УДК 521.21/22 *A.N. Marchenko, N.P. Yarema*

ESTIMATION OF THE EARTH'S TENSOR OF INERTIA FROM RECENT GEODETIC AND ASTRONOMICAL DATA

The transformation of the second-degree harmonic coefficients $\bar{C}_{2m}$ and $\bar{S}_{2m}$ in the case of a finite commutative rotation was derived instead of the traditional Lambeck's approach based on an infinitesimal rotation. The modified Lambeck's formulae avoid uncertainty in the deviatoric part of inertia tensor and allow simple transformation of the 2nd-degree harmonic coefficients and zonal coefficients of an arbitrary degree (including their temporal changes) via orthogonal matrixes. These formulae together with exact solution of the eigenvalue-eigenvector problem are applied to determine static components and accuracy of the Earth's tensor of inertia from the adjustment in the principal axes system of $\bar{C}_{2m}$, $\bar{S}_{2m}$ from recent four gravity field models (EGM2008, GGM03S, ITG-GRACE03S, and EIGEN-GL04S1) and eight values H_D of the dynamical ellipticity all reduced to the common MHB2000 precession constant at the epoch J2000. The second solution contains the same parameters based on these four sets of $\bar{C}_{2m}, \bar{S}_{2m}$ and only one H_D from the MHB2000 model and corresponds better to the IERS Conventions 2003 and latest gravity field determinations. Two solutions for static components consist of the adjusted five 2nd-degree harmonic coefficients related to the IERS reference pole given by the conventional mean pole coordinates at the epoch 2000 (IERS Conventions 2003), the orientation of principal axes in this system, the principal moments (*A*, *B*, *C*) of inertia, and other associated parameters. The evolution with time of the above-mentioned static parameters was estimated in the principal axes system from the GRACE time series of $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ derived in five different centers of analysis over the time interval from 2002 to 2008. Special attention is given to the direct computation of temporally varying principal axes and moments of inertia based on $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ and the estimation of their mean values together with periodic constituents on given time-period. Stability of the positions of the equatorial inertia axes ($\bar{A}$, $\bar{B}$) and the angle between two quadrupole axes located in the plane of the axes $\bar{A}$ and $\bar{C}$ of inertia is found. The estimated longitude $\lambda_{\bar{A}}$ of the principal axis $\bar{A}$ as the parameter of the Earth's triaxiality in the precession-nutation theory and $\dot{J}_2$ precession rate $\dot{p}_A$ of the precession constant are recommended for the Earth's rotation theory. Additionally to some permanent constituents periodic components at seasonal and shorter time scale were evaluated.

Key words: the earth's inertia tensor; principal axes and moments of inertia; Lambeck's approach.

Introduction

Estimation of the Earth's fundamental parameters including elements of the tensor of inertia is the traditional area of interest of the IAG [Bursa, 1995; Groten, 2000; Groten 2004]. Suitable solutions for the Earth's principal moments of inertia (*A*, *B*, *C*), principal axes ($\bar{A}$, $\bar{B}$, $\bar{C}$), and other fundamental constants were obtained in [Marchenko, Schwintzer, 2003; Marchenko, 2007] from the adjustment (in the principal axes system) at one chosen epoch of several sets of the second degree harmonic coefficients $\bar{C}_{2m}$, $\bar{S}_{2m}$ of the Earth's gravity models all referred to different epochs with a spacing of 18 years in between and values of the dynamical ellipticity H_D. Derived from GRACE observations recent gravity field models give more accurate solutions for the time-dependent coefficients $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$. In addition, latest determinations of the dynamical ellipticity H_D are based on the non-rigid Earth's rotation theory including the MHB2000 precession-nutation model [Mathews et al., 2002] estimated from VLBI observations during the time-period of 20 years, adopted by the IAU, and recommended by the IERS Conventions 2003 [McCarthy and Petit, 2004]. After the launch of CHAMP and GRACE satellites the combination of new gravity field models, Earth's orientation series, and geophysical fluids data have led to a number of important contributions with the treatment of $H_D = H_D(t)$ and $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ as the sum of constant and variable (secular or/and periodic) parts caused by mass redistribution within the Earth's system [Marchenko and Schwintzer, 2001; Bourda and Capitaine, 2004; Chen et al., 2005; Fernández, 2007; Gross et al., 2007]. The consistency of such investigations and the modeling of the time evolution require additionally to the consistent set of fundamental constants more precise theories to determine the dynamic figure of the Earth, the orientation of the principal axes in the Earth's-fixed system and its evolution with time from geodetic $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ and astronomical $H_D(t)$ parameters.

This study aims to derive more accurate expressions for the transformation of the second-degree coefficients and zonal coefficients of an arbitrary degree through a finite commutative rotation instead of the most widely used approximate Lambeck's approach based on an infinitesimal rotation [Lambeck, 1971; Reigber, 1981]. The modified Lambeck's formulae for polar coordinates considered at the sphere avoid uncertainty in the deviatoric part of inertia tensor in comparison with the usual planar approximation and allow simple reduction of the 2nd-degree harmonic coefficients and zonal coefficients of an arbitrary degree together with their temporal changes to the figure axis $\overline{C}$. On the other hand, various solutions of the coefficients $\overline{C}_{2m}$, $\overline{S}_{2m}$ transformed in the ($\overline{A}$, $\overline{B}$, $\overline{C}$) system and H_D-estimates (expressed through (*A*, *B*, *C*)-values) represent initial information for the determination of the principal moments (*A*, *B*, *C*) via simultaneous adjustment by iterations providing in this way their agreement with different sets of geodetic and astronomical constants [Marchenko and Schwintzer, 2003]. The last approach is analyzed additionally to select initial values for iterations, which can be slightly differed from the mean moment of inertia of a homogeneous planet.

In contrast to the previous papers [Marchenko and Schwintzer, 2003; Marchenko, 2007] the fully normalized coefficients $\overline{C}_{2m}$, $\overline{S}_{2m}$ are selected from the recent four gravity field models EGM2008, GGM03S, ITG-GRACE03S, and EIGEN-GL04S1 constructed in different centers of analysis, based on different data sets, and referred to various epochs with a spacing of 5 years in between. The secular change in the 2nd-degree zonal coefficient $\dot{C}_{20} = 1.1628 \cdot 10^{-11}\,\mathrm{yr}^{-1}$ is adopted for these gravity fields together with the simple linear model for $\dot{C}_{21}$, $\dot{S}_{21}$ represented by the mean pole's drift with the reference mean pole coordinates $\overline{x}_p(t_0) = 0.054''$, $\overline{y}_p(t_0) = 0.357''$ at the epoch t_0=2000 according to the IERS Conventions 2003 [McCarthy and Petit, 2004]. It has to be pointed out that $\overline{C}_{2m}$, $\overline{S}_{2m}$ of the conventional solution EGM96 given at epoch 1986 (IERS Conventions 2003) were replaced by $\overline{C}_{2m}$, $\overline{S}_{2m}$ of the new gravity field model EGM2008 based on surface gravity data only [Pavlis et al., 2008] and referred to epoch J2000 with $\overline{C}_{21}$, $\overline{S}_{21}$ selected in agreement with this epoch [Pavlis, 2008]. The Earth's fundamental parameters were estimated from the weighted least squares adjustment of the new set of $\overline{C}_{2m}$, $\overline{S}_{2m}$ of four gravity field models and eight values H_D of the dynamical ellipticity [Williams, 1994; Souchay and Kinoshita, 1996; Hartmann et al., 1997; Bretagnon et al., 1998; Roosbeek and Dehant, 1998; Mathews et al., 2002; Fukushima, 2003; Capitaine et al., 2003] all reduced to the common value $p_A = 50.2879225''/\mathrm{yr}$ of the MHB2000 precession constant at epoch J2000.

Because the modified Lambeck's approach allows simple transformation of $\overline{C}_{2m}, \overline{S}_{2m}$ via orthogonal matrixes based on a finite commutative rotation the corresponding formulae were applied in the adjustment of the geodetic-only parameters $\overline{C}_{2m}, \overline{S}_{2m}$ of the four gravity field models to the IERS reference pole. Hence, the solution for static components consists of the adjusted $\overline{C}_{2m}, \overline{S}_{2m}$-coefficients related to the reference IERS pole at the epoch 2000, the orientation of principal axes in this system, the principal moments of inertia (*A*, *B*, *C*) of the Earth, H_D, the coefficients in the Eulerian dynamical equations, and other associated values. Another solution contains the same parameters based on these four sets of $\overline{C}_{2m}, \overline{S}_{2m}$ and only one H_D from the MHB2000 theory recommended by the IERS Conventions 2003. In this way the second solution for the time-independent principal moments of inertia and other associated parameters as a by-product of this adjustment at epoch corresponds better to the frequently used IERS Conventions 2003 and latest gravity field determinations instead of the old conventional model EGM96.

Secular changes of dynamical ellipticity $\dot{H}_D$ and precession constant were estimated via $\dot{C}_{20}$ temporal variation preliminary transformed via modified Lambeck's formulae to the figure axis $\overline{C}$. These estimates were compared with other results. Temporally varying components of the tensor of inertia were found from adjusted value of the dynamical ellipticity H_D, the secular variation $\dot{H}_D$, and the GRACE time series of $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$ derived in five different centers of analysis on the period from 2002 to 2008: 1) CNES-GRGS; 2) CSR Release 04; 3) GFZ Release 04; 4) JPL Release 04.1; 5) ITG-GRACE03S. Special attention is given not only to the direct computation of temporally varying principal axes and moments of inertia based on these time series of $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$ but to the estimation of their mean values and periodic components on given time-period from time-frequency analysis at seasonal and shorter time scale. As a result, additionally to some permanent constituents of discussed parameters as mean values at mean epoch their periodic stable changes were also detected.

Transformation of 2nd degree harmonic coefficients based on the Lambeck's approach

Simultaneous adjustment of appropriate sets of the harmonic coefficients ($\overline{C}_{2m}, \overline{S}_{2m}$) to the adopted reference pole based on the standard approach [Lambeck, 1971; Reigber, 1981] was considered in [Marchenko and Schwintzer, 2003] by means of the equation:

$$\mathbf{g} = \mathbf{P}_{xy} \cdot \mathbf{g}_{Z'}, \qquad (1)$$

where the matrix $\mathbf{P}_{xy}$ depends only on the coordinates $\bar{x}_p, \bar{y}_p$ of the mean pole at chosen epoch including also order 2 terms; the vector

$$\mathbf{g} = \left[\bar{C}_{20}, \bar{C}_{21}, \bar{S}_{21}, \bar{C}_{22}, \bar{S}_{22}\right]^{\mathrm{T}}, \qquad (2a)$$

(hereafter the symbol T denotes transposition) of the fully normalized second degree coefficients $\bar{C}_{2m}$ and $\bar{S}_{2m}$, adopted in the Earth body-fixed frame *XYZ*, shall be denoted by

$$\mathbf{g}_{Z'} = \left[\bar{A}_{20}, \bar{A}_{21}, \bar{B}_{21}, \bar{A}_{22}, \bar{B}_{22}\right]^{\mathrm{T}}, \qquad (2b)$$

if given in the coordinate system $X'\,Y'\,Z'$, which is close to *XYZ* but with a difference in the orientation of the third axis with *Z-Z′* being equal to the mean pole coordinates.

According to [Lambeck, 1971] the pole coordinates x_p, y_p are connected in the planar approximation with the so-called amplitude θ_p and azimuth λ_p as

$$x_P = \theta_p \cos\lambda_p, \quad y_P = -\theta_p \sin\lambda_p, \qquad (3)$$

that leads to the expressions for θ_p, λ_p in the following form

$$\theta_p = \sqrt{x_p^{\,2} + y_p^{\,2}}, \quad \tan\lambda_p = \frac{-y_p}{x_p}. \qquad (4)$$

To avoid the planar approximation (3) and the corresponding non-orthogonal matrix $\mathbf{P}_{xy}$ we will consider the angles θ_p, λ_p and x_p, y_p at the unit sphere for further determination θ_p, λ_p from the solution of associated spherical triangles. It is easy to verify that after some simple algebra the following relationships are valid

$$\tan\theta_p = \sqrt{\tan^2 x_p + \tan^2 y_p}, \qquad (5a)$$

$$\tan\lambda_p = \frac{-\tan y_p}{\tan x_p}, \qquad (5б)$$

$$\tan x_P = \tan\theta_p \cos\lambda_p, \qquad (6a)$$

$$\tan y_P = -\tan\theta_p \sin\lambda_p, \qquad (6б)$$

$$\cos\theta_p = \frac{\cos x_p \cos y_p}{\sqrt{1 - \sin^2 x_p \sin^2 y_p}}, \qquad (7)$$

which give exact expressions for the polar coordinates θ_p, λ_p. Eqs. (5 – 7) will get a special importance for similar to Eq. (1) transformation, where the non-orthogonal matrix $\mathbf{P}_{xy}$ will replace by some orthogonal matrix $\mathbf{R}_{\theta\lambda}$, which is depended on the polar coordinates θ_p, λ_p adopted now in spherical approximation.

