SYSTEMS APPROACH TO PROBLEMS OF THE DESIGN OF AUTOMATED TEST COMPLEXES.
REPORT 2. DETERMINATION OF SYSTEM PARAMETERS

V. G. Grishko

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At the present time, sufficient experience has been accumulated in the development of automated test complexes (ATC). Howsoever large the practical successes in developing ATC of some classes of investigations involving the mechanical properties of materials (MPM), they should not exclude the fact that further effective development of ATC is possible with the condition that the final design theory, which does not have well-defined formulations at the present time even with respect to basic positions, is created.

In familiar literature sources, problems of ATC design are treated as problems involving the engineering design of test machines, measuring systems, ASP, and systems for automated data processing independently of one another. Where, in this case, rational (in some sense) parameters are selected from some of these systems, this problem does not arise with other systems. One usually speaks of the selection of a serviceable system without a quantitative estimate of its quality. In those cases where optimization is accomplished, it is usually done so, as a rule, with a single dominating criterion. This approach gives rise to significant loss of efficiency as compared with the potentially possible efficiency.

In our first report, we proposed a method of selecting a rational ATC structure.

Optimization of ATC parameters, which is carried out from the position of a multicriterial evaluation within the framework of the solution of the following problem, is considered below:

the selection of a large number of criteria serving to estimate the ATC

$$D = \{d_1, d_2, \dots, d_N\}, \qquad D \neq \varnothing;$$

the formation of a large number of quality-intensity levels for D criteria

$$Q_{d\in D} = \{q_1, q_2, \dots, q_m\}, \qquad Q \neq \varnothing;$$

reflection of the quality-intensity levels in a large number of estimates

$$\delta: Q \to O, \text{where } O = \{o_1, o_2, \dots, o_n\}, \qquad O \neq \varnothing;$$

and, determination of the optimality principle and derivation of ATC parameters that satisfy this principle.

The problem of selecting rational (optimal) ATC parameters consists in defining a variant of the $^{\circ}\phi \in \Phi$ system in terms of a multicriterial estimate:

$$O(\Phi) = \{O(\Phi/d)\}_{d\in D}. \tag{1}$$

From the mathematical standpoint, the problem of multicriterial optimization is incorrect if the detection of an extremum is understood for the optimization, since the attainment of an extremum for a single criteria does not make it possible to find an extremum for the remaining criteria [1]. The optimization concept must therefore be defined more precisely.

In our study, the detection of an extremum for estimates of a set of ATC criteria in a preferred scalar with certain constraints will be understood as optimization. The latter is attained by solving the above-indicated problems.

Selection of Criteria. A large number of ATC-estimate criteria can be ordered in a hierarchic structure, one of the variants of which is shown in Fig. 1. The first level is represented by the criteria d_1, d_2, and d_3, which characterize the universality of the testing machines that enter into the ATC (d_1), the level of improvement in technical and program

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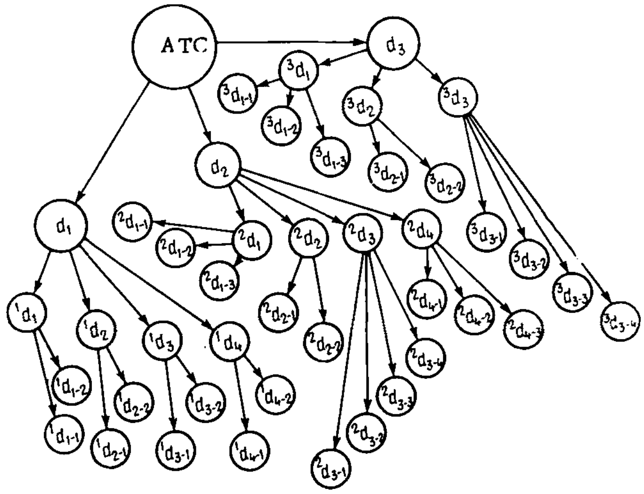

