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STUDY OF THE STRESS–STRAIN STATE OF RIBBED CYLINDRICAL SHELLS BY THE FINITE ELEMENTS METHOD

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UDC 539.3:629.7.036.3

It is proposed that the finite elements method (FEM) be used in conjunction with the "SPRINT" program pack* to design the casings of gas-turbine engines (GTE), which are thin-walled shells reinforced by annular stiffening ribs – rings – subjected in service to concentrated forces.

The FEM is an approximate numerical method the accuracy of which depends on the design scheme, the density of the grid, and the quality of the finite elements (FE). The denser the grid, the more accurate the results obtained. However, such grids increase computer operating time and thereby increase machine errors and result in some loss in accuracy. At the same time, it is difficult to perform calculations with a large number of elements, and a large computer capacity is required. Subdivision into elements is usually done on the basis of cumulative experience and is checked in a repeat computation with a denser (or less dense) grid.

The present work reports on numerical experiments conducted with a computer to determine the convergence, accuracy, and quality of calculations with different grid densities. For the structure shown in Fig. 1 seven variants of the claculation were performed.

Planar rectangular elements were used to model the shells and ribs. Half of the shell was subdivided into 16 annular elements in the first variant and 48 such elements in the seventh variant, there being 16 and 30 elements in these variants in the longitudinal direction, respectively (Fig. 2). The number of elements in the seventh variant represented an increase in the number of elements compared to the first variant by a factor of 5.4.

Table 1 shows the width of the stiffness matrix band, the order of the system, and the computing time for all design variants. The computations were performed on an ES 1040 computer.

*N. N. Shaposhnikov, V. B. Babaev, G. V. Poltorak, et al., Instructions for the SPRINT Program of Calculation of Combined Systems by the Method of Finite Elements, TsNIIproekt, Moscow (1982).

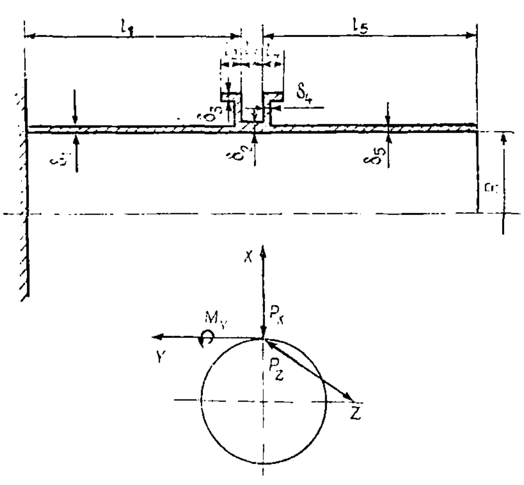

Fig. 1. Longitudinal section and loading scheme of shell.

Moscow Aviation Institute. Translated from Problemy Prochnosti, No. 7, pp. 109-112, July, 1985. Original article submitted October 8, 1984.

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TABLE 1. Main Parameters in the Solution of the System of Equations

No. of variant	No. of nodes about circumference	No. of nodes along axis	Total No. of nodes	Width of band	Order of system	Computing time, min
1	17	16	272	120	1632	12
2	25	20	500	144	3000	40
3	25	30	750	204	4500	67
4	33	24	792	168	4752	46
5	49	20	980	144	5880	60
6	49	24	1176	168	7056	75
7	49	30	1470	204	8820	120

TABLE 2. Displacement of Shell from the Load P_x

No. of variant	No. of node	Displacement along X axis 10^{-1} cm	Displacement along Z axis 10^{-2} cm	No. of node	Displacement along X axis 10^{-2} cm	Displacement along Y axis 10^{-1} cm	Displacement along Z axis 10^{-2} cm	No. of node	Displacement along X axis 10^{-3} cm	Displacement along Z axis 10^{-3} cm
1	16	4,555	2,595	144	1,547	0 7041	0,8886	272	8,626	1,8532
2	20	4,867	2,654	260	1,861	0,6616	0,8707	500	5,126	1,679
3	30	4,952	2,677	390	1,922	0,6383	0,8738	750	4,1682	1,605
4	24	4,991	2,676	408	1,978	0,6411	0,8786	792	3,803	1,6076
5	20	4,970	2,661	500	1,984	0,6522	0,8786	980	4,0511	1,6433
6	24	5,043	2,678	600	2,029	0,6370	0,8735	1176	3,0696	1,5933
7	30	5,049	2,678	750	2,046	0,6283	0,8701	1470	3,0899	1,5692

