STABILITY OF MULTILAYERED COILED SHELLS LOADED BY EXTERNAL PRESSURE

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Studies devoted to the calculation and experimental investigation of the stability of multilayered shells [1-3] have examined shells consisting of alternating stiff and flexible layers. The stability of multilayered shells connected to each other only by frictional forces was not studied. It is possible to evaluate the stability of multilayered cylindrical steel shells by following the general method expounded in [1] and schematizing the contact approach and slip of the layers by means of very fine flexible interlayers. In this case, the flexible interlayers should have the same elastic moduli in the annular and radial directions, and the elastic constants of the interlayers — including the shear modulus — should depend on the contact pressure. An exact solution obtained in this manner is very complicated and requires the use of a computer.

Thus, the experimental results presented below are analyzed by means of engineering estimates that have not been rigorously substantiated but are attractive in their simplicity and illustrative character.

The critical pressure was determined for two multiple-layered coiled shells with an inside diameter of 300 mm made by winding 26 layers of 1-mm-thick steel 10G2S1 about a central course of steel 20 that was 4 mm thick. The end of the coiled strip of steel 10G2S1 was lap-welded to the underlying layer. Both shells were 320 mm long, but one of them was made of two multilayered shells 150 mm long welded to each other over the entire thickness of the wall with an annular weld 20 mm wide.

The test shell 1, assembled with coupling bolts 2 with end pieces 3, was placed inside a high-pressure vessel 4 (Fig. 1). The shell was made hermetic with rubber seals 5.

The inside cavity of the shell was connected to the atmosphere by means of control pipe 6. The outlet of the pipe was made hermetic with end piece 7 which compressed stuffing-box packing 8.

The external pressure on the shell 1 was created by water pumped into the vessel 4. When the shells lost stability, we heard a sharp metallic bang, and water burst through the control pipe. Pressure in the vessel was monitored with manometer 9.

The multilayered shell without an annular weld lost stability at a pressure of 58 MPa, while the shell with an annular weld became unstable at a pressure of 80.0 MPa. The shells are shown in Fig. 2 after loss of stability. It can be seen that the outer shell does not lose stability after loss of stability in the structure as a whole when it contains an annular weld, and the stability of the structure is determined by the short multilayered course.

Proceeding on the basis of the fact that all of the layers of the multilayered shell lose their stability at the same time, for an engineering estimate of the result obtained we will assume that the upper critical pressure q_u for the entire shell is equal to the sum of the critical pressures q_{ui} of each layer:

$$q_u = \sum_{i=1}^{n} q_{ui},$$

where i is the number of the layer; n is the number of layers.

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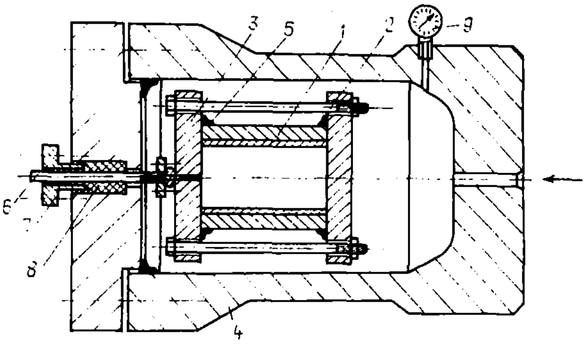

Fig. 1. Diagram of unit for testing shells for stability.

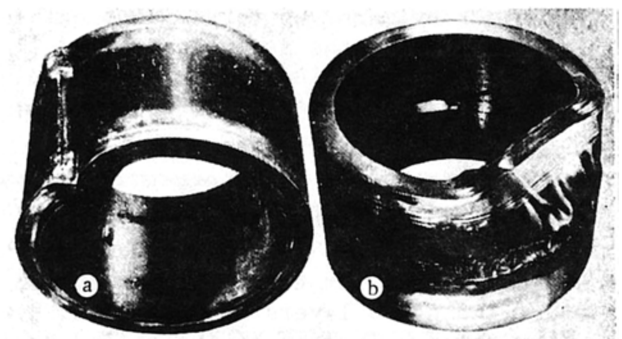

Fig. 2. Shells without annular weld (a) and with annular weld (b) after loss of stability.