Thus, we will consider a transformation of the coefficients ($\bar{C}_{2m}, \bar{S}_{2m}$), defined in the coordinate system (X, Y, Z), into the coordinate system $X'\,Y'\,Z'$, which is obtained by a certain *finite* rotation of the *XYZ* – system around the origin. Hence, the potential V_2 of the 2nd degree may be written in the following forms

$$V_2(P) = \frac{1}{2}\frac{GMa^2}{r^5}\mathbf{r}^{\mathrm{T}}\mathbf{H}\mathbf{r} \;\Rightarrow XYZ \text{ system} \qquad (8a)$$

$$V_2(P) = \frac{1}{2}\frac{GMa^2}{r^5}\mathbf{r}'^{\mathrm{T}}\mathbf{H}'\mathbf{r}' \;\Rightarrow X'Y'Z' \text{ system} \qquad (8b)$$

where

$$\mathbf{H} = \begin{pmatrix} \sqrt{15}\bar{C}_{22} - \sqrt{5}\bar{C}_{20} & \sqrt{15}\bar{S}_{22} & \sqrt{15}\bar{C}_{21} \\ \sqrt{15}\bar{S}_{22} & -\sqrt{15}\bar{C}_{22} - \sqrt{5}\bar{C}_{20} & \sqrt{15}\bar{S}_{21} \\ \sqrt{15}\bar{C}_{21} & \sqrt{15}\bar{S}_{21} & 2\sqrt{5}\bar{C}_{20} \end{pmatrix}, \qquad (9a)$$

$$\mathbf{H}' = \begin{pmatrix} \sqrt{15}\bar{A}_{22} - \sqrt{5}\bar{A}_{20} & \sqrt{15}\bar{B}_{22} & \sqrt{15}\bar{A}_{21} \\ \sqrt{15}\bar{B}_{22} & -\sqrt{15}\bar{A}_{22} - \sqrt{5}\bar{A}_{20} & \sqrt{15}\bar{B}_{21} \\ \sqrt{15}\bar{A}_{21} & \sqrt{15}\bar{B}_{21} & 2\sqrt{5}\bar{A}_{20} \end{pmatrix}. \qquad (9b)$$

The matrices $\mathbf{H}$ and $\mathbf{H}'$ are defined in the geocentric coordinate systems (X, Y, Z) and $(X'\,Y'\,Z')$, respectively, representing the deviatoric part of inertia tensor; the vectors $\mathbf{r}^{\mathrm{T}}$ and $\mathbf{r}'^{\mathrm{T}}$ contain the Cartesian coordinates of the current point *P* in these systems. GM is the product of the gravitational constant G and the planet's mass M; a is the semimajor axis of the ellipsoid of revolution; r is the distance from the origin of a coordinate system to the current point *P*.

It should be pointed out that the rotation of the system *XYZ* around the origin can be expressed via the three matrixes of elementary rotations $\mathbf{R}_1(\alpha_1)$, $\mathbf{R}_2(\alpha_2)$, $\mathbf{R}_3(\alpha_3)$. According to [Madelund, 1957] there are only two kinds of *commutative* rotations. First one is an infinitesimal rotation. Second one is a finite rotation about the fixed axis. An infinitesimal rotation was considered in [Marchenko and Schwintzer, 2003] for the adjustment of $\bar{C}_{2m}, \bar{S}_{2m}$-coefficients. To resolve a possible ambiguity for various sequences of finite rotations we will use this second type of a commutative rotation with the following transformation of the coordinate vector

$$\mathbf{Q} = \begin{pmatrix} \cos^2\lambda_P(\cos\theta_P - 1) + 1 & \sin\lambda_P\cos\lambda_P(\cos\theta_P - 1) & -\cos\lambda_P\sin\theta_P \\ \sin\lambda_P\cos\lambda_P(\cos\theta_P - 1) & \cos^2\lambda_P(1 - \cos\theta_P) + \cos\theta_P & -\sin\lambda_P\sin\theta_P \\ \cos\lambda_P\sin\theta_P & \sin\lambda_P\sin\theta_P & \cos\theta_P \end{pmatrix}, \qquad (10)$$

$$\mathbf{r}' = \mathbf{Q} \cdot \mathbf{r}, \qquad (11)$$

is the rotation matrix depended on the polar coordinates of the axis Z' in the system XYZ: θ_P is the polar distance of the axis Z' and λ_P is the longitude of this axis defined by the Eqs. (5–7).

It is easy to verify that the matrix **Q** can be constructed in the following way

$$\mathbf{Q} = \mathbf{R}_3(-\lambda_P)\mathbf{R}_2(\theta_P)\mathbf{R}_3(\lambda_P), \qquad (12)$$

where

$$\mathbf{R}_2(\alpha_2) = \begin{pmatrix} \cos(\alpha_2) & 0 & -\sin(\alpha_2) \\ 0 & 1 & 0 \\ \sin(\alpha_2) & 0 & \cos(\alpha_2) \end{pmatrix}, \qquad (13a)$$

$$\mathbf{R}_3(\alpha_3) = \begin{pmatrix} \cos(\alpha_3) & \sin(\alpha_3) & 0 \\ -\sin(\alpha_3) & \cos(\alpha_3) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (13b)$$

by means of the rotation about the angles $\alpha_2 = \pm\theta_P$ and $\alpha_3 = \pm\lambda_P$ around the *nodes line* of the XYZ and $X'Y'Z'$ systems. Clearly, the inverse transformation reads

$$\mathbf{r} = \mathbf{Q}^{\mathrm{T}} \cdot \mathbf{r}' = \mathbf{R}_3(-\lambda_P)\mathbf{R}_2(-\theta_P)\mathbf{R}_3(\lambda_P)\mathbf{r}', \qquad (14)$$

due to the orthogonality of the rotation matrix **Q**. By inserting (11)and (14) into (8) we get

$$V_2(P) = \frac{1}{2}\frac{GMa^2}{r^5}\mathbf{r}'^{\mathrm{T}}\left[\mathbf{QHQ}^{\mathrm{T}}\right]\cdot\mathbf{r}', \qquad (15a)$$

$$V_2(P) = \frac{1}{2}\frac{GMa^2}{r^5}\mathbf{r}^{\mathrm{T}}\left[\mathbf{Q}^{\mathrm{T}}\mathbf{H}'\mathbf{Q}\right]\cdot\mathbf{r} \qquad (15b)$$

Eq. (15a) represents now the potential V_2 with reference to the $X'Y'Z'$ system and the harmonic coefficients $\overline{C}_{2m}, \overline{S}_{2m}$ given in the XYZ system. Eq. (15b) describes the potential V_2 in the XYZ system with the harmonic coefficients $\overline{A}_{2m}, \overline{B}_{2m}$ related to the $X'Y'Z'$ system.

It has to be noted that the tesseral coefficients $\overline{C}_{21}(IERS)$ and $\overline{S}_{21}(IERS)$ related to the IERS reference pole are based on the [Lambeck, 1971; Reigber, 1981] formulae

$$\overline{C}_{21}(IERS) = (\sqrt{3}\overline{C}_{20} - \overline{C}_{22})\overline{x}_p + \overline{S}_{22}\overline{y}_p, \quad (16a)$$

$$\overline{S}_{21}(IERS) = -(\sqrt{3}\overline{C}_{20} + \overline{C}_{22})\overline{y}_p - \overline{S}_{22}\overline{x}_p, \quad (16б)$$

used also in the approximate form

$$\overline{C}_{21}(IERS) \approx \sqrt{3}\overline{C}_{20}\overline{x}_p, \qquad (17a)$$

$$\overline{S}_{21}(IERS) \approx -\sqrt{3}\overline{C}_{20}\overline{y}_p. \qquad (17б)$$

Thus, Eq. (16) is recommended by IERS Conventions 2003 [MacCarthy and Petit, 2004] for the computation of $\overline{C}_{21}(IERS)$, $\overline{S}_{21}(IERS)$. But Lambeck's standard approach may be developed to the expressions for all 2nd degree coefficients

$$\overline{C}_{20}(IERS) = \overline{C}_{20} + \sqrt{3}\overline{C}_{22}(\overline{x}_p^2 - \overline{y}_p^2)/2 - $$
$$- \overline{C}_{20}(\overline{x}_p^2 + \overline{y}_p^2)/2 - \sqrt{3}\overline{S}_{22}\overline{x}_p\overline{y}_p, \quad (18)$$

$$\overline{C}_{22}(IERS) = \overline{C}_{22} + \overline{C}_{20}(\overline{x}_p^2 - \overline{y}_p^2)/\sqrt{3}, \quad (19a)$$

$$\overline{S}_{22}(IERS) = \overline{S}_{22} - 2\overline{C}_{20}\overline{x}_p\overline{y}_p/\sqrt{3}, \qquad (19б)$$

and we can verify Eqs. (16 – 17) by considering the characteristic equation of the matrices $\mathbf{H}'$ (or **H**) and deriving the first invariant $I_1' = \mathrm{Trace}(\mathbf{H}')$ for new harmonic coefficients $\overline{A}_{2m} = \overline{C}_{2m}(IERS)$, $\overline{B}_{2m} = \overline{S}_{2m}(IERS)$ through Eqs. (16 – 19). Of course, the equality $I_1 = 0$ is satisfied by Eqs. (9) *trivially for arbitrary* sets of $\overline{C}_{2m}, \overline{S}_{2m}$ or $\overline{A}_{2m}, \overline{B}_{2m}$. Nevertheless, after some easy algebra we may get using Eqs. (18 – 19):

$$I_1(IERS) = \overline{x}_p^2\left(\sqrt{5}\overline{C}_{20} + \sqrt{15}\overline{C}_{22}\right) + $$
$$+ \overline{y}_p^2\left(\sqrt{5}\overline{C}_{20} - \sqrt{15}\overline{C}_{22}\right) - 2\overline{x}_p\overline{y}_p\sqrt{15}\overline{S}_{22}, \quad (20)$$

as a rule non-zero value in Eq. (20), if the planar approximation [Eqs. (16 – 19)] was used.

For example, the application of Eq. (20) to the conventional EGM96 gravity model leads to $I_1(IERS) \approx -0.2\cdot10^{-14}$ instead of the trivial case and we note again that $I_1(IERS) = 0$ can be obtained only by the direct computation of the first invariant based on Eq. (9). Hence, Eq. (20) allows us to demonstrate a level of accuracy of the planar approximation. Transformation in Eqs. (15) via the matrix **Q** represents here an *exception*, because all $\overline{C}_{2m}$, $\overline{S}_{2m}$ or $\overline{A}_{2m}, \overline{B}_{2m}$ are results of the commutative orthogonal rotation that always gives zero value of $I_1 = \mathrm{Trace}(\mathbf{H}) = \mathrm{Trace}(\mathbf{QH} \times \mathbf{Q}^{\mathrm{T}}) = \mathrm{Trace}(\mathbf{H}') = 0$.

Thus, in contrast to the Lambeck's formulae in planar approximation, the transformation (15) of V_2 from XYZ to $X'Y'Z'$ system by applying the matrix **Q** makes available to keep the first invariant I_1 of the deviatoric part **H** of inertia tensor. In this case all elements of the matrix **H**, expressed through $\overline{C}_{2m}$, $\overline{S}_{2m}$, or all elements of the matrix $\mathbf{H}'$, expressed via $\overline{A}_{2m}, \overline{B}_{2m}$, are connected by the commutative orthogonal rotation that leads to $I_1 = 0$ in both cases. If the non-orthogonal matrix $\mathbf{P}_{xy}$ is used instead of the matrix **Q**, we get the non-zero first invariant $I_1 = \mathrm{Trace}(\mathbf{H}') \neq 0 \approx \approx 10^{-15}$.