TABLE 3. Displacement of Shell from the Load P_z

No. of variant	No. of node	Displacement along X axis 10^{-1} cm	Displacement along Z axis 10^{-1} cm	No. of node	Displacement along X axis 10^{-3} cm	Displacement along Y axis 10^{-2} cm	Displacement along Z axis 10^{-4} cm	No. of node	Displacement along X axis 10^{-3} cm	Displacement along Z axis 10^{-4} cm
1	1 (16)	8,914	0,542	2 (144)	0,947	1,785	1,592	3 (272)	2,832	3,918
2	1 (20)	8,486	0,528	2 (270)	1,571	1,694	1,555	3 (500)	2,168	3,569
3	1 (30)	8,490	0,529	3 (390)	1,772	1,660	1,547	3 (750)	1,924	3,425
4	1 (24)	8,385	0,523	2 (408)	1,805	1,668	1,541	3 (792)	1,899	3,424
5	1 (20)	8,288	0,517	2 (500)	1,731	1,695	1,538	3 (980)	1,996	3,484
6	1 (24)	8,301	0,518	2 (600)	1,875	1,670	1,534	3 (1176)	1,831	3,391
7	1 (30)	8,344	0,519	2 (750)	1,930	1,658	1,530	3 (1470)	1,756	3,340

TABLE 4. Displacement of Shell from the Load M_y

No. of variant	No. of node	Displacement along X axis 10^{-1} cm	Displacement along Z axis 10^{-2} cm	No. of node	Displacement along X axis 10^{-3} cm	Displacement along Y axis 10^{-3} cm	Displacement along Z axis 10^{-4} cm	No. of node	Displacement along X axis 10^{-3} cm	Displacement along Z axis 10^{-4} cm
1	1 (16)	2,694	0,882	2 (144)	1,294	1,744	3,613	3 (272)	0,570	0,603
2	1 (20)	2,689	0,837	2 (260)	1,211	2,148	3,488	3 (500)	0,616	0,576
3	1 (30)	2,911	0,835	2 (390)	0,936	2,444	3,244	3 (750)	0,818	0,588
4	1 (24)	2,712	0,825	2 (408)	1,076	2,168	3,407	3 (792)	0,706	0,583
5	1 (20)	2,584	0,811	2 (500)	1,211	2,006	3,887	3 (980)	0,589	0,567
6	1 (24)	2,654	0,813	2 (600)	1,073	2,074	3,432	3 (1176)	0,699	0,569
7	1 (30)	2,713	0,808	2 (750)	0,934	2,075	3,337	3 (1470)	0,796	0,577

TABLE 5. Displacement of the Top Generatrix from the Load P_x

No. of variant	No. of node	Displacement along X axis 10^{-1} cm	Displacement along Z axis 10^{-2} cm	No. of node	Displacement along X axis 10^{-1} cm	Displacement along Z axis 10^{-2} cm	No. of node	Displacement along X axis 10^{-1} cm	Displacement along Z axis 10^{-2} cm
1	1 (6)	2,690	2,897	2 (9)	2,845	2,767	3 (16)	4,555	2,595
2	1 (8)	2,845	3,006	2 (11)	3,008	2,829	3 (20)	4,867	2,654
3	1 (13)	2,879	3,059	2 (16)	3,045	2,866	3 (30)	4,952	2,677
4	1 (10)	2,918	3,058	2 (13)	3,085	3,855	3 (24)	4,991	2,676
5	1 (8)	2,954	3,030	2 (11)	3,121	2,835	3 (20)	4,970	2,661
6	1 (10)	2,976	3,075	2 (13)	3,144	2,857	3 (24)	5,043	2,678
7	1 (13)	2,989	3,091	2 (16)	3,159	2,870	3 (30)	5,049	2,678

The studies were done with the loads P_x, P_z, and M_y (Fig. 1).

Tables 2-4 show calculated data from the seven variants of subdivision of the region into finite elements, i.e., values of the displacements at certain characteristic points. The numbers 1, 2, 3 in the column headed "No. of node" respectively signify the points of intersection of the top generatrix of the shell with the ribs and the unfastened base of the shell. The numbers in parentheses show the numbers of nodes for different variants.

