing any vertex in $U-\{u_i\}$. Let S_i be a subset of $V(T_{u_i})$ as follows. If u_i is a support vertex and t_i is the leaf adjacent to u_i, then $t_i \in S_i$, else if u_i is not a support vertex and t_i is an arbitrary vertex of $N(u_i)$ in T_{u_i}, then $t_i \in S_i$. Let S_i contains all the vertices of T_{u_i} such that they are at distance at least two from u_i. Let $S = \cup_{i=1}^{k} S_i$. The set S is an outer connected dominating set of T. Since $|V(T)-S| = \Sigma_{i=1}^{k} d(u_i) - 2(k-1)$, so $|S| < n(T)-\Delta(T)$, therefore $\widetilde{\gamma}_c(T) < n(T)-\Delta(T)$. By Theorem 22 we have $\widetilde{\gamma}_{gc}(T) = \widetilde{\gamma}_c(T)$. □

The converse of the Theorem 24 is not true. For counterexample consider Figure 1. The set $V(T)-\{a, b, c, d\}$ is a $\widetilde{\gamma}_{gc}$-set and the sets $V(T)-\{a, b, c, d\}$ and $V(T)-\{e, f, g, h\}$ are $\widetilde{\gamma}_c$-sets, so $\widetilde{\gamma}_{gc}(T) = \widetilde{\gamma}_c(T)$ but there is not any path with the properties mentioned in theorem.

We have the following result from [7].

Theorem 25. ([7] Theorem) *Let G be a graph with* $\operatorname{diam}(G) \geqslant 5$ *and* $\delta(G) \geqslant 3$, *then $\widetilde{\gamma}_{gc}(G) \leqslant n(G) - \operatorname{diam}(G) - 1$.*

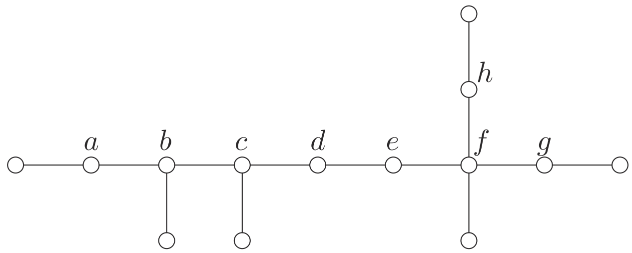

Figure 1.

Theorem 26. *Let T be a tree and $P(T)$ be the set of leaves of T. If* $\operatorname{diam}(T) \geqslant 5$ *and* $d(u) \geqslant 3$, *for every* $u \in V(T) - P(T)$, *then* $\widetilde{\gamma}_{gc}(T) \leqslant n(G) - \operatorname{diam}(T) + 1$.

Proof. Let $k = \operatorname{diam}(T)$ and $d(x, y) = k$ and $P = xu_0u_1 \dots u_{k-2}y$ be the longest path in T. It is clear that $d(x) = d(y) = 1$ and $d(u_i) \geqslant 3$, $i = 0, 1, \dots, k-2$. We show that $S = V(T) - \{u_0, u_1, \dots, u_{k-2}\}$ is a global outer connected dominating set of size $n(T) - \operatorname{diam}(T) + 1$. Since P is the shortest path between x and y, and $d(u_i) \geqslant 3$, $i = 0, 1, \dots, k-2$, so S is a dominating set of T. Since $\operatorname{diam}(T) \geqslant 5$ so $V(T) - S$ has at least 4 vertices, by Theorem 18, S is a dominating set of $\overline{T}$. Since $\langle V(T) - S\rangle \neq P_t$, $t \geqslant 4$, so $\langle V(T) - S\rangle$ and $\langle \overline{V(T) - S}\rangle$ are connected. $\square$

Conclusion

A graph G is said to be a unicyclic graph, if it is connected and has one and only one cycle. For example any cycle is a unicyclic graph. Or if we add an edge to a tree we obtain a unicyclic graph. We now make a future research works as follows.

Let $\mathfrak{F}$ be a family of unicyclic graphs. Does there exist some conditions such that, we have the equality of outer connected domination numbers and the global outer connected domination numbers for this family?

Let G be a 2-cyclic graph, that is G has exact two cycles. If we delete an edge from one of cycles, then the new graph will be changed to a unicyclic graph. Do we extend the result on unicyclic graphs to 2-cyclic graphs?

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