Basic relationships for the adjustment of 2nd degree harmonic coefficients to adopted reference pole

Let us now consider the vector $\mathbf{g}_{Z'}$ consisting of the harmonic coefficients $\overline{A}_{2m}, \overline{B}_{2m}$ in the $X'Y'Z'$ system and taking into account Eq. (15b) we find the following auxiliary matrix

$$\mathbf{H}_{\lambda} = \mathbf{R}_3(\lambda_P)\mathbf{H}'\mathbf{R}_3(-\lambda_P)\,. \qquad (21)$$

After simple manipulations in Eq. (21), we come to the possibility of direct transformation of the vector $\mathbf{g}_{Z'}$ to some vector $\mathbf{g}_{\lambda} = \left[\overline{C}_{20}^{\lambda}, \overline{C}_{21}^{\lambda}, \overline{S}_{21}^{\lambda}, \overline{C}_{22}^{\lambda}, \overline{S}_{22}^{\lambda}\right]^{\mathrm{T}}$ of harmonic coefficients

$$\mathbf{g}_{\lambda} = \mathbf{R}_{\lambda}(\lambda_P)\cdot\mathbf{g}_{Z'}\,,$$

$$\mathbf{R}_{\lambda}(\lambda_P) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos\lambda_P & \sin\lambda_P & 0 & 0 \\ 0 & -\sin\lambda_P & \cos\lambda_P & 0 & 0 \\ 0 & 0 & 0 & \cos 2\lambda_P & \sin 2\lambda_P \\ 0 & 0 & 0 & -\sin 2\lambda_P & \cos 2\lambda_P \end{pmatrix}, \qquad (22)$$

where $\mathbf{R}_{\lambda}(\lambda_P)$ is the (5x5)-orthogonal matrix of rotation about the angle λ_P. Making our manipulations in the same manner we can get some new auxiliary matrix

$$\mathbf{H}_{\lambda\theta} = \mathbf{R}_2(-\theta_P)\mathbf{H}_{\lambda}\mathbf{R}_2(\theta_P)\,, \qquad (23)$$

or the auxiliary vector

$$\mathbf{g}_{\lambda\theta} = \left[\overline{C}_{20}^{\lambda\theta}, \overline{C}_{21}^{\lambda\theta}, \overline{S}_{21}^{\lambda\theta}, \overline{C}_{22}^{\lambda\theta}, \overline{S}_{22}^{\lambda\theta}\right]^{\mathrm{T}}$$

of harmonic coefficients

$$\mathbf{g}_{\lambda\theta} = \mathbf{R}_{\theta}(-\theta_P)\cdot\mathbf{g}_{\lambda}\,, \qquad (24)$$

where

$$\mathbf{R}_{\theta}(-\theta_P) = \begin{pmatrix} \frac{3\cos 2\theta_P}{4}+\frac{1}{4} & \frac{\sqrt{3}\sin 2\theta_P}{2} & 0 & -\frac{\sqrt{3}\cos 2\theta_P}{4}+\frac{\sqrt{3}}{4} & 0 \\ -\frac{\sqrt{3}\sin 2\theta_P}{2} & \cos 2\theta_P & 0 & \frac{\sin 2\theta_P}{2} & 0 \\ 0 & 0 & \cos\theta_P & 0 & \sin\theta_P \\ -\frac{\sqrt{3}\cos 2\theta_P}{4}+\frac{\sqrt{3}}{4} & -\frac{\sin 2\theta_P}{2} & 0 & \frac{\cos 2\theta_P}{4}+\frac{3}{4} & 0 \\ 0 & 0 & -\sin\theta_P & 0 & \cos\theta_P \end{pmatrix}, \qquad (25)$$

is the (5x5)-orthogonal matrix of rotation about the angle θ_P. Taking into consideration Eq. (14) finally we come to the following transformation of the vector $\mathbf{g}_{Z'}$ given in the $X'Y'Z'$ system, to the vector $\mathbf{g} = \left[\overline{C}_{20}, \overline{C}_{21}, \overline{S}_{21}, \overline{C}_{22}, \overline{S}_{22}\right]^{\mathrm{T}}$ adopted in the XYZ system

$$\mathbf{g} = \mathbf{R}_{\lambda}(-\lambda_P)\cdot\mathbf{g}_{\lambda\theta} \qquad (26a)$$

$$\mathbf{g} = \mathbf{R}_{\lambda}(-\lambda_P)\mathbf{R}_{\theta}(-\theta_P)\mathbf{R}_{\lambda}(\lambda_P)\cdot\mathbf{g}_{Z'}\,. \qquad (26б)$$

Then taking into account some properties of these orthogonal matrixes, the inverse transformation from the vector $\mathbf{g}$ (XYZ system) to the vector $\mathbf{g}_{Z'}$ ($X'Y'Z'$ system) reads

$$\mathbf{g}_{Z'} = \mathbf{R}_{\lambda}(-\lambda_P)\mathbf{R}_{\theta}(\theta_P)\mathbf{R}_{\lambda}(\lambda_P)\cdot\mathbf{g}\,. \qquad (27)$$

Eq. (27) can be considered as the *observational equations* for further adjustment of different sets of the 2nd degree harmonic coefficients $\mathbf{g}_{Z'}$ to the IERS reference pole fixed by the conventional mean pole coordinates. Additional conditions for the harmonic coefficients $\overline{A}_{21} = \overline{B}_{21} = 0$ can be obtained from Eq. (27), if the axis Z' will coincide with the figure axis C.

The harmonic coefficients of the degree n=2 can be derived from Eq. (26) and represented now in the matrix form

$$\mathbf{g} = \mathbf{R}_{\theta\lambda}\cdot\mathbf{g}_{Z'} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} \\ -r_{12} & r_{22} & r_{23} & r_{24} & r_{25} \\ -r_{13} & r_{23} & r_{33} & r_{34} & r_{35} \\ r_{14} & -r_{24} & -r_{34} & r_{44} & r_{45} \\ r_{15} & -r_{25} & -r_{35} & r_{45} & r_{55} \end{pmatrix}\cdot\mathbf{g}_{Z'}\,, \qquad (28)$$

with the elements (29), (30). Then, according to Eq. (27) the inverse transformation admits the representation with the orthogonal matrix $\mathbf{R}_{\theta\lambda}^{\mathrm{T}}$ obtained by the transposition of the orthogonal matrix $\mathbf{R}_{\theta\lambda}$ in Eq. (28) with elements given by Eqs. (29 – 30):

$$\left.\begin{aligned}
r_{11} &= (3\cos^2\theta_P - 1)/2, \\
r_{12} &= -\sqrt{3}\cos\lambda_P\sin\theta_P\cos\theta_P, \\
r_{13} &= -\sqrt{3}\sin\lambda_P\sin\theta_P\cos\theta_P, \\
r_{14} &= (\sqrt{3}\sin^2\theta_P u_1)/2, \\
r_{15} &= \sqrt{3}\cos\lambda_P\sin\lambda_P\sin^2\theta_P, \\
r_{22} &= \cos^2\lambda_P u_2 + \cos\theta_P\sin^2\lambda_P, \\
r_{23} &= \cos\lambda_P\sin\theta_P(u_2 - \cos\theta_P), \\
r_{24} &= -\cos\lambda_P\sin\theta_P u_4, \\
r_{25} &= -\sin\lambda_P\sin\theta_P(2\cos^2\lambda_P u_3 + 1), \\
r_{33} &= \cos^2\lambda_P\cos\theta_P + \sin^2\lambda_P u_2, \\
r_{34} &= \sin\lambda_P\sin\theta_P(\cos\theta_P - 2\cos^2\lambda_P u_3), \\
r_{35} &= -\cos\lambda_P\sin\theta_P u_5, \\
r_{44} &= -(4\cos^2\lambda_P\sin^2\lambda_P u_3^2 - \cos^2\theta_P - 1)/2, \\
r_{45} &= \cos\lambda_P\sin\lambda_P u_3^2 u_1, \\
r_{55} &= 2\cos^2\lambda_P\sin^2\lambda_P u_3^2 + \cos\theta_P,
\end{aligned}\right\} \qquad (29)$$

Where

$$\left.\begin{aligned}
u_1 &= 2\cos^2\lambda_P - 1, \\
u_2 &= 2\cos^2\theta_P - 1, \\
u_3 &= \cos\theta_P - 1, \\
u_4 &= \cos\theta_P u_1 + 2\sin^2\lambda_P, \\
u_5 &= 2\cos^2\lambda_P + 2\cos\theta_P\sin^2\lambda_P - 1.
\end{aligned}\right\} \qquad (30)$$

$$\mathbf{g}_{Z'} = \mathbf{R}_{\theta\lambda}^{\mathrm{T}}\cdot\mathbf{g}\,. \qquad (31)$$

The last relationship together with Eqs. (29 – 30) will be considered as basic equation for the adjustment to the adopted IERS reference pole of different 2nd degree harmonic coefficients $\mathbf{g}_{Z'}$ chosen as observations according to various gravity field models.

In particular, making further manipulations, it is easy to verify that the degree n zonal harmonic coefficients in these two coordinate systems can be formed as

$$\overline{C}_{n0} = \sum_{m=0}^{n} (-1)^m \left(\overline{A}_{nm} \cos m\lambda_P + \overline{B}_{nm} \sin m\lambda_P\right) \cdot \widetilde{P}_{nm}(\cos\theta_P), \tag{32a}$$

$$\overline{A}_{n0} = \sum_{m=0}^{n} (\overline{C}_{nm} \cos m\lambda_p + \overline{S}_{nm} \sin m\lambda_p) \widetilde{P}_{nm}(\cos\theta_p), \tag{32b}$$

where $\widetilde{P}_{nm}(\cos\theta_P)$ are A. Schmidt's quasi-normalized by the factor $\sqrt{(2-\delta_{m0})\frac{(n-m)!}{(n+m)!}}$ associated Legendre functions of the first kind (δ_{m0} is the Kronecker delta). If m=0 these functions coincide with $P_{nm}(\cos\theta_P)$. If m>0 we have for the fully normalized Legendre functions $\overline{P}_{nm}(\cos u_P)$ the following relationship: $\overline{P}_{nm}(\cos u_P) = \sqrt{2n+1}\widetilde{P}_{nm}(\cos u_P)$.

Then we will split up the matrix (18) onto two parts

$$\mathbf{R}_\theta(-\theta_P) = \mathbf{R}^\theta_{const} + \mathbf{R}^\theta, \tag{33}$$

$$\mathbf{R}^\vartheta_{const} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{\sqrt{3}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{3}}{4} & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{34}$$

$$\mathbf{R}^\theta = \begin{pmatrix} \frac{3\cos 2\theta_P}{4} & \frac{\sqrt{3}\sin 2\theta_P}{2} & 0 & -\frac{\sqrt{3}\cos 2\theta_P}{4} & 0 \\ -\frac{\sqrt{3}\sin 2\theta_P}{2} & \cos 2\theta_P & 0 & \frac{\sin 2\theta_P}{2} & 0 \\ 0 & 0 & \cos\theta_P & 0 & \sin\theta_P \\ -\frac{\sqrt{3}\cos 2\theta_P}{4} & -\frac{\sin 2\theta_P}{2} & 0 & \frac{\cos 2\theta_P}{4} & 0 \\ 0 & 0 & -\sin\theta_P & 0 & \cos\theta_P \end{pmatrix}, \tag{35}$$

that leads to extracting in Eqs. (26 – 27) some constant terms, longitude – only terms, and longitude – polar distance terms. The constant terms exist in the expressions for $\overline{C}_{20}$, $\overline{C}_{22}$, $\overline{S}_{22}$ coefficients only.

If the coefficients $\overline{C}_{2m}$ and $\overline{S}_{2m}$ are given, Eq. (28) to Eq. (30) can be applied to compute $\overline{A}_{2m}$ and $\overline{B}_{2m}$ related to the axis Z'. $\overline{A}_{21}$ and $\overline{B}_{21}$ then read

$$\overline{A}_{21} = r_{12}\overline{C}_{20} + r_{22}\overline{C}_{21} + r_{23}\overline{S}_{21} - r_{24}\overline{C}_{22} - r_{25}\overline{S}_{22}, \tag{36a}$$

$$\overline{B}_{21} = r_{13}\overline{C}_{20} + r_{23}\overline{C}_{21} + r_{33}\overline{S}_{21} - r_{34}\overline{C}_{22} - r_{35}\overline{S}_{22}, \tag{36b}$$

where the harmonic coefficients $\overline{A}_{21}$ and $\overline{B}_{21}$ must be zero by definition, if the axis Z' and the figure axis $\overline{C}$ are coinciding at t_0. By this, Eqs. (36) give a tool to test whether gravity field models are referred to a common axis $\overline{C}$.

Transformation of 2nd degree harmonic coefficients from initial to principle axes coordinate system

Assuming our initial information consisting of the vector **g** (Eq. (2a)) of 2nd-degree coefficients and their variance-covariance matrix, we will use for the transformation of $(\overline{C}_{2m}, \overline{S}_{2m})$ to the principal axes system the exact closed solution of the eigenvalue problem with accuracy estimation by rigorous error propagation. Let us give briefly according to [Marchenko and Schwintzer, 2003; Marchenko, 2003] the corresponding closed expressions for the transformation of $(\overline{C}_{2m}, \overline{S}_{2m})$, defined in an adopted Earth's-fixed coordinate system (X, Y, Z), to the vector $\widetilde{\mathbf{g}} = \left[\overline{A}_{20},\ 0,\ 0,\ \overline{A}_{22},\ 0\right]^{\mathrm{T}}$ of the two nonzero harmonic coefficients $\overline{A}_{20}$, $\overline{A}_{22}$ in the coordinate system of the Earth's principal axes of inertia ($\overline{A}$, $\overline{B}$, $\overline{C}$). The potential V_2 of the second degree may be written in the following way

$$V_2(P) = \frac{\sqrt{15}}{2}\frac{GMa^2}{r^5}\widetilde{\mathbf{r}}^{\mathrm{T}}\widetilde{\mathbf{H}}\widetilde{\mathbf{r}} \tag{37a}$$

$$V_2(P) = \frac{\sqrt{15}}{2}\frac{GMa^2}{r^5}\mathbf{r}^{\mathrm{T}}\mathbf{H}\mathbf{r} \tag{37б}$$

where the deviatoric matrix **H** is defined by Eq. (9a) and

$$\widetilde{\mathbf{H}} = \begin{pmatrix} \overline{A}_{22} - \frac{\overline{A}_{20}}{\sqrt{3}} & 0 & 0 \\ 0 & -\overline{A}_{22} - \frac{\overline{A}_{20}}{\sqrt{3}} & 0 \\ 0 & 0 & 2\frac{\overline{A}_{20}}{\sqrt{3}} \end{pmatrix} \tag{38}$$

The matrix $\widetilde{\mathbf{H}}$ is adopted in the system of principal axes of inertia ($\overline{A}$, $\overline{B}$, $\overline{C}$); the vector $\widetilde{\mathbf{r}}^{\mathrm{T}}$ contains the Cartesian coordinates of the current point P in this system; $(\overline{A}_{20}, \overline{A}_{22})$ are fully normalized harmonic coefficients in the Earth's principal axes of inertia system ($\overline{A}$, $\overline{B}$, $\overline{C}$).