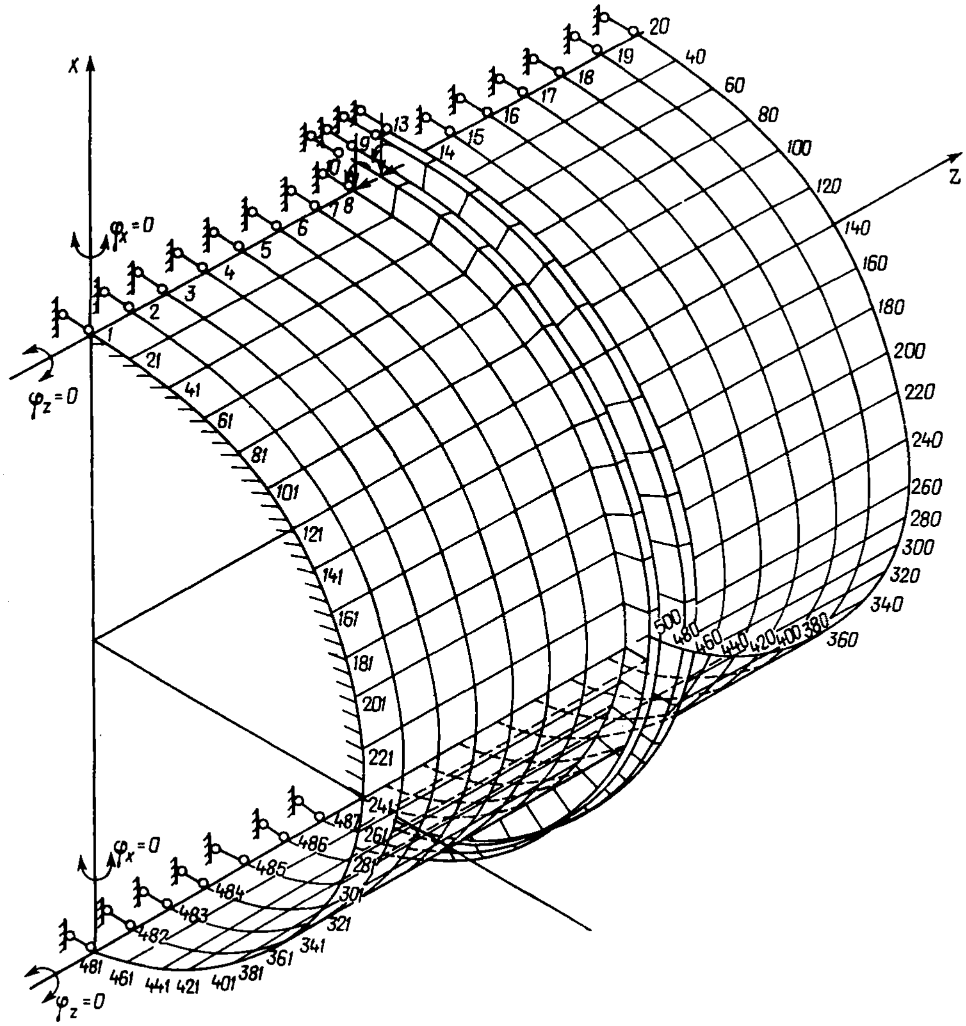

Fig. 2. Shell design scheme (the FE subdivision corresponds to the second variant).

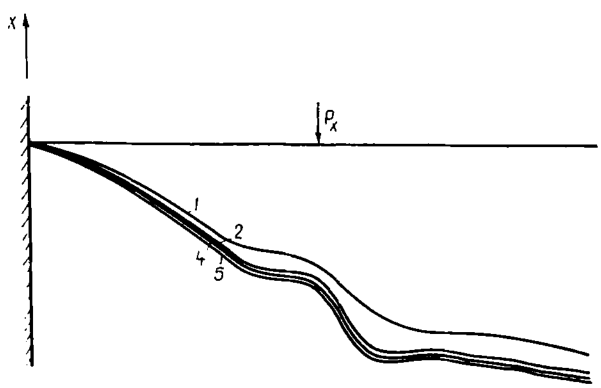

Fig. 3. Displacement of the top generatrix of the shell for variants Nos. 1, 2, 4, and 5 as a result of the load P_x.