Fig. 1. Hierarchy of ATC-estimate criteria: d_1) universality of ATC; $^1d_1$) mechanical loads; $^1d_2$) thermal loads; $^1d_3$) chemical effects; $^1d_4$) radiation effects; $^1d_{i-1}$) dynamic range of test loads; $^1d_{i-2}$) range of loading rates; d_2) level of improvement in technical and program facilities of ASNI; d_3) degree of automation of control exercised over testing apparatus; $^2d_1$ ($^3d_1$)) data losses; $^2d_{1-1}$ ($^3d_{1-1}$)) error generated by measurement converters; $^2d_{1-2}$ ($^3d_{1-2}$)) computational error; $^2d_{1-3}$) error generated by method of data processing (inadequacy of model); $^3d_{1-3}$) ASP error; $^2d_2$ ($^3d_2$)) operational properties; $^2d_{2-1}$, $^3d_{2-1}$) production losses due to poor component reliability; $^2d_{2-2}$ ($^3d_{2-2}$)) probability of repeat run of algorithm; $^2d_3$ ($^3d_3$)) economic indicators; $^2d_{3-1}$ ($^3d_{3-1}$)) cost of technical facilities; $^2d_{3-2}$ ($^3d_{3-2}$)) cost of special software; $^2d_{3-3}$ ($^3d_{3-3}$)) cost of ATC reconfiguration with emergence of new research problems; $^2d_{3-4}$ ($^3d_{3-4}$)) operating expense; $^2d_4$) correspondence between ASNI parameters and characteristics of process under investigation; $^2d_{4-1}$) average value during which requirement should expect accommodation in turn; $^2d_{4-1}$) average value of number of requirements in system at any point in time.

facilities of the automated data-processing system for the ATC (d_2), and the degree of improvement in the automated-control system for the loading conditions of the testing machines ASP (d_3).

These criteria are detailed on the second and third levels of the hierarchy. The second level of criteria can be represented in the form of the following groups.

The criterion d_1, which determines the testing machine's potential in conducting investigations, is partitioned into elements:

mechanical effects ($^1d_1$), which are subdivided into uniaxial, biaxial, and triaxial effects in view of the stressed state in the specimen. In each stressed state, there are basic (tension, compression) and combined effects, which are derivatives of the basic effects (bending, shear, etc.);

physical effects ($^1d_2$), which include the effect of thermal stresses and electromagnetic radiation;

the external medium ($^1d_3$), which in terms of its own chemical properties is subdivided into chemically neutral, acidic, basic, and special (vapors of hydrogen, nitrogen, carbon, etc.) parts; and,

reactor radiation ($^1d_4$): γ-radiation and neutron radiation.

The criteria d_2 and d_3 include the following elements:

data losses $^2d_1$ and $^3d_1$;

operational properties ${}^2d_2$ and ${}^3d_2$;

economic indicators ${}^2d_3$ and ${}^3d_3$; and,

correspondence between the ASNI parameters and the characteristics of the process under investigation ${}^2d_4$.

Components of the second-level criteria are determined on the third level of the hierarchy.

Thus, each of the forms of energy effects can be characteized by two independent indicators: the dynamic range of the test loads (${}^1d_{i-1}$) and the range of variation in the loading rate (${}^1d_{i-2}$).

The error generated by the measurement converters (${}^2d_{1-1}$, ${}^3d_{1-1}$), the computational error (${}^2d_{1-2}$, ${}^3d_{1-2}$), the error induced by the data-processing method (${}^2d_{1-3}$), and the ASP error (${}^3d_{1-3}$) will be treated as components of the criteria ${}^2d_1$ and ${}^3d_1$.

The operational problems can be evaluated from two components: productivity losses owing to poor component reliability (${}^2d_{2-1}$, ${}^3d_{2-1}$), and the probability of a repeat run of the algorithm (${}^2d_{2-2}$, ${}^3d_{2-2}$).

The economic indicators include: the cost of technical facilities (${}^2d_{3-1}$, ${}^3d_{3-1}$), the cost of special software (${}^2d_{3-2}$, ${}^3d_{3-2}$), the cost of ATC reconfiguration with the emergence of new research problems (${}^2d_{3-3}$, ${}^3d_{3-3}$), and operating expenses (${}^2d_{3-4}$, ${}^3d_{3-4}$).

Correspondence between the ASNI parameters and the characteristics of the process under investigation: the average time during which a requirement should expect accommodation in turn (${}^2d_{4-1}$); and, the average value of the number of requirements in the system at any point in time (${}^2d_{4-2}$).