The computation of the harmonic coefficients $\overline{A}_{20}$, $\overline{A}_{22}$ requires a transformation of the matrix **H** [Eq. (9a)] into the diagonal form $\widetilde{\mathbf{H}}$ [Eq. (38)]. Solving the *eigenvalue* problem for the corresponding *deviatoric* tensor (Eq. (9a)) in the case of the given quadratic form $\mathbf{r}^{\mathrm{T}}\mathbf{H}\mathbf{r}$ we get eigenvalues Λ_i in the following non-linear form [Marchenko and Schwintzer, 2003]:

$$\left.\begin{matrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{matrix}\right\} = 2\sqrt{\frac{k_2}{3}} \cdot \left\{\begin{matrix} \sin\left(\frac{\widetilde{\varphi}}{3} + \frac{\pi}{3}\right) \\ -\sin\frac{\widetilde{\varphi}}{3} \\ \sin\left(\frac{\widetilde{\varphi}}{3} - \frac{\pi}{3}\right) \end{matrix}\right., \tag{39}$$

where the auxiliary angle $\widetilde{\varphi}$ is expressed by means of the invariants $I_2 = -k_2$ and I_3:

$$\tilde{\varphi} = \sin^{-1}\left(\frac{3\sqrt{3}}{2}\cdot\frac{I_3}{\sqrt{k_2^3}}\right), \; -\frac{\pi}{2} \le \tilde{\varphi} \le \frac{\pi}{2},$$

$$(I_1 = \mathrm{Trace}(\mathbf{H}) = 0), \qquad (40)$$

with

$$k_2 = -I_2 = \sum_{m=0}^{2}\left(\bar{C}_{2m}^2 + \bar{S}_{2m}^2\right) = \bar{A}_{20}^2 + \bar{A}_{22}^2, \quad (41)$$

$$I_3 = \det(\mathbf{H}) = \frac{2\bar{C}_{20}^3}{3\sqrt{3}} + \frac{\bar{C}_{20}}{\sqrt{3}}\left(\bar{C}_{21}^2 + \bar{S}_{21}^2 - 2\bar{C}_{22}^2 - 2\bar{S}_{22}^2\right) +$$

$$+ \bar{C}_{22}\left(\bar{C}_{21}^2 - \bar{S}_{21}^2\right) + 2\bar{C}_{21}\bar{S}_{21}\bar{S}_{22} = \det(\tilde{\mathbf{H}}). \quad (42)$$

Here the 2nd degree variance k_2 and I_3 represent the invariant characteristics of the gravity field, which are independent of linear transformations of the coordinate system (*X*, *Y*, *Z*).

Thus, Eqs. (39) to (42) provide the computation of the harmonic coefficients ($\bar{A}_{20}, \bar{A}_{22}$) in the principle axes coordinate system via the simple expressions

$$\bar{A}_{20} = \frac{\sqrt{3}\Lambda_3}{2}, \; \bar{A}_{22} = \frac{\Lambda_1 - \Lambda_2}{2}. \qquad (43)$$

The matrix $\tilde{\mathbf{H}}$ can be used also in the following way

$$\begin{pmatrix} B+C-2A & 0 & 0 \\ 0 & A+C-2B & 0 \\ 0 & 0 & A+B-2C \end{pmatrix} =$$

$$= \sqrt{15}\tilde{\mathbf{H}} = \sqrt{15}\begin{pmatrix} \Lambda_1 & 0 & 0 \\ 0 & \Lambda_2 & 0 \\ 0 & 0 & \Lambda_3 \end{pmatrix}, \quad (44)$$

where *A*, *B*, and *C* are the Earth's principal moments of inertia normalized by the factor $1/Ma^2$. As a result, if the eigenvalues Λ_i are found, we come after some easy algebra to the following relationships for differences between these *normalized* moments of inertia

$$B - A = \frac{2\sqrt{15}}{3}\bar{A}_{22}, \; C - A = \frac{\sqrt{15}\bar{A}_{22}}{3} - \sqrt{5}\bar{A}_{20}, \quad (45a)$$

$$C - B = -\frac{\sqrt{15}\bar{A}_{22}}{3} - \sqrt{5}\bar{A}_{20}, \qquad (45б)$$

represented by means of the harmonic coefficients ($\bar{A}_{20}, \bar{A}_{22}$) in the principal axes system. Similarly, these differences can be expressed also through parameters of the Earth's gravitational quadrupole [Marchenko, 1979; Marchenko, 1998]:

$$C - A = \tilde{M}_2 = \frac{M_2}{Ma^2} = \frac{\sqrt{15}\bar{A}_{22}}{3} - \sqrt{5}\bar{A}_{20}, \quad (46)$$

$$\frac{C-B}{C-A} = \frac{1-\cos\tilde{\gamma}}{2} = \sin^2\frac{\tilde{\gamma}}{2}, \qquad (47a)$$

$$\cos\tilde{\gamma} = \frac{3\bar{A}_{22} + \sqrt{3}\bar{A}_{20}}{\bar{A}_{22} - \sqrt{3}\bar{A}_{20}}, \qquad (47б)$$

where M_2 is the moment of the quadrupole and $\tilde{\gamma}$ is the angle between two quadrupole axes, located in the plane of the axes $\bar{A}$ and $\bar{C}$. The parameter $\cos\tilde{\gamma}$ of the Earth's *triaxiality* as the cosine of an angle has a bounded range of variation, $-1 \le \cos\tilde{\gamma} \le 1$, and enables us via Eqs. (46, 47) to obtain "limiting" relationships between the principal moments of inertia, 2nd degree harmonic coefficients in the principal axes system $0 \le \bar{A}_{22} \le -\bar{A}_{20}/2$, and the polar f_p and equatorial f_e flattenings $0 \le f_e < f_p$ [Marchenko, 1979].

The estimation of the *normalized* principal moments of inertia can be obtained now by involving the dynamical ellipticity H_D:

$$C = -\frac{\sqrt{5}\bar{A}_{20}}{H_D} \text{ with}$$

$$H_D = \frac{2C - A - B}{2C} = -\frac{\sqrt{5}\bar{A}_{20}}{C} \qquad (48)$$

Substitution of Eq. (48) into Eq. (45) gives

$$A = \sqrt{5}\bar{A}_{20} - \frac{\sqrt{15}\bar{A}_{22}}{3} - \frac{\sqrt{5}\bar{A}_{20}}{H_D}, \qquad (49a)$$

$$B = \sqrt{5}\bar{A}_{20} + \frac{\sqrt{15}\bar{A}_{22}}{3} - \frac{\sqrt{5}\bar{A}_{20}}{H_D}. \qquad (49б)$$

Therefore, with H_D known, the computation of the polar moment of inertia (normalized by the factor $1/Ma^2$), $C = -\sqrt{5}\bar{A}_{20}/H_D$, the trace $\mathrm{Tr}(\mathbf{I})$:

$$\mathrm{Tr}(\mathrm{I}) = A + B + C = \sqrt{5}\bar{A}_{20}\left(2 - \frac{3}{H_D}\right) = 3I_m, \quad (50)$$

of the Earth's tensor of inertia **I** considered in the principal axes

$$\mathbf{I} = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix}, \qquad (51)$$

and functions of the principal moments of inertia (α, β, γ - dynamical flattenings) used in the integration of the Eulerian dynamical equations [Bretagnon et al., 1998; Hartmann et al., 1999]:

$$\alpha = \frac{C-B}{A}, \quad \beta = \frac{C-A}{B}, \quad \gamma = \frac{B-A}{C}, \quad (52)$$

are straightforward, if the fully normalized harmonic coefficients, $\bar{A}_{20}, \bar{A}_{22}$ are computed through Eqs. (39 – 43). Then the orientation of the principal axes in the XYZ frame is based on the exact solution of eigenvector problem, using $\bar{C}_{2m}$, $\bar{S}_{2m}$ only without the dynamical ellipticity H_D [Marchenko and Schwintzer, 2003].

Estimation of the Earth's fundamental parameters in the principal axes coordinate system

The harmonic coefficients of 2nd degree and their temporal variations are selected from the following four gravity field models derived in various centers of analysis: three solutions resulting from satellite tracking data and GRACE observations for different time-periods, GGM03S [Tapley et al., 2007], ITG-GRACE03S [Mayer-Gürr, 2007], and EIGEN-GL04S1 [Förste et al., 2008], and one gravity field model of high resolution, EGM2008 (Pavlis et al., 2008), based on surface gravimetry only. The time variable coefficients in these models are referred to different epochs with a spacing of 5 years in between. Among these models the harmonic coefficients of ITG-GRACE03S have non-calibrated errors, which were multiplied on the factor 10 according to the recommendation of [Mayer-Gürr, 2008]. To be consistent, the following transformations were applied to values given (after reductions) in Table 1: (a) prediction of $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ for a common epoch 2000, (b) reduction of $\bar{C}_{20}$ to a common permanent tide system, and (c) scaling of these coefficients to common values of GM=398600.4415 km^3/s^2 and a=6378136.49 m.

Table 1.

Geodetic parameters in the zero-frequency system (GM=398600.4415 km^3/s^2; a=6378136.49 m; epoch: t_0=2000; $\bar{x}_p = 0.054''$, $\bar{y}_p = 0.357''$).

Model	$\bar{C}_{20}\cdot 10^6$	$\bar{C}_{21}\cdot 10^6$	$\bar{S}_{21}\cdot 10^6$	$\bar{C}_{22}\cdot 10^6$	$\bar{S}_{22}\cdot 10^6$
EGM2008	-484.16928852 ±0.000007	-0.00020662 ±0.000007	0.00138441 ±0.000007	2.43938343 ±0.000007	-1.40027362 ±0.000007
ITG-GRACE03	-484.16928857 ±0.000006	-0.00026548 ±0.000006	0.00147539 ±0.000006	2.43938345 ±0.000006	-1.40027368 ±0.000006
GGM03S	-484.16929290 ±0.000047	-0.00020659 ±0.000008	0.00138442 ±0.000008	2.43934997 ±0.000008	-1.40029646 ±0.000008
EIGEN-GL04S1	-484.16944263 ±0.000025	-0.00024172 ±0.000016	0.00137671 ±0.000016	2.43936442 ±0.000017	-1.40028586 ±0.000017

For the transformation of $\bar{C}_{20}$ from the tide-free system $\bar{C}_{20}^{f}$ to the zero-frequency tide system $\bar{C}_{20}^{Z}$ the following relation was used :

$$\bar{C}_{20}^{Z} = \bar{C}_{20}^{f} - 3.1108\cdot 10^{-8}\cdot 0.3/\sqrt{5}\,. \quad (53)$$

The IERS Conventions 2003 recommends the simple linear model representing the mean pole's drift as

$$\left.\begin{matrix}\bar{x}_p\\ \bar{y}_p\end{matrix}\right\} = \begin{Bmatrix}\bar{x}_p(t_0)\\ \bar{y}_p(t_0)\end{Bmatrix} + \begin{Bmatrix}\dot{\bar{x}}_p(t_0)\\ \dot{\bar{y}}_p(t_0)\end{Bmatrix}(t-t_0)\,, \quad (54)$$

where $\bar{x}_p(t_0) = 0.054''$, $\bar{y}_p(t_0) = 0.357''$ are the mean pole coordinate at the reference epoch t_0=2000; $\dot{\bar{x}}_p(t_0) = 0.00083$ [$''$/yr], $\dot{\bar{y}}_p(t_0) = 0.00395$ [$''$/yr] are the secular variations in $\bar{x}_p(t_0), \bar{y}_p(t_0)$ valid in the vicinity of t_0. The linear model (54) can be applied only for the transformation of the harmonic coefficients $\bar{C}_{21}$ and $\bar{S}_{21}$ caused by a linear drift of the mean pole [Eq. (17)], involving into the temporal variations $\dot{\bar{C}}_{21}$ and $\dot{\bar{S}}_{21}$:

$$\bar{C}_{21}(t) = \bar{C}_{21}(t_0) + (t-t_0)\cdot\dot{\bar{C}}_{21}\,, \quad (55a)$$

$$\bar{S}_{21}(t) = \bar{S}_{21}(t_0) + (t-t_0)\cdot\dot{\bar{S}}_{21}\,, \quad (55б)$$

$$\dot{\bar{C}}_{21} = \sqrt{3}\bar{C}_{20}\cdot\dot{\bar{x}}_p(t_0) = -0.337\cdot 10^{-11}\,\text{yr}^{-1}\,, \quad (56a)$$

$$\dot{\bar{S}}_{21} = -\sqrt{3}\bar{C}_{20}\cdot\dot{\bar{y}}_p(t_0) = 1.606\cdot 10^{-11}\,\text{yr}^{-1}\,, \quad (56б)$$

because for other coefficients we get from Eq. (54) $\dot{\bar{C}}_{20} = \dot{\bar{C}}_{22} = \dot{\bar{S}}_{22} \approx 0$. Additionally to Eqs. (55 – 56) we will take into account the non-tidal secular drift in the zonal coefficient

$$\bar{C}_{20}(t) = \bar{C}_{20}(t_0) + (t-t_0)\cdot\dot{\bar{C}}_{20}\,, \quad (57a)$$

$$\dot{\bar{C}}_{20} = 1.1628\cdot 10^{-11}\,\text{yr}^{-1}\,. \quad (57б)$$

In order to determine the Earth's normalized principal moments of inertia A, B, C we use Eqs. (48 – 49). Table 2 lists eight estimations of H_D and the values of the underlying precession constant p_A. The first five H_D were discussed in Dehant et al. (1999) as 'the best values to be used' in the rigid nutation theory in the year 1999. Another three solutions for the dynamical flattening correspond to the non-rigid Earth's rotation theory including the MHB2000 precession-nutation model [Mathews et al., 2002] estimated from VLBI observations during the time-period of 20 years, adopted by the IAU, and recommended by the IERS Conventions 2003 [McCarthy and Petit, 2004]. The value H_D by [Krasinsky, 2006] has a large deviation from other determinations H_D and for this reason was omitted. For only three selected H_D accuracy estimates are found in the literature.