We will determine the upper critical pressure of each layer for shells of moderate length, for which in the case of the condition

$$0.8 < l/R < 2$$

the critical pressure is equal to [4, 5]

$$q_{\mathrm{u}i} = \frac{\varepsilon_i h_i}{R_i}\left[\frac{h_i^2 (m^2-1)}{R_i^2 \cdot 12(1-\nu^2)} + \frac{r^4}{m^4(m^2-1)}\right],$$

where ε_i is the elastic modulus of the i-th layer, MPa; ν is the Poisson's ratio; h_i is the thickness of the i-th layer, mm; R_i is the mean radius of the i-th layer, mm; m is the number of complete waves about the circumference; $r = \pi R_i/L$; L is the length of the shell, mm.

Henceforth, to simplify the calculations we will assume that the radii of all layers are equal to the mean radius of the multilayered shell R. The left side of Tables 1 and 2 shows results of calculation of the critical pressures for shells of different thicknesses with a length L = 320 mm or L = 150 mm and different numbers of waves m.

The upper critical pressure for each layer is determined as the minimum value for all possible values of m. It is evident from the tabular data that the number of waves corresponding to the critical pressure is different for layers of different thickness and that it decreases with an increase in layer thickness. If there were no friction between the layers, then during bending of the shell that would normally lead to instability each layer would work individually, and the upper critical pressure would be determined from the formula

$$q_{\mathrm{u}}^{\mathrm{I}} = \frac{\mathrm{E}}{R}\left[\frac{\left(\sum_{i=1}^{n} h_i^3\right)(m^2-1)}{R^2 \cdot 12(1-\nu^2)} + \frac{r^4\left(\sum_{i=1}^{n} h_i\right)}{m^4(m^2-1)}\right].$$

Results of calculations using this formula are shown in column I on the right side of the tables. It can be seen that loss of stability occurs with a smaller number of waves than in the case of a layer of thickness h_i = 1 mm but with a greater number of waves than in the case of a layer of thickness h_i = 4 mm. Here, the theoretical critical pressure

TABLE 1. Theoretical Upper Critical Pressures q_u, MPa, for Shells of Length L = 320 mm

No. of complete waves m	Thickness of layer h, mm						Design variants					
	1	4	5	6	7	8	I	II	III	IV	V	VI
3	13,6	56,3	71,90	88,50	106,4	123,6	410	412	414	417	421	425
4	2,34	13,2	19,42	27,54	37,97	51,09	74,0	77,9	81,8	87,5	95,4	106,4
5	0,687	8,91	15,80	25,68	39,30	57,23	26,8	32,9	39,1	48,4	61,2	78,5
6	0,343	10,35	19,70	33,48	52,60	78,16	19,3	28,2	37,2	50,7	69,5	94,7
7	0,281	13,44	26,04	44,80	—	—	20,5	33,1	45,4	—	—	—
8	0,303	17,38	33,84	—	—	—	25,3	41,5	57,6	—	—	—

Note: The upper critical pressures are underlined here and in Table 2.

TABLE 2. Theoretical Upper Critical Pressures q_u, MPa, for a Shell of Length L = 150 mm

No. of complete waves m	Thickness of layer h, mm					Design variants				
	1	4	5	6	7	I	II	III	IV	V
5	12,2	55,0	73,3	94,8	119,9	372	378	384	394	407
6	4,15	25,6	38,7	56,3	79,3	133,5	142,5	151,4	164,9	183,8
7	1,78	19,4	33,5	53,8	81,4	65,7	78,0	90,3	108,8	134,6
8	0,975	20,1	37,2	62,4	97,3	45,4	61,6	78,7	102,0	135,9
9	0,687	23,3	44,5	67,6	119,7	41,2	61,7	82,2	104,6	156,0
10	0,607	—	—	—	—	43,6	69,0	—	—	—
11	0,617	—	—	—	—	49,3	80,2	—	—	—

is considerably less than the experimental value (19.3 instead of 58 MPa for L = 320 mm and 41.2 instead of 80 MPa for L = 150 mm). Thus, the frictional forces between the layers cannot be ignored.