Construction of Criteria Scale. Practical use of the criteria under consideration is coupled with the need to determine the allowable transformations, which are solved for each criteria. The type of allowable transformations should therefore be considered in constructing the scales. The values of the criteria d_1, d_2, and d_3 can be represented as discrete ordinal scales. As we know, the principle of superposition is invalid for an ordinal scale since the distance between the values of the criteria X_1 and X_2 does not equal the distance between the values X_2 and X_3. The type of scale in question permits, however, certain statistical operations, to wit: determination of the frequencies, modes, median, and coefficient of rank correlation.

The d_1 scale will be represented by the range 0-1; the scale values can be defined as the ratio of the number of load forms that can be realized to the number of load forms desirable for the class of problems being solved.

The scale of criterion d_2 can be ordered in the form of the following gradations: ${}^2q_1$ denotes manual processing of experimental data, ${}^2q_2$ recording of experimental data on a data-storage medium with subsequent processing on a computer, ${}^2q_3$ rapid analysis of experimental data, which can be performed in real time, ${}^2q_4$ secondary processing of experimental data, and ${}^2q_5$ complex processing, which can be carried out in the cycle of a test series.

The form of processing in question assumes the construction of models, the prediction of strength properties of the material under investigation in regions of factored space, which are not encompassed by the experiment, confirmation of the range of applicability of existing strength theories, etc.

The ordinal d_3 scale can be ordered in the form of the following values: ${}^3q_1$ denotes manual control of the experimental apparatus, ${}^3q_2$ automatic stabilization of the parameters of the loading factors, ${}^3q_3$ automatic program control of certain forms of energy effects, ${}^3q_4$ automatic program control of basic forms of energy effects of the testing machines, ${}^3q_5$ adaptive control of certain parameters of the energy effects, ${}^3q_6$ finite control of certain forms of energy effects, etc.

The criteria d_1, d_2, and d_3 actually determine the possibility of attaining or not attaining the goal of the investigations. Criteria of the second and third levels of the hierarchy characterize the quality of goal attainment for each scale value of the criteria d_1, d_2, and d_3.

Two scales will be formulated for each criteria ${}^1d_1$-${}^1d_4$: the first defines the dynamic range of the loads, and the second the range of variation in the loading rate. Rather welldeveloped engineering methods, which are described in many publications, exist for the construction of scales for the criteria ${}^1d_1$–${}^1d_4$, ${}^2d_1$–${}^2d_4$, and ${}^3d_1$–${}^3d_3$; therefore, we shall not dwell on them here.

The scales of these criteria are cardinal, and procedures for finding the mathematical expectancy, standard deviation, coefficient of asymmetry, and mixed moments are applicable to these scales.

Reflection of Criteria in Estimation Scale. The estimation scale must be obtained using analytical methods. Solution of this problem is a heuristic procedure, which can be carried out with consideration of the ASNI goals. For commensurability of the criteria, the values of the latter must be represented as dimensionless quantities — estimates indicating the degree of correspondence between actual values of the criteria and standard values.

The construction of estimation scales requires the acquisition of additional information and cannot be performed by formal methods, since there are no objectively correct estimates. This procedure is therefore carried out directly by the processor, or group of processors, who reflect the actual cardinal or ordinal scales of the criteria in the estimation scale.

Selection of a rational variant of the ATC parameters from a large number of possible variants reduces to selection of rational estimates from a large number of practicable estimates:

$$O = \psi(Q) = \{{}^lO \in \psi({}^lq),\ {}^lq \in Q\}. \tag{2}$$

In other words, this is equivalent to assigning a scalar function of the criteria scales, which is defined in estimate space and which exhibits the following properties: the function ψ increases monotonically as the quality intensity of the criteria varies. The first derivative of the estimation function, which is called the limiting substitution factor (the weighting factor of the criterion) between the i-th and j-th criteria at point ${}^lq \in Q$, exists for any ${}^1q_i,\ {}^2q_i \in d_i$; ${}^1q_i \neq {}^2q_i$ when ${}^1q_i \geqslant {}^2q_i$, $\psi({}^1q_i) > \psi({}^2q_i)$, $i = \overline{1, I}$; $\forall {}^lq \in d_l$.