From the initial values of the dynamical ellipticity H_D given in Table 2 (also assumed to refer to J2000) seven values differ in the adopted according to IERS Conventions 2003 (IAU2000 Precession-Nutation model) precession constant $p_A = 50.2879225''/\text{yr}$. To transform the associated quantities from different p_A to the common value $p_A = 50.2879225''/\text{yr}$ the differential relationship of Souchay and Kinoshita (1996) was used

$$dH_D = \frac{\delta H_D}{\delta p_A} dp_A = 6.4947 \cdot 10^{-7} dp_A, \quad (58)$$

where dp_A is expressed in arcseconds per Julian century and we get the values H_D given as 'transformed H_D to the MHB2000 precession constant' in Table 2. Eqs. (48 – 50) reflect a direct dependence of A, B, C, and of the mean moment of inertia $I_m = \mathrm{Tr}(\mathbf{I})/3$

Table 2.

Determinations of the dynamical ellipticity H_D

Reference	Initial value of the precession constant p_A [″/yr], J2000	Initial value of the dynamical ellipticity H_D	Transformed H_D to the MHB2000 precession constant $p_A = 50.2879225''/\text{yr}$
Williams, 1994	50.287700	0.0032737634	0.003273777851
Souchay and Kinoshita, 1996	50.287700	0.0032737548	0.003273769251
Hartmann et al., 1999	50.288200	0.003273792489	0.003273774466
Bretagnon et al., 1998	50.287700	0.003273766818 ±0.000000000023	0.003273781269
Roosbeek and Dehant, 1998	50.287700	0.0032737674	0.003273781851
Mathews et. Al., 2002, (MHB2000)	50.2879225 ±0.000018	0.0032737949 ±0.0000000012	0.003273794900
Fukushima, 2003	50.287955 ±0.000003	0.0032737804 ±0.0000000003	0.003273778289
Capitaine et al., 2003	50.28796195	0.00327379448	0.003273791918

The parameter $B - A = 2\sqrt{15}\bar{A}_{22}/3$ is also slightly depending on the adopted permanent tide system because $\bar{C}_{20}$ enters into the computation of the coefficient $\bar{A}_{22}$ through Eq. (43). The indirect effect of the permanent tide may either be included in the $\bar{C}_{20}$-coefficient (zero-frequency tide system) or excluded (tide-free system). It is assumed that the H_D values are related to the zero-frequency tide system [Bursa, 1995; Groten, 2000].

With given variance-covariance matrices of $\bar{A}_{20}$, $\bar{A}_{22}$, the Earth's principal moments of inertia A, B, C are determined from a weighted least-squares adjustment of the astronomical and geodetic parameters, all referred to a common permanent tide system and one epoch 2000. As 'observations' generally are taken (a) the eight values for H_D (Table 2) and (b) the four sets of $\bar{A}_{20}$, $\bar{A}_{22}$ in the principal axes system, computed from the coefficients given in Table 1 by applying Eq. (43). Using Eq. (45) and Eq. (48) we get the over-determined system of non-linear observation equations

$$\left.\begin{aligned} \frac{2C - A - B}{2C} &= H_D^{(i)} + \varepsilon_H^{(i)}, \\ \frac{1}{2\sqrt{5}}(A + B - 2C) &= \bar{A}_{20}^{(j)} + \varepsilon_{20}^{(j)}, \\ \frac{3}{2\sqrt{15}}(B - A) &= \bar{A}_{22}^{(j)} + \varepsilon_{22}^{(j)}, \end{aligned}\right\} \quad (59)$$

with respect to the normalized principal moments (A, B, C). $H_D^{(i)}$ (i=1,2,..k), $\bar{A}_{20}^{(j)}$, and $\bar{A}_{22}^{(j)}$ (j=1,2,..l) are treated as observations with ε being an error component. For k values of $H_D^{(i)}$ and for l sets of degree 2 harmonic coefficients $\bar{A}_{20}^{(j)}$, $\bar{A}_{22}^{(j)}$ of l gravity field models we get according to [Marchenko and Schwintzer, 2003] the system of (k+2l) observation equations,

$$\begin{pmatrix} -\frac{1}{2C_0} & -\frac{1}{2C_0} & \frac{A_0 + B_0}{2C_0^2} \\ \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ -\frac{3}{2\sqrt{15}} & \frac{3}{2\sqrt{15}} & 0 \end{pmatrix} \cdot \begin{pmatrix} \Delta A \\ \Delta B \\ \Delta C \end{pmatrix} = \begin{pmatrix} \Delta H_D^{(i)} \\ \Delta \bar{A}_{20}^{(j)} \\ \Delta \bar{A}_{22}^{(j)} \end{pmatrix} + \begin{pmatrix} \varepsilon_{H_D}^{(j)} \\ \varepsilon_{\bar{A}_{20}}^{(j)} \\ \varepsilon_{\bar{A}_{22}}^{(j)} \end{pmatrix}, (60)$$

where A_0, B_0, C_0 are some approximate values of A, B, C; $\Delta H_D^{(j)} = H_D^{(j)} - H_D^0$,

$\Delta\overline{A}_{20}^{(j)} = \overline{A}_{20}^{(j)} - \overline{A}_{20}^{0}$, $\Delta\overline{A}_{22}^{(j)} = \overline{A}_{22}^{(j)} - \overline{A}_{22}^{0}$; and $\Delta A, \Delta B, \Delta C$ are the corrections provided by the solution of the normal equation system following from Eq. (60) through iterations.

A number of iterations depends on the initial values A_0, B_0, C_0 in Eq. (60). Traditional characteristic for such an adjustment of astronomical and geodetic parameters is a high close to +1 correlation between the solved parameters, the three moments of inertia. Nevertheless, the selection of the value I_m=0.4 of the mean moment of inertia of a homogeneous planet as initial values for A_0=B_0=C_0=0.4 leads also to the convergence process but requires about 10 iterations. Finally in zero approximation were adopted A_0=B_0=0.3 and C_0=0.35. Usually with the last amounts of A_0, B_0, and C_0 it is enough to make 4 – 5 iterations. For each of the 8 values H_D an identical standard deviation σ_{8H}=±0.799·10^{-8} derived from the scattering about the mean value was assumed for the weighting in the subsequent adjustment by applying weights two times greater for the last three values H_D from Table 2, corresponding to the non-rigid rotation theory, than for other H_D.

Table 3.

Results of the simultaneous adjustment of the astronomical H_D and geodetic $\overline{A}_{20}$, $\overline{A}_{22}$ parameters (zero-frequency-tide system; *GM*=398600.4415 km^3/s^2; *a*=6378136.49 m, epoch: 2000)

Parameter	S1: 8 H_D + 4 gravity field models	S2: 1 H_D + 4 gravity field models
Solved		
A	0.329612131 ± 0.00000073	0.329611131 ± 0.00000019
B	0.329619393 ± 0.00000073	0.329618393 ± 0.00000019
C	0.330698397 ± 0.00000073	0.330697398 ± 0.00000019
Derived		
I_m	0.329976640 ± 0.00000073	0.329975641 ± 0.00000019
H_D	0.0032737850 ± 0.0000000072	0.0032737949 ± 0.0000000019
$(C-A)\cdot 10^6$	1086.266646 ± 0.000049	1086.266646 ± 0.000049
$(C-B)\cdot 10^6$	1079.004263 ± 0.000049	1079.004263 ± 0.000049
$(B-A)\cdot 10^6$	7.262383 ± 0.000043	7.262383 ± 0.000043
$\alpha = (C-B)/A$	(3273.5575 ± 0.072) ·10^{-6}	(3273.5674 ± 0.019) ·10^{-6}
$\beta = (C-A)/B$	(3295.5180 ± 0.073) ·10^{-6}	(3295.5280 ± 0.019) ·10^{-6}
$\gamma = (B-A)/C$	(21.9607 ± 0.001) ·10^{-6}	(21.9608 ± 0.0001) ·10^{-6}
$\overline{A}_{20}\cdot 10^6$	–484.1692942 ± 0.000009	–484.1692942 ± 0.000012
$\overline{A}_{22}\cdot 10^6$	2.8127085 ± 0.000013	2.8127085 ± 0.000017
$1/f$	298.256508 ± 0.000008	298.256508 ± 0.000008
$1/f_e$	91434.77 ± 0.4	91434.77 ± 0.6

The variance-covariance matrices of ($\overline{A}_{20}$, $\overline{A}_{22}$)-sets are also taken into account. RMS differences before and after iterations are equal to 0.05 and 0.6·10^{-8}, respectively. Simultaneous adjustment of the eight values of $H_D^{(i)}$ and four models of the 2nd degree harmonic coefficients, taken from the Table 1 and transformed to the principal moments systems ($\overline{A}_{20}$, $\overline{A}_{22}$) is given in the first column of Table 3 as the solution S1. The second solution S2 represents the adjustment of only one H_D from the MHB2000 theory and four sets of the same harmonic coefficients from Table 1.

Thus, two solutions, computed for the epoch 2000, are derived from two combinations of eight (S1) and one (S2) values of $H_D^{(i)}$ plus the 2nd degree harmonics of the gravity field models EGM2008, GGM03S, ITG-GRACE03S, and EIGEN-GL04S1. Apart from the solved parameters, the other fundamental parameters of the Earth derived from the three moments of inertia are given in Table 3 together with their accuracy estimates from error propagation [Marchenko and Schwintzer, 2003]. Better accuracy of the S2 solution reflects a level of agreement of geodetic parameters since only one H_D was adopted in this case. In general both sets of parameters from Table 3 have small differences on the level of accuracy estimates. Nevertheless, the second solution S2 corresponds better to the frequently used IERS Conventions 2003 and latest gravity field models instead of the conventional EGM96.

Time-independent constituent adjusted to the IERS reference pole

Let us now will examine values of $\overline{A}_{21}$ and $\overline{B}_{21}$ which must be zero by definition, if the axis Z' and the figure axis $\overline{C}$ are coinciding at t_0. Eqs. (36) give a good opportunity to test whether the adopted here gravity field models are referred to a common axis $\overline{C}$.

Table 4 lists the obtained differences about zero for adopted $\overline{x}_p = 0.054''$ and $\overline{y}_p = 0.357''$ (taken from IERS Conventions 2003 at epoch 2000) and leads to the conclusion that the reference systems of considered models do not exactly match. We get differences up to one order greater than the standard deviations given in Table 1 for $\overline{C}_{21}, \overline{S}_{21}$. However these differences are smaller than the same values in [Marchenko and Schwintzer, 2003] given for old gravity field models.

To avoid the differences in Table 4 when fixing a unique figure axis $\overline{C}$ we determine one set of the coefficients $\overline{C}_{2m}$ and $\overline{S}_{2m}$ at epoch 2000 from a least squares adjustment of the given six sets, taking into account their variance-covariance matrices and the two natural conditions for the left-hand sides of Eqs. (36): $\overline{A}_{21} = \overline{B}_{21} = 0$.

For l adopted gravity models we initially compute the harmonic coefficients $\overline{A}_{2m}^{(j)}$, $\overline{B}_{2m}^{(j)}$ (j=1,2,...l) treated further as observations.

Table 4.