Figure 3 shows displacements of the top generatrix of the shell due to the vertical load for different grids. It is evident from Table 5, which shows values of these displacements, that the convergence is good with an increase in grid density.

The following can be concluded from the data obtained:

a) the "SPRINT" program complex gives stable results for different grid densities and can be used for three-dimensional calculation of the stress–strain state of ribbed GTE casings;

b) to obtain results accurate to within about 10%, the minimum grid density should be about 1/3 of the circumference in the annular direction and about 1/10 of the diameter of the shell in the axial direction;

c) use of the method developed significantly reduces the amount of work necessary at the design stage and produces results with the required degree of accuracy.

USE OF THE FINITE ELEMENTS METHOD TO CALCULATE THE STRENGTH OF CONICAL SHELLS WITH NOTCHES

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There are serious mathematical difficulties in determining the stress–strain state of conical shells with notches, but these obstacles can be surmounted by using numerical methods. Mainly shells with small notches in the case of uniform loading have been examined in the well-known solutions of this problem. The stress-strain state has been assumed momentless when the notches are large.

The present study determines the moment stress–strain state of conical shells with notches of arbitrary size and form under complex loading. The problem is solved by the finite elements method in displacements. As the finite element we chose a curvilinear arbitrary tetragonal element of natural curvature (Fig. 1) with 24 degrees of freedom.

We are examining a conical shell of length L and thickness h with a cone angle γ and radius R_0 at $x = x_0$. The shell is weakened by a notch with a radius $r = r(\varphi)$ and is loaded by a system of surface loads $q_i(x, y)$, local forces P_{lI}, and moments $M_{lI}(y)$, as well as lineal contour forces $P_{kI}(y)$ and moments $M_{kI}(y)$. The indices I = 1, 2, 3 correspond to the directions of the axes x, y, z (Fig. 1).

We subdivide the shell into n parts along the boundary of the notch and into m parts along a line connecting the contour of the hole with the outer contour of the shell (Fig. 1). The shell is thus replaced by a set of $m \times n$ arbitrary tetragonal curvilinear elements of natural curvature.

We approximate the displacements of points of a finite element with the polynomials

$$\left.\begin{aligned} u &= \alpha_1\xi\eta + \alpha_2\xi + \alpha_3\eta + \alpha_4;\\ v &= \alpha_5\xi\eta + \alpha_6\xi + \alpha_7\eta + \alpha_8;\\ w &= \alpha_9\xi^3\eta^3 + \alpha_{10}\xi^3\eta^2 + \alpha_{11}\xi^3\eta + \alpha_{12}\xi^3 +\\ &+ \alpha_{13}\xi^2\eta^3 + \alpha_{14}\xi^2\eta^2 + \alpha_{15}\xi^2\eta + \alpha_{16}\xi^2 +\\ &+ \alpha_{17}\xi\eta^3 + \alpha_{18}\xi\eta^2 + \alpha_{19}\xi\eta + \alpha_{20}\xi + \alpha_{21}\eta^3 +\\ &+ \alpha_{22}\eta^2 + \alpha_{23}\eta + \alpha_{24}.\end{aligned}\right\} \qquad (1)$$

We write (1) as follows in matrix form:

$$\mathbf{u} = \mathbf{P}\alpha, \qquad (2)$$

where $\mathbf{P}$ is a coupling matrix of order 3 × 24; $\mathbf{u}$ = u, v, w is the column vector of the displacements of points of the element; $\alpha = \{\alpha_1, \ldots, \alpha_{24}\}$ is the vector of the unknown coefficients of the polynomials.

Using the solutions in [1], for the displacements, strains, and forces of a finite element, we obtain the expressions

$$\mathbf{u} = \mathbf{P}_1\bar{\mathbf{u}}; \quad \mathbf{P}_1 = \mathbf{P}\mathbf{B}^{-1}; \quad \varepsilon = \mathbf{A}\mathbf{u}; \quad \mathbf{T} = \mathbf{D}\varepsilon, \qquad (3)$$

Novosibirsk. Translated from Problemy Prochnosti, No. 8, pp. 112-116, August, 1985. Original article submitted July 4, 1983.

0039-2316/85/1708-1169$09.50