Determination of Weighting Factors for Criteria. All of the criteria under consideration can be structured in the following groups:

$$D_1 = \{d_1, d_2, d_3\};\quad D_2 = \{{}^1d_1, {}^1d_2, {}^1d_3, {}^1d_4\};$$

$$D_3 = \{{}^2d_1, {}^2d_2, {}^2d_3, {}^2d_4\},\quad D_4 = \{{}^3d_1, {}^3d_2, {}^3d_3\}.$$

The groups of criteria D_1–D_4 are mutually independent by preference (each of the groups of criteria is independent of its complement); this makes it possible to determine the weighting factors of the criteria in each group independently of the other groups. To determine these factors, let us construct estimation scales, using the method outlined in [2].

The essence of the method is reduced to the following procedures:

find the average estimate of the interval of the criterion scale and consider that

$$o({}^{0.5}d) = 0.5;$$

find the average estimate of the interval $[{}^{0.75}d,\ {}^1d]$ and assume that

$$o({}^{0.75}d) = 0.75;$$

find the average estimate of the interval $[{}^0d,\ {}^{0.5}d]$ and consider that

$$o({}^{0.25}d) = 0.25;$$

construct the $o(d) = \psi(d)$ diagram.

The $o(d_i) = \psi(d_i)$ function for $i = \overline{1, N}$ criteria is plotted in a manner similar to the case described.

This method of plotting the estimation scales yields a scale of an order (ordinal) with several constraints based on the distance between elements. These distances are insufficient, however, to provide for an interval scale.

Let us examine the determination of weighting factors for the criteria d_1, d_2, and d_3 using the method proposed by Kinny and Raifa [2].

Let us designate the worst value for each of the criteria by a_i, and best by b_i. We will then have $a_i \leqslant X_i \leqslant b_i$ for positively oriented scales. Let N be the set of all numbers of the criteria; in our example, $N = \{1, 2, 3\}$. Let L be a subset of the set N and $\bar{L}$ be its complement, i.e., $\bar{L} = N - L$. Let us also designate such values of the criteria that $d_i = d_i\{\max\}$ $i \in L$ and $d_i = d\{\min\}$ for all $i \in \bar{L}$ by d^L.

Let us rank the combinations of criteria $\{a_1, X_2, a_3\}$ and $\{b_1, a_2, a_3\}$. The quantity X_2 can be varied until there is no difference in the selection between them. Let us designate $X_2 = q^e{}_2$. We then have $o(a_1, q_2^e a_3) = o(b_1 a_2 a_3)$, or $\nu_2 o(d_2^e) = \nu_1$. And, since the estimation function $o(d_2)$ is plotted, the relationship between ν_1 and ν_2 is determined. The relationship between ν_3 and ν_2 can be found in a similar manner. Considering the relationship $\nu_1 + \nu_2 + \nu_3 = 1$, let us determine the values of the weighting factors (solution of a linear system of equations).

Let us proceed similarly with the criteria that enter into groups D_2, D_3, and D_4.

The function ν, which is determined in the subsets of the set D, possesses conventional properties of a probabilistic measure [3]: $\nu(L) \geqslant 0$; $\nu(D) = 1$ for $L \subset D$; $\nu(S \cup L) = \nu(S) + \nu(L)$ if S and L do not intersect.

Thus, the search for ν is related to the problem of establishing the appropriate probability distribution in finite random space. Considering what has been stated, the weighting factors of the criteria of the second and third levels of the hierarchy are defined as the conditional measures

$$\begin{cases} {}^1\nu_1 = {}^1\nu_1^*({}^1o_1)/d_1, \ldots, {}^1\nu_4 = {}^1\nu_4^*({}^1o_4)/d_1 \\ {}^2\nu_1 = {}^2\nu_1^*({}^2o_1)/d_2, \ldots, {}^2\nu_4 = {}^2\nu_4^*({}^2o_4)/d_2 \\ {}^3\nu_1 = {}^3\nu_1^*({}^3o_1)/d_3, \ldots, {}^3\nu_3 = {}^3\nu_3^*({}^3o_4)/d_3 \end{cases}$$

$$\begin{cases} {}^1\nu_{1-1} = {}^1\nu_{1-1}^*({}^1o_{1-1})/d_1, {}^1d_1, \ldots, {}^1\nu_{1-2} = \\ = {}^1\nu_{1-2}^*({}^1o_{1-2})/d_1, {}^1d_1 \\ \ldots\ldots\ldots\ldots\ldots \\ {}^3\nu_{3-1} = {}^3\nu_{3-1}^*({}^3o_{3-1})/d_3, {}^3d_3, \ldots, {}^3\nu_{3-4} = \\ = {}^3\nu_{3-4}^*({}^3o_{3-4})/d_3, {}^3d_3, \end{cases} \tag{3}$$