Harmonic coefficient $\overline{A}_{21}$ and $\overline{B}_{21}$ at t_0=2000 based on Eqs. (36) and adopted $\overline{x}_p = 0.054''$ and $\overline{y}_p = 0.357''$ (from IERS Conventions 2003)

Parameter	EGM2008	ITG-GRACE03	GGM03S	EIGEN-GL04S1
$A_{21}\cdot 10^{10}$	0.160	-0.429	-0.160	-0.191
$B_{21}\cdot 10^{10}$	-0.632	0.278	-0.632	-0.709

Applying Eq. (31) we get the observation equations in the linear form

$$\mathbf{R}_{\theta\lambda}^{\mathrm{T}} \cdot \begin{pmatrix} \overline{C}_{20} \\ \overline{C}_{21} \\ \overline{S}_{21} \\ \overline{C}_{22} \\ \overline{S}_{21} \end{pmatrix} = \begin{pmatrix} \overline{A}_{20}^{(j)} \\ \overline{A}_{21}^{(j)} \\ \overline{B}_{21}^{(j)} \\ \overline{A}_{22}^{(j)} \\ \overline{B}_{21}^{(j)} \end{pmatrix} + \begin{pmatrix} \varepsilon_1^{(j)} \\ \varepsilon_2^{(j)} \\ \varepsilon_3^{(j)} \\ \varepsilon_4^{(j)} \\ \varepsilon_5^{(j)} \end{pmatrix}, \qquad (61)$$

with the 5 unknown elements of the vector $\mathbf{g} = \left[\overline{C}_{20}, \overline{C}_{21}, \overline{S}_{21}, \overline{C}_{22}, \overline{S}_{22}\right]^{\mathrm{T}}$; $\varepsilon_i^{(j)}$ are error components.

Table 5.

Results of a simultaneous adjustment of the $\overline{C}_{2m}$, $\overline{S}_{22}$ parameters to the IERS reference pole fixed by the mean pole coordinates $\overline{x}_p = 0.054''$, $\overline{y}_p = 0.357''$ at epoch 2000 (zero-frequency-tide system; GM=398600.4415 km^3/s^2; a=6378136.49 m)

Parameter	4 models: EGM2008, ITG-GRACE03S, GGM03S, EIGEN-GL04S1	2 models: EGM2008, ITG-GRACE03S
$\overline{C}_{20}\cdot 10^6$	-484.16929419 ± 0.000020	-484.169288549 ± 0.000023
$\overline{C}_{21}\cdot 10^6$	-0.00022261 ± 3.1 ·10^{-11}	-0.00022261 ± 4.0 ·10^{-11}
$\overline{S}_{21}\cdot 10^6$	0.00144761 ± 6.6·10^{-11}	0.00144761 ± 7.7·10^{-11}
$\overline{C}_{22}\cdot 10^6$	2.43937396 ± 0.000016	2.439383442 ± 0.000022
$\overline{S}_{22}\cdot 10^6$	-1.40028032 ± 0.000017	-1.40027366 ± 0.000022

The orthogonal matrix $\mathbf{R}_{\theta\lambda}^{\mathrm{T}}$ of this system depends only through Eqs. (3-7) on the mean pole $\overline{x}_p = 0.054''$ and $\overline{y}_p = 0.357''$ at epoch 2000. The vector $\mathbf{g}$ results from the solution of the normal system following from Eq. (61) with the two additional conditions, i.e. zero left hand sides in Eqs. (36).Taking for all (l=4) gravity models the harmonic coefficients $\overline{A}_{2m}^{(j)}$ and $\overline{B}_{2m}^{(j)}$ in the $X'\,Y'\,Z'$ frame as observations, we get in this way our basic set of adjusted $\overline{C}_{2m}$, $\overline{S}_{2m}$ - coefficients to IERS Reference pole (at the epoch 2000), given in Table 5 in the first column. Second solution from Table 5 (l=2, second column) based on the two EGM2008 and ITG-GRACE03S gravity field models was developed in the same manner only for the comparison of adjusted to the IERS 2003 pole sets of $\overline{C}_{2m}$, $\overline{S}_{2m}$ and corresponding accuracy estimates. Note that initial set of harmonic coefficients for the construction of the EGM2008 model was taken from the ITG-GRACE03S solution and small differences between $\overline{C}_{2m}$, $\overline{S}_{2m}$ in Table 1 (excluding $\overline{C}_{21}$ and $\overline{S}_{21}$ of EGM2008 adopted according to the IERS Conventions 2003) reflect the influence of ‘the inclusion of the surface gravity data into the least-squares adjustment’.

Both sets of these coefficients from Table 5 restore exactly the adopted mean pole coordinates $\overline{x}_p = 0.054''$ and $\overline{y}_p = 0.357''$, if inserted into the following expressions based on Eqs. (5 – 7):

$$\overline{x}_p = \frac{(\sqrt{3}\overline{C}_{20} + \overline{C}_{22})\overline{C}_{21} + \overline{S}_{22}\overline{S}_{21}}{3\overline{C}_{20}^2 - \overline{C}_{22}^2 - \overline{S}_{22}^2}, \quad (62a)$$

$$\overline{y}_p = -\frac{(\sqrt{3}\overline{C}_{20} - \overline{C}_{22})\overline{S}_{21} + \overline{S}_{22}\overline{C}_{21}}{3\overline{C}_{20}^2 - \overline{C}_{22}^2 - \overline{S}_{22}^2}. \quad (62б)$$

Applying the exact equations to the first set of adjusted $\overline{C}_{2m}$, $\overline{S}_{2m}$, the orientation of the principal axes $\overline{A}$, $\overline{B}$, and $\overline{C}$ are computed for each individual gravity field model and for the adjusted second-degree coefficients. The results are given in spherical coordinates in Table 6 and for the axis *C* also in polar coordinates (Table 7).

Table 6.

Spherical coordinates of the principal axes and their accuracy (epoch 2000)

Gravity field model	Lat. $\overline{A}$ [degree]	Lon. $\overline{A}$ [degree]	Lat. $\overline{B}$ [degree]	Lon. $\overline{B}$ [degree]	Lat. $\overline{C}$ [degree]	Lon. $\overline{C}$ [degree]
EGM2008	-0.000038 ±0.0000005	345.0715 ±0.0001	0.000088 ±0.000005	75.0715 ±0.0001	89.999904 ±0.0000005	278.3486 ±0.2885
ITG-GRACE03S	-0.000043 ±0.0000004	345.0715 ±0.0001	0.000093 ±0.0000004	75.0715 ±0.0001	89.999897 ±0.0000004	280.053074 ±0.2328
GGM03S	-0.000038 ±0.0000005	345.0711 ±0.0001	0.000088 ±0.0000005	75.0711 ±0.0001	89.999904 ±0.0000005	278.3476 ±0.3180
EIGEN-GL04S1	-0.000040 ±0.000001	345.0713 ±0.0002	0.000087 ±0.000001	75.0713 ±0.0002	89.999904 ±0.000001	279.8118 ±0.6604
Adjusted $\overline{C}_{2m}$, $\overline{S}_{2m}$ (4 models)	-0.000040 $\pm 0.3\cdot 10^{-9}$	345.0714 ±0.0002	0.000092 $\pm 0.1\cdot 10^{-9}$	75.0714 ±0.0002	89.999900 $\pm 0.6\cdot 10^{-11}$	278.6014 $\pm 0.2\cdot 10^{-5}$

Table 7.

Polar coordinates of the principal axis $\overline{C}$ and their accuracy (epoch 2000)

Gravity field model	x_C [0.001"]	y_C [0.001"]
EGM2008	50.1±1.7	341.4±1.8
ITG-GRACE03S	64.5±1.5	363.8±1.6
GGM03S	50.1±1.9	341.4±1.9
EIGEN-GL04S1	58.7±4.0	339.5±4.0
Adjusted $\overline{C}_{2m}$, $\overline{S}_{2m}$ (Table 5, 4 models)	54.0 $\pm 0.1\cdot 10^{4}$	357.0 $\pm 0.2\cdot 10^{4}$

It should be pointed out, that such 'high' accuracy of $\overline{C}_{21}$, $\overline{S}_{21}$ in Table 5 are result from the application of the mentioned conditions $\overline{A}_{21} = \overline{B}_{21} = 0$ [Eqs. (36)]. Accuracy of x_c and y_c in Table 7 for the adjusted harmonic coefficients $\overline{C}_{2m}$, $\overline{S}_{2m}$ and accuracy of the corresponding latitudes of the principal axes $\overline{A}$, $\overline{B}$, and $\overline{C}$ (Table 6) again reflect the mentioned influence of conditions $\overline{A}_{21} = \overline{B}_{21} = 0$ which were initially introduced via adjustment to the adopted IERS reference pole fixed at epoch 2000 by the mean pole coordinates in Eq. (36).

After transformation of adjusted $\overline{C}_{2m}$, $\overline{S}_{2m}$ based on four models to the principal axes system in view of accuracy estimation we get comparable numerical values with the coefficients $\overline{A}_{20}$, $\overline{A}_{22}$ (S1) of Table 3. Hence, their combination with the adjusted $H_D = 0.0032737850 \pm 0.0000000072$ gives similar values for other parameters of the solution S1 in Table 3. Therefore, the first columns of Table 3 and Table 5 can be considered as one consistent set of the Earth's fundamental parameters at epoch 2000 given in the principal axes and the Earth's-fixed systems, respectively. In comparison with previous results [Marchenko and Schwintzer, 2003] based on Eq. (1) we get generally slightly better accordance between the adjustment of astronomical and geodetic constants and the separate adjustment of the 2nd harmonic coefficients only to the IERS reference pole. But differences between adjusted $\overline{C}_{2m}$, $\overline{S}_{2m}$ based on Eqs. (1 – 3) and Eqs. (61) have values about 10^{-15} that corresponds to the non-zero $I_1 = \mathrm{Trace}(\mathbf{H}')$ in the case of the traditional Lambeck's approach [Eqs. (1 – 3)].

Earth's time-dependent parameters from GRACE

The time-dependent 2nd-degree harmonic coefficients $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$ were taken from the International Center for Global Earth's Models of the IAG and extracted for the following GRACE time series: CNES-GRGS, CSR Release 04, GFZ Release 04, JPL Release 04.1, and ITG-GRACE03S time-dependent solution [Mayer-Gürr, 2007]. These $\overline{C}_{2m}$, $\overline{S}_{2m}$ with a step size from 10 days (CNES-GRGS) to one month (other solutions) were applied to the direct computation of temporally evolving components of the Earth's inertia tensor and other associated parameters on the time period from 2002.3 to 2008.5 years.

To be consistent the following transformations were used to values $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$ as in the case of time-independent constituent: (1) reduction of $\overline{C}_{20}$ to a common zero-frequency tide system (Eq. (53)), and (2) scaling of these coefficients to common values of GM=398600.4415 km^3/s^2 and a=6378136.49 m. Taking into account the adjusted dynamical ellipticity $H_D = 0.0032737850$, all parameters listed in Table 3, Table 6, and Table 7 were determined now as time-dependent for each related moment of time according to these four solutions on the total period from 2002.3 to 2008.5 years.

Because of a great number of various parameters computed for each moment of time t we give only their evolution for the axes $\overline{A}$, $\overline{B}$, and $\overline{C}$ of inertia. For other illustrations it is sufficient to give only mean values of some time-dependent quantities obtained by averaging their instant values on the given time-period from 2002.3 to 2008.5 years. Fig. 1 and Fig.2 show temporal changes from GRACE of longitudes of the axes $\overline{A}$, $\overline{B}$, and $\overline{C}$.

Table 8 demonstrates mean longitudes of these axes and mean values of the angle $\tilde{\gamma}$ (Eq. (47)) between two quadrupole axes, located in the plane of the axes $\overline{A}$ and $\overline{C}$ of inertia. Table 9 lists obtained average values of polar coordinates of the figure axis $\overline{C}$ for the same period related to the corresponding mean epochs about 2005.

A comparison of each initial $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$ taken from various centers of analysis leads to the conclusion about systematic differences existing in these series. Fig. 2, Table 8, and Table 9 reflect these probable systematic trends in five deter-

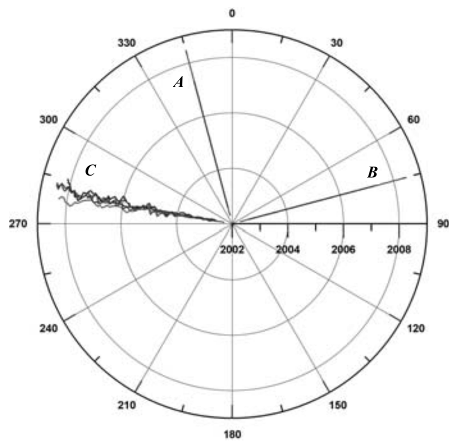

Fig. 1. Longitude of the axes $\overline{A}$, $\overline{B}$, $\overline{C}$ of inertia from CNES-GRGS (—), CSR (—), GFZ (—), JPL (—), and ITG-GRACE03S (—) time series for the period from 2002.3 to 2008.5 years

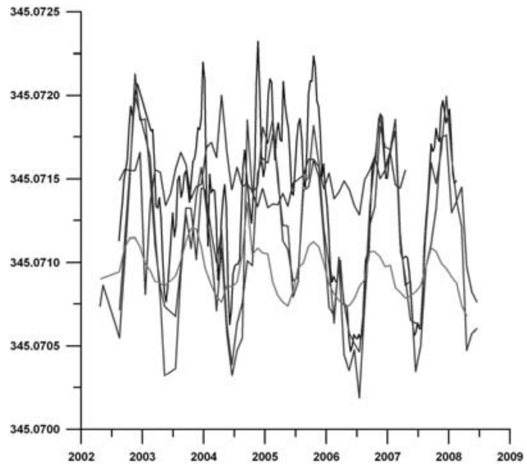

Fig. 2. Longitude of the axis $\overline{A}$ of inertia from CNES-GRGS (—), CSR (—), GFZ (—), JPL (—), and ITG-GRACE03S (—) time series for the period from 2002.3 to 2008.5 years

Table 8.