It can be suggested that several external layers of the shell work together during bending, depending on the amount of external pressure and the tightness of fit of the layers achieved during manufacturing. In other words, the frictional forces between several layers completely take up the shear stresses created during bending of the shell. Columns II-VI of Table 1 and II-V of Table 2 show results of calculation of q_u for a multilayered shell when the thickness of the jointly working layers is 4-8 mm.

Considering that the lower critical pressure q_w is about 0.75 of the upper critical pressure [5], we obtain $q_w = q_u^{VI} \cdot 0.75 = 78.5 \cdot 0.75 = 58.8$ MPa for a shell of length L = 320 mm with eight tight external layers. This agrees with the empirical findings. It should be noted that, here, m = 5, and the length of the half-wave corresponding to buckling is about 100 mm. The same result was obtained experimentally.

The lower critical pressure of a short shell 150 mm long with six tight layers is $q_w = q_u^{IV} \cdot 0.75 = 102.0 \cdot 0.75 = 76.5$ MPa, while in the case of seven external tight layers $q_w = q_u^{V} \cdot 0.75 = 134.6 \cdot 0.75 = 101$ MPa. (The experimental value was 80 MPa.)

It follows from analysis of the experimental results that the tightness of the layers and the friction between them are the main factors determining the stability of a multilayered shell. It can also be stated that such a shell can be made appreciably more stable by providing it with a thicker outer casing or by welding several outer layers with annular welds.

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STUDY OF THE STRESS–STRAIN STATE OF RIBBED CYLINDRICAL SHELLS BY THE FINITE ELEMENTS METHOD

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It is proposed that the finite elements method (FEM) be used in conjunction with the "SPRINT" program pack* to design the casings of gas-turbine engines (GTE), which are thin-walled shells reinforced by annular stiffening ribs – rings – subjected in service to concentrated forces.

The FEM is an approximate numerical method the accuracy of which depends on the design scheme, the density of the grid, and the quality of the finite elements (FE). The denser the grid, the more accurate the results obtained. However, such grids increase computer operating time and thereby increase machine errors and result in some loss in accuracy. At the same time, it is difficult to perform calculations with a large number of elements, and a large computer capacity is required. Subdivision into elements is usually done on the basis of cumulative experience and is checked in a repeat computation with a denser (or less dense) grid.

The present work reports on numerical experiments conducted with a computer to determine the convergence, accuracy, and quality of calculations with different grid densities. For the structure shown in Fig. 1 seven variants of the claculation were performed.

Planar rectangular elements were used to model the shells and ribs. Half of the shell was subdivided into 16 annular elements in the first variant and 48 such elements in the seventh variant, there being 16 and 30 elements in these variants in the longitudinal direction, respectively (Fig. 2). The number of elements in the seventh variant represented an increase in the number of elements compared to the first variant by a factor of 5.4.

Table 1 shows the width of the stiffness matrix band, the order of the system, and the computing time for all design variants. The computations were performed on an ES 1040 computer.

*N. N. Shaposhnikov, V. B. Babaev, G. V. Poltorak, et al., Instructions for the SPRINT Program of Calculation of Combined Systems by the Method of Finite Elements, TsNIIproekt, Moscow (1982).

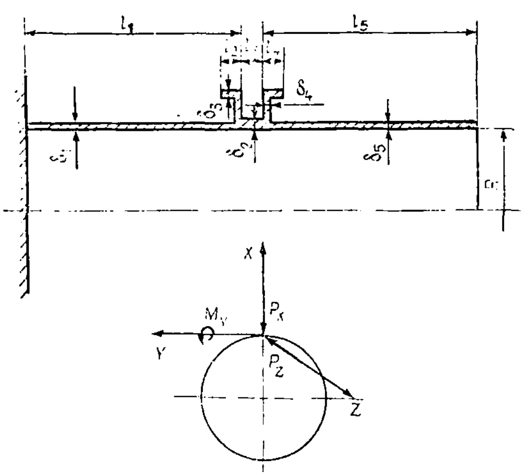

Fig. 1. Longitudinal section and loading scheme of shell.

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