where ν^* is an unconditional measure, or

$$\begin{cases} {}^1\nu_1 = {}^1\nu_1^* \cdot \nu_1, \ldots, {}^1\nu_4 = {}^1\nu_4^* \cdot \nu_1 \\ \ldots\ldots\ldots\ldots \\ {}^3\nu_1 = {}^3\nu_1^* \cdot \nu_3, \ldots, {}^3\nu_3 = {}^3\nu_3^* \cdot \nu_3 \end{cases}$$

and, respectively,

$$\begin{cases} {}^1\nu_{1-1} = {}^1\nu_{1-1}^* \cdot {}^1\nu_1 \cdot \nu_1, \ldots, {}^1\nu_{4-2} = {}^1\nu_{4-2}^* \cdot \nu_4 \cdot \nu_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ {}^3\nu_{3-1} = {}^3\nu_{3-1}^* \cdot {}^3\nu_3 \cdot \nu_3, \ldots, {}^3\nu_{3-4} = {}^3\nu_{3-4}^* \cdot {}^3\nu_3 \cdot \nu_3. \end{cases} \tag{4}$$

Considering what has been stated, the overall ATC estimate can be represented as an additive package of estimates of criteria with allowance for their weight:

$$O = \sum_{i=1}^{N} \nu_i o_i(d_i) \to \max. \tag{5}$$

Formal proof of the possibility of (5) is given in [4, 5].

In certain cases, the analytically complete solution can be obtained by finding the extremum of relationship (5) using the apparatus of indefinite Lagrangian multipliers. In this case, we can speak of the synthesis of optimal ATC parameters.

Let us represent relationship (5) in the form

$$O = \sum_{i=1}^{N} \nu_i o_i(d_1, d_2, \ldots, d_N). \tag{6}$$

As a result of differentiation, we obtain the following system of differential equations in partial derivatives:

$$\left.\begin{aligned}\frac{\partial O}{\partial d_1} &= \sum_{i=1}^{N} \nu_i \frac{\partial o_i(d_1, d_2, \ldots, d_N)}{\partial d_1} = 0;\\ \frac{\partial O}{\partial d_2} &= \sum_{i=1}^{N} \nu_i \frac{\partial o_i(d_1, d_2, \ldots, d_N)}{\partial d_2} = 0;\\ &\ldots\ldots\ldots\ldots\ldots\\ \frac{\partial O}{\partial d_N} &= \sum_{i=1}^{N} \nu_i \frac{\partial o_i(d_1, d_2, \ldots, d_N)}{\partial d_N} = 0.\end{aligned}\right\} \quad (7)$$

If o_i are functions of all d_i $(i = \overline{1,N})$, we have $\frac{n^2+n}{2}$ partial derivatives.

It is obvious that if $\frac{n^2+n}{2} - n = \frac{n^2-n}{2}$ equations are introduced for the relation between $d_1, \ldots, d_N$, a system of $\frac{n^2-n}{2}$ equations with $\frac{n^2-n}{2}$ unknowns can be constructed using Lagrangian multipliers.

The result of solution of the given system of equations will be $o_i = \psi_i(d_1, \ldots, d_N)$ in explicit form. Using D. Sylvester's criteria, it is possible to establish the conditional extremum of relationship (6), which actually defines the possible limit of ATC refinement under available constraints.

CONCLUSION

An increase in the complexity of solvable problems must be related, in final account, to the potentials of ATC; this, in turn, leads to the need for a more detailed investigation of both the parameters of the problem and also the corresponding characteristics of the projected ATC. This requires solutions in a multicriterial statement, i.e., modeling of the optimal selection in indicators.

In plotting estimates of the criteria under consideration, the possibility of their mutual compensation is proposed. Although not rigid, the assumption in question is, in our opinion, however, completely acceptable in practical situations.

Further progress in engineering is impossible without the creation of new automated testing complexes, the requirements for which are growing more and more; this governs the need for the development of methods free from erroneous design solutions. In the evolution of our study, we considered the creation of a package of applied programs and dialogue procedures that can be used in automated systems invoked to assist developers in searches for optimal solutions.

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