Mean longitudes of the principal axes $\overline{A}$, $\overline{C}$ and mean values of the angle $\tilde{\gamma}$ between two quadrupole axes (Eq. (47)) (period from 2002.3 to 2008.5 years)

Mean values	Longitude $\overline{A}$	Longitude $\overline{C}$	Angle $\tilde{\gamma}$ [Eq. 47]
CNES-GRGS	345.0714 ±0.00005	281.0880 ± 0.16	170.61988° ±0.000008
CSR Release 04	345.0711 ±0.00002	279.4887 ± 0.09	170.61988° ±0.000003
GFZ Release 04	345.0712 ±0.00001	280.5852 ± 0.05	170.61988° ±0.000002
JPL Release 04.1	345.0709 ±0.00001	278.7286 ± 0.02	170.61985° ±0.000001
ITG-GRACE03	345.0715 ±0.00006	280.0541 ± 0.23	170.61986° ±0.000010

Table 9.

Mean coordinates of the figure axis $\overline{C}$ for the period from 2002.3 to 2008.5 years

Gravity field model	Mean epoch [year]	x_C [0.001"]	y_C [0.001"]
CNES-GRGS	2005.46	70.2 ±1.0	358.5 ±1.0
CSR Release 04	2005.48	56.2 ±0.5	336.6 ±0.5
GFZ Release 04	2005.72	61.3 ±0.3	328.4 ±0.3
JPL Release 04.1	2005.44	52.7 ±0.2	343.0 ±0.2
ITG-GRACE03	2004.96	64.5 ±1.5	363.8 ±1.6

minations of the time-dependent coefficients $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$. Nevertheless some derived parameters illustrated by Table 8 and Fig. 1 are generally permanent taking into account accuracy estimation of their static part. In contrast to the evident temporal change of the figure axis $\bar{C}$ (Fig. 1) we get a remarkable stability in time of the position of the inertia axes $\bar{A}$ and $\bar{B}$ derived from GRACE (Fig. 1). Processing of the CHAMP quarterly solutions [Reigber et al., 2003] for $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ (period from 2000.9 to 2003.4 years) produces the same conclusion about stability of the axis $\bar{A}$ (and $\bar{B}$) with the mean longitude $\lambda_{\bar{A}} = 345^{\circ}.0706\mathrm{E}$. In addition, we get a similar accordance with previous results [Marchenko, 2007] based on such GRACE time series as CSR Release 01, GFZ Release 03, and JPL Release 02, which allow the same general conclusion excluding small differences in relation to values from Table 8. It has to be pointed out, that the direction of the principal axis $\bar{A}$ is considered in the precession-nutation theory [Bretagnon et al., 1998; Roosbeek and Dehant, 1998] as the parameter of the Earth's triaxiality or the longitude $\lambda_{\bar{A}}$ of the major axis of the equatorial ellipse. Thus, the adjusted to the IERS reference pole at the epoch 2000 numerical value $\lambda_{\bar{A}} = 345^{\circ}.0714\mathrm{E} \pm 0^{\circ}.0002$ (Table 6) in terms of accuracy estimation agrees perfectly with those from Table 8 and may be recommended for the Earth's rotation theory: $\lambda_{\bar{A}} = = 14^{\circ}.9286\mathrm{W} \pm 0^{\circ}.0002$. But a most stable value represents another parameter of the Earth's triaxiality, $\tilde{\gamma} = 170.6199^{\circ}$, the angle between the quadrupole axes.

If orientation of the Earth's principal axes of inertia, the angle $\tilde{\gamma}$, and some other parameters depend only on the $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ coefficients, the determination of temporal changes of the Earth's tensor of inertia requires according to Eqs. (48 – 49) the dynamical ellipticity H_D. To compute the principal moments of inertia *A*, *B*, and *C* (Eqs. (48 – 49)) from adjusted $H_D = 0.0032737850$ (related to J2000) and GRACE $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ at each given moment *t*, which is different from the standard epoch 2000, an additional correction δH to H_D should be applied. Special study of the $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ GRACE series led to a non-stable determination of the secular variation of $\dot{\bar{A}}_{22}$ adopted finally as $\dot{\bar{A}}_{22} = 0$. Therefore, we assume the non-tidal variation δC in the moment of inertia *C* as a function of $\dot{\bar{C}}_{20}$ only [Yoder et al., 1983] as zonal forces do not change the revolution shape of the body' [Melchior, 1978] and come to $\delta A = \delta B = -\delta C/2$ from the condition for the trace $\mathrm{Tr}(\mathbf{I}) = \mathrm{const}$ [Rochester and Smylie, 1974]. By this we get from Eqs. (48) and (49) the secular change of H_D:

$$\dot{H}_D = \frac{\dot{\bar{A}}_{20}}{\bar{A}_{20}} H_D (1 - \frac{2}{3} H_D), \quad (63a)$$

$$\dot{H}_D = -\sqrt{5}\dot{\bar{A}}_{20} \frac{\mathrm{Trace}(\mathbf{I})}{3C^2} \approx \frac{\dot{\bar{C}}_{20}}{\bar{C}_{20}} H_D (1 - \frac{2}{3} H_D), \quad (63б)$$

if secular variations in different coordinate system are equal $\dot{\bar{A}}_{20} = \dot{\bar{C}}_{20}$. To verify this equality we will use Eq. (32b) written for the time-dependent harmonic coefficients $\bar{C}_{nm}(t)$, $\bar{S}_{nm}(t)$, and $\bar{A}_{n0}$. Differentiation of $\bar{A}_{n0}$, $\bar{C}_{nm}(t)$, $\bar{S}_{nm}(t)$ in Eq. (32b) with respect to time *t* gives

$$\dot{\bar{A}}_{n0} = \sum_{m=0}^{n} (\dot{\bar{C}}_{nm} \cos m\lambda_C + \dot{\bar{S}}_{nm} \sin m\lambda_C) \tilde{P}_{nm}(\cos\theta_C), \quad (64)$$

the equation for the reduction of the given $\dot{\bar{C}}_{2m}, \dot{\bar{S}}_{2m}$ to the unknown $\dot{\bar{A}}_{20}$ through the polar coordinates θ_C and λ_C of the figure axis $\bar{C}$ which are considered in Eq. (64) as time-independent and known at fixed epoch. With $\dot{\bar{C}}_{20}$, $\dot{\bar{C}}_{21}$, $\dot{\bar{S}}_{21}$ taken from Eqs. (56 – 57), $\dot{\bar{C}}_{22} = \dot{\bar{S}}_{22} = 0$, and the position θ_C and λ_C of the axis $\bar{C}$ supposed to coincide with the mean pole coordinates at epoch 2000 (IERS Conventions 2003), we get from Eq. (64) the estimation $\dot{\bar{A}}_{20} = 1.162795 \cdot 10^{-11}\, 1/\mathrm{yr} \approx \dot{\bar{C}}_{20}$ which is slightly differed from $\dot{\bar{C}}_{20}$ (Eq. (57)) on the smaller value $\delta = -0.5 \cdot 10^{-16}$ than accuracy estimates of $\dot{\bar{C}}_{20}$ and other temporal variations.

That is why we neglect this correction δ and get numerically $\dot{H}_D = -7.8453 \cdot 10^{-11}\,\mathrm{yr}^{-1}$ with $\dot{\bar{C}}_{20}$ taken from Eq. (57). This amounts to $\delta H = \dot{H}_D (t - t_0)$ for the reduction of H_D from the year t_0=2000 to each moment *t* related to $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ of GRACE time series. (Note that $\delta H = -7.1 \cdot 10^{-10}$ for the reduction of H_D from the year 2000 to 2009). Then, applying for parameters connected with $\dot{\bar{C}}_{20} = 1.1628 \cdot 10^{-11}\,\mathrm{yr}^{-1}$ the following linear dependence

$$\delta F(t) = \dot{F}(t - t_0), \quad (65)$$

where $\dot{F} = \frac{dF(t)}{dt}$ and t_0 is chosen reference epoch, we give in Table 10 using Eq. (65) the corresponding estimates of different secular changes according to (Marchenko, 2007).

It has to be pointed out that similar estimates of secular changes $\dot{H}_D = -7.86 \cdot 10^{-11}\,\mathrm{yr}^{-1}$ and

Table 10.
Secular changes in some astronomical and geodetic parameters corresponding to the secular drift in the coefficient $\bar{A}_{20} \approx \bar{C}_{20}$ (t_0=2000) [Marchenko, 2007]

Parameter	$\delta F(t) = \dot{F}(t-t_0)$	$\dot{F}$
$\bar{A}_{20} \approx \bar{C}_{20}$	$\delta\bar{A}_{20} = \dot{\bar{A}}_{20}(t-t_0)$	$\dot{\bar{A}}_{20}$ =1.1628·10^{-11} [1/yr]
H_D	$\delta H_D = -\sqrt{5}\dot{\bar{A}}_{20}\frac{\mathrm{Trace}(\mathbf{I})}{3C^2}(t-t_0)$	$\dot{H}_D$ =–7.8453·10^{-11}[1/yr]
p_A	$\delta p_A = \left[\dot{H}_D \Big/ \frac{\delta H_D}{\delta p_A}\right](t-t_0)$	$\dot{p}_A$ =–0.0121 [″/cy^2]
A	$\delta A = \frac{\sqrt{5}\dot{\bar{A}}_{20}}{3}(t-t_0)$	$\dot{A}$ =0.8667·10^{-11} [1/yr]
B	$\delta B = \frac{\sqrt{5}\dot{\bar{A}}_{20}}{3}(t-t_0)$	$\dot{B}$ =0.8667·10^{-11} [1/yr]
C	$\delta C = -\frac{2\sqrt{5}\dot{\bar{A}}_{20}}{3}(t-t_0)$	$\dot{C}$ =–1.7334·10^{-11} [1/yr]
$\alpha = \frac{C-B}{A}$	$\delta\alpha = -\frac{\sqrt{5}\dot{\bar{A}}_{20}(C-B+3A)}{3A^2}(t-t_0)$	$\dot{\alpha}$ =–7.8970·10^{-11} [1/yr]
$\beta = \frac{C-A}{B}$	$\delta\beta = -\frac{\sqrt{5}\dot{\bar{A}}_{20}(C-A+3B)}{3B^2}(t-t_0)$	$\dot{\beta}$ =–7.8968·10^{-11} [1/yr]
$\gamma = \frac{B-A}{C}$	$\delta\gamma = \frac{\sqrt{5}\dot{\bar{A}}_{20}(B-A)}{3C^2}(t-t_0)$	$\dot{\gamma}$ =5.7552·10^{-16} [1/yr]
f	$\delta f = -\frac{3\sqrt{5}\dot{\bar{A}}_{20}}{2}(t-t_0)$	$\dot{f}$ =–3.9001·10^{-11} [1/yr]

Table 11.
Mean values of the principal moments of inertia *A*, *B*, and *C* from the GRACE series of $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$, adjusted $H_D = 0.0032737850$, and $\dot{H}_D = -7.8453\cdot10^{-11}\,\mathrm{yr}^{-1}$ (for the period from 2002.3 to 2008.5 years)

Mean values	Mean epoch [year]	Principal moment *A*	Principal moment *B*	Principal moment *C*
CNES-GRGS	2005.46	0.32961228	0.32961954	0.33069855
CSR Release 04	2005.48	0.32961220	0.32961946	0.33069846
GFZ Release 04	2005.72	0.32961215	0.32961941	0.33069841
JPL Release 04.1	2005.44	0.32961220	0.32961947	0.33069847
ITG-GRACE03	2004.96	0.32961217	0.32961943	0.33069843

$\dot{H}_D = -7.4\cdot10^{-11}\,\mathrm{yr}^{-1}$ were found under the same condition to conserve changes in the trace Trace(**I**) of inertial tensor by [Marchenko and Schwintzer, 2003] and [Bourda and Capitaine, 2004] respectively. Small differences in all $\dot{H}_D$-values are explained by the application of various sets of chosen constants entering in Eq. (63).

Among parameters from Table 10 all secular changes have the same order as variation $\dot{\bar{A}}_{20}$ excluding $\dot{\gamma}$ and $\dot{p}_A$. According to [Marchenko, 2007] the variation $\dot{p}_A$ was called by the $\dot{J}_2$ precession rate with the estimated range (–11.6 to –16.8)×10^{-3} [″/centuries^2], which is depended on the adopted $\dot{J}_2 = -\sqrt{5}\dot{\bar{C}}_{20}$ having the value 'about 0.7% classical acceleration induced by ecliptic motion and two orders of magnitude larger than tidally induced accelerations'. Williams' $\dot{J}_2$ precession rate $\dot{p}_A$ = –0.014 [″/cy^2] was based on the old determinations of the variation $\dot{J}_2$. Nevertheless his estimation given in 1994 agrees well with those from Table 10. Because the derived value $\dot{p}_A = -0.012$ [″/cy^2] was based on Eq. (58) and the secular variation $\dot{\bar{C}}_{20} = 1.1628\cdot10^{-11}\,\mathrm{yr}^{-1}$ adopted for recent gravity field models this parameter also may be recommended for the Earth's rotation theory.

Table 11 illustrates mean values of the principal moments of inertia *A*, *B*, and *C* derived from the $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$ GRACE series, the adjusted dynamical ellipticity $H_D = 0.0032737850$, and the secular change $\dot{H}_D = -7.8453\cdot10^{-11}\,\mathrm{yr}^{-1}$ by averaging the instant values *A*(*t*), *B*(*t*), and *C*(*t*) on given time-period. Taking into account the previous results by [22] based on such GRACE time series as CSR Release 01, GFZ Release 03, and JPL Release 02, the comparison of the GRACE only principal moments of inertia from Table 11 with the adjusted quantities *A*, *B*, and *C* given in Table 3 leads to a good agreement in terms of accuracy estimation in all cases of $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$ GRACE series. Nevertheless, only secular variations in the 2nd degree time-dependent GRACE coefficients are not sufficient for the description of $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$-changes. For example, another representation for the series of $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$ was introduced by adopting

Table 12.

Contribution of nearly annual time variations of time-dependent parameters given in percentages to common periodic changes

Parameter	CNES-GRGS	CSR-r104	GFZ-r104	JPL-r104.1	ITG-GRACE03
Parameters in the principal axes system					
A	54%	24%	33%	39%	45%
B	54%	25%	33%	39%	46%
C	54%	22%	33%	39%	48%
$\overline{A}_{20}$	54%	23%	33%	38%	50%
$\overline{A}_{22}$	41%	37%	44%	85%	32%
$\widetilde{\gamma}$	42%	40%	48%	85%	51%
Longitudes of the principal axes $\overline{A}$ and $\overline{C}$					
$\lambda_{\overline{A}}$	51%	59%	45%	–	5%
$\lambda_{\overline{C}}$	72%	24%	41%	60%	38%

the model of secular, annual, and semi-annual periodic variations, based on the EIGEN-GL04S static gravity field model and the GRACE 10-days solutions [Lemoine et al., 2007]. Taking into account that time-dependent parameters from Table 8 and Table 11 depend on $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$-coefficients, these are then analyzed after removing a linear trend for the detection of basic periods derived from a spectral analysis using the following model

$$F(t) = F_0 + \dot{F}(t-t_0) + \sum_i A_i \cos\left(\frac{2\pi}{P_i}(t-t_0) - \phi_i\right), \quad (66)$$

for time-dependent function $F(t)$ with the simultaneous determination of all components A_i, ϕ_i, and P_i of an oscillation, including periods P_i. As a result, close to annual and semi-annual terms among estimated periods were observed with common contributions more than 50% in all determinations. Table 12 reflects the contribution of nearly annual variations only into common periodic changes, which are different for various centers of analysis. Thus, although exist some basic part of discussed parameters given in Table 8 and Table 11 we detect their small deviations having annual, semi-annual, and other terms. On the other hand, mean values of these parameters agreed well with their 'static' values from Table 3 and can be considered as some permanent constituents given at the corresponding mean epochs.

Conclusions

In order to avoid uncertainty in the deviatoric part **H** of inertia tensor the transformation of the second-degree harmonic coefficients $\overline{C}_{2m}, \overline{S}_{2m}$ was developed especially for the case of a finite commutative rotation via modified Lambeck's formulae applied to polar coordinates considered at the sphere. The modified Lambeck's approach allows simple transformation of the 2nd-degree harmonic coefficients and zonal coefficients of an arbitrary degree (including their temporal changes) via orthogonal matrixes. This transformation was used in the two individual adjustments of the geodetic only parameters $\overline{C}_{2m}, \overline{S}_{2m}$ of four gravity field models adopted in the Earth's-fixed system to the IERS reference pole given by the conventional mean pole coordinates $\overline{x}_p = 0.054''$ and $\overline{y}_p = 0.357''$ at epoch 2000 (IERS Conventions 2003). The same sets of $\overline{C}_{2m}, \overline{S}_{2m}$-coefficients together with eight values of the dynamical ellipticity H_D all reduced to the common MHB2000 precession constant $p_A = 50.2879225''/\mathrm{yr}$ were used in the two general adjustments given in the principal axes system with respect to the Earth's principal moments of inertia *A*, *B*, and *C*. Results of the first adjustment of geodetic and astronomical 'constants' represent one set (S1) of consistent parameters given in Table 3, Table 5, and Table 6 at one chosen epoch J2000 as time-independent constituent of the orientation of principal axes in the Earth's-fixed system, the principal moments (*A*, *B*, *C*) of inertia, H_D, coefficients in the Eulerian dynamical equations, and other associated parameters. The second solution contains the same parameters based on the same four sets of $\bar{C}_{2m}, \bar{S}_{2m}$ and only one H_D from the MHB2000 model and corresponds better to the frequently used IERS Conventions 2003 and latest gravity field determinations.

Time-dependent components of the Earth's tensor of inertia were found from the time-dependent $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ GRACE time series of the following five solutions: CNES-GRGS; CSR Release 04; GFZ Release 04; JPL Release 04.1; ITG-GRACE03S. The condition $\delta A = \delta B = -\delta C/2$ to conserve Trace(**I**) of the inertia tensor when changing the dynamical ellipticity H_D from the reference epoch t_0=2000 to a current moment of time t was applied via variation $\dot{H}_D$ for the estimation of $\dot{H}_D$, $\dot{J}_2$ precession constant rate, and other parameters. These estimations are based on the modified Lambeck's approach and derived closed expression for the reduction of the $\dot{\bar{C}}_{2m}, \dot{\bar{S}}_{2m}$ secular variations related to the standard Earth-fixed system to the unknown $\dot{\bar{A}}_{20}$ related to the figure axis $\bar{C}$ through the polar coordinates θ_C and λ_C of the axis $\bar{C}$, which were fixed for the epoch 2000. Estimation of $\dot{\bar{A}}_{20} = 1.162795 \cdot 10^{-11}$ 1/yr secular variation leads to a slightly different from $\dot{\bar{C}}_{20}$ value. It has to be pointed out that mean values of the principal moments *A*, *B*, and *C* of inertia given at the epoch about 2005 based only on the $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ GRACE series, H_D, and $\dot{H}_D$ agree well with the adjusted quantities *A*, *B*, and *C* at the epoch J2000.

A stability in time of the position of the axes $\bar{A}$ and $\bar{B}$ of inertia and the angle $\tilde{\gamma}$ between two quadrupole axes, located in the plane of the axes $\bar{A}$ and $\bar{C}$, was observed from the time-dependent $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ GRACE time series. Since the longitude $\lambda_{\bar{A}}$ of the principal axis $\bar{A}$ is considered in the nutation theory as the parameter of the Earth's triaxiality, the estimated value $\lambda_{\bar{A}} = 14^{\circ}.9286\mathrm{W} \pm 0^{\circ}.0002$ can be recommended for the Earth's rotation theory together with the $\dot{J}_2$ precession rate $\dot{p}_A = -0.012$ [″/cy^2] of the precession constant p_A. Nevertheless, periodic components at seasonal and shorter time scale were evaluated for the detection of basic periods derived from a spectral analysis. As a result, nearly annual and semi-annual terms among estimated periods were observed with common contributions more than 50% in all determinations. Hence, although exist some permanent constituents of discussed parameters (as mean values at mean epoch) their small deviations have also stable terms with about annual and semi-annual periods, which are different for various centers of analysis.

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ВИЗНАЧЕННЯ ТЕНЗОРА ІНЕРЦІЇ ЗЕМЛІ ЗА СУЧАСНИМИ ДАНИМИ АСТРОНОМІЇ ТА ГЕОДЕЗІЇ

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Перетворення гармонічних коефіцієнтів другого порядку $\bar{C}_{2m}$ та $\bar{S}_{2m}$ було обчислено через скінченний поворот замість традиційного наближення Ламбека, основаного на нескінченно малих поворотах. Модифікована формула Ламбека виключає невизначеність в девіаторній частині тензора інерції і дозволяє найпростіше перетворення гармонічних коефіцієнтів другого порядку та зональних коефіцієнтів довільних порядків (включаючи часові варіації) через ортогональні матриці. Ці формули разом з строгим розв'язком задачі на власні числа застосовані для визначення статичної складових тензора інерції Землі та їх точності з врівноваження гармонічних коефіцієнтів $\bar{C}_{2m}$, $\bar{S}_{2m}$ другого порядку в головних осях інерції для чотирьох моделей гравітаційного поля (EGM2008, GGM03S, ITG-GRACE03S та EIGEN-GL04S1) та восьми величин динамічного стиску H_D, приведених до єдиної прецесійної константи MHB2000 на епоху J2000. Другий розв'язок складається з кількох параметрів, основаних на цих чотирьох наборах $\bar{C}_{2m}, \bar{S}_{2m}$ і тільки одному значенні H_D з MHB2000 моделі. З двох розв'язків для статичної компоненти отримано п'ять врівноважених гармонічних коефіцієнтів другого порядку, напрям головних осей, головні моменти інерції (A, B, C) та інші параметри. Зміну з часом цих статичних параметрів оцінено в головних осях системи з часових рядів $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ супутника GRACE, які отримані з п'яти різних центрів аналізу на інтервалі з 2002 по 2008р. Особлива увага надається визначенню часових варіацій головних осей та моментів інерції, обчислених на основі $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ та оцінці їх середніх значень разом з періодичними складовими на даний період. Стабільність положення екваторіальної осей інерції ($\bar{A}$, $\bar{B}$) та кут між двома квадрупольними осями, який розміщений в площині осей інерції $\bar{A}$ та $\bar{C}$. Оскільки довгота $\lambda_{\bar{A}}$ головної осі інерції $\bar{A}$ розглядається в теорії нутації як параметр трьохосності Землі, то отримане значення можна рекомендувати для теорії обертання Землі разом з динамічним параметром другого порядку $\dot{J}_2$ прецесії p_A .

Ключові слова: тензор інерції Землі; головні осі інерції; головні моменти інерції, наближення Ламбека.

ОПРЕДЕЛЕНИЕ ТЕНЗОРА ИНЕРЦИИ ЗЕМЛИ ПО СОВРЕМЕННЫМ ДАННЫМ АСТРОНОМИИ И ГЕОДЕЗИИ

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Преобразование гармонических коэффициентов второго порядка $\bar{C}_{2m}$ та $\bar{S}_{2m}$ в случае конечного коммутативного поворота было вычислено вместо традиционного приближения Ламбека, основанного на бесконечно малых поворотах. Модифицированная формула Ламбека исключает неопределенность в девиаторной части тензора инерции и делает возможным простое преобразование гармонических коэффициентов второго порядка и зональных коэффициентов разных степеней (в том числе временные вариации) через ортогональные матрицы. Эти формулы вместе с точным решением задачи на

собственные матрицы-векторы применяют для определения постоянных компонент тензора инерции Земли, а также их точности с уравнения гармонических коэффициентов второго порядка $\overline{C}_{2m}, \overline{S}_{2m}$, полученных по четырем моделям гравитационного поля (EGM2008, GGM03S, ITG-GRACE03S та EIGEN-GL04S1) и восьми значениям динамического сжатия H_D, приведенных к прецессийной константе MHB2000 на эпоху J2000.

Второе решение состоит из тех же параметров, полученных с уравнивания четырех наборов гармонических коэффициентов второго порядка $\overline{C}_{2m}, \overline{S}_{2m}$ и одного значения динамического сжатия H_D по модели MHB2000. Два решения для постоянной составной состоят из пяти уравненных гармонических коэффициентов второго порядка, направлений главных осей, главных моментов инерции (*A*, *B*, *C*) и других параметров. Изменение во времени постоянных параметров оценено в главных осях системы по часовым рядам $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$ GRACE, полученных в пяти разных центрах анализа на интервале с 2002 по 2008г. Особенное внимание приделяется определению временных вариаций главных осей и моментов инерции, вычисленных по $\overline{C}_{2m}(t)$, $\overline{S}_{2m}(t)$ и оценке точности средних значений вместе с периодическими составными. Найдена стабильность положения экваториальных осей инерции ($\overline{A}$, $\overline{B}$) и угол между двумя квадропольными осями, расположенный в плоскости осей инерции $\overline{A}$ и $\overline{C}$. Так как долгота $\lambda_{\overline{A}}$ главной оси инерции рассматривается в теории нутации как параметр трехосности Земли, то полученное значение можно рекомендовать для теории вращения Земли вместе с динамическим параметром второго порядка $\dot{J}_2$ прецессии p_A .

Ключевые слова: тензор инерции Земли; главные оси инерции; главные моменты инерции, приближение Ламбека.

Національний університет "Львівська політехніка", м. Львів Надійшла 25.05.2009