UDC 517 . 9

SOLITONS LIKE EXCITATIONS IN THE ONE-DIMENSIONAL ELECTRICAL TRANSMISSION LINE

СОЛІТОНОПОДІБНІ ЗБУДЖЕННЯ В ОДНОВИМІРНІЙ ЛІНІЇ ПЕРЕДАЧ

F. B. Pelap

Univ. Dschang
PO Box: 69 Dschang, Cameroon
e-mail: fbpelap@yahoo.fr

M. M. Faye

Univ. Cheikh Anta Diop de Dakar
BP 5005 Dakar – Fann, Sénégal

Dynamics of modulated waves are studied in the one-dimensional discrete nonlinear electrical transmission line. Contribution of the linear dispersive capacitance is appreciated and it is shown via a reductive perturbation method that evolution of such waves in this system is governed by a higher order nonlinear Schrödinger equation. Passing through the Stokes analysis, a generalized criterion for the Benjamin – Feir instability in the network is established and the exact solutions of the obtained wave equation are determined by the means of the Pathria and Morris's approach.

Вивчається динаміка модульованої хвилі в одновимірній дискретній нелінійній лінії електропередач. Враховано внесок лінійної дисперсійної ємності та показано за допомогою методу редуктивного збурення, що еволюція таких хвиль у системі описується нелінійним рівнянням Шрьодінгера вищого порядку. За допомогою методу Стокса встановлено узагальнений критерій нестійкості за Бенджаміном – Фейрі у мережі і знайдено точні розв'язки хвильового рівняння за методом Патріа та Морріса.

1. Introduction. It is well known that exactly integrable nonlinear differential equations have soliton solutions that travel stationary and collide elastically. Many wave spread phenomena can be explained by integrable equations in some ideal conditions and there is a great variety of applications of the concept of solitons in Condensed Matter Physics [1]. It is possible to divide these applications into two parts: In one part continuum media are treated, e.g. in hydrodynamics [2], and solitons arise as solutions of partial differential equations (pde's). In the other part, intrinsically discrete models are considered, e.g. chains of magnetic ions or hydrogen-bonded chains in proteins [1, 3]. Here differential-difference equations have to be solved instead of the pde's of the first part. However, apart from very few exceptions like Toda lattice [4], the differential-difference equations cannot be solved exactly. Therefore, several soliton perturbation theories have been developed to study the effect of small perturbations on integrable equations. In these theories, the reductive perturbation method [5, 6] is well known. Within this method, we have the semi discrete approximation that consists of considering the continuum approximation to describe the envelope of the signal and to treat the carrier wave with its discrete character.

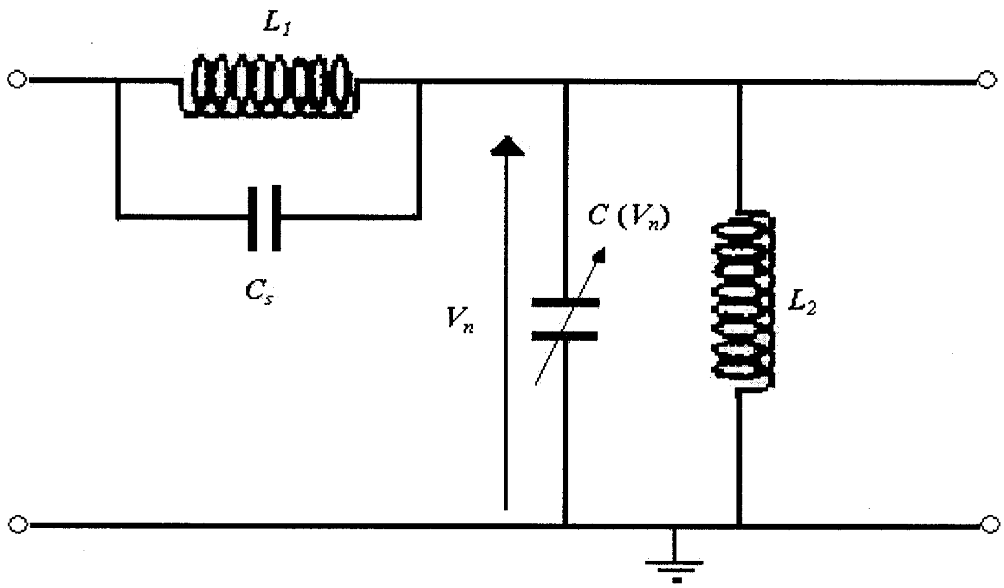


Fig. 1. Schematic representation of one unit cell of a discrete nonlinear electrical transmission line. The network is composed of N identical cells.

The main purpose of this paper is to study the dynamics of modulated wave trains in the discrete nonlinear electrical transmission line using the semi discrete approximation. This paper is organized as follows. In Section 2, we present a nonlinear electrical network representing a bandpass filter with a linear dispersive capacitance C_s. In Section 3, we use the reductive perturbation method to derive the higher order nonlinear Schrödinger (HONLS) equation describing the propagation of modulated waves in the semi discrete limit. The impact of C_s on the dispersion relation is presented. In Section 4, the resulting HONLS equation is utilized to determine the condition for instability of slowly modulated waves. A detailed calculation to predict the modulational instability is given. In Section 5, the Pathria and Morris method is exploited to establish that the HONLS equation possesses solitary wave solutions showing that solitons can propagate in the network. Finally, Section 6 is devoted to concluding remarks.

2. Model description. The model under consideration is a lossless discrete nonlinear transmission line made of N identical unit cells as illustrated in Fig. 1. Each cell contains a linear inductance L_1 in parallel with a linear capacitance C_s in the series branch, and a linear inductance L_2 in parallel with a nonlinear capacitance $C(V)$ in the shunt branches. This nonlinear capacitance consists of a reversed-biased diode with differential capacitance function of the voltage V_n across the nth capacitor [7] and biased by a constant voltage $V_0 : C(V_0 + V_n) = dQ_n/dV_n$ in which Q_n is the corresponding nonlinear charge. For low voltages chosen around V_0 the quantity $Q_n(V_n)$ can be approximated by [8]:

$$Q_n(V_n) = C_0 \left(V_n - \alpha V_n^2 + \beta V_n^3\right), \tag{2.1}$$

where $C_0 = C(V_0)$, α and β are positive constants. From the Kirchhoff's laws applied to the circuit of Fig. 1, we derive the system of nonlinear equations for the voltage $V_n(t)$:

$$\frac{d^2V_n}{dt^2} + u_0^2\left(2V_n - V_{n-1} - V_{n+1}\right) + \lambda\frac{d^2}{dt^2}\left(2V_n - V_{n-1} - V_{n+1}\right) + + \omega_0^2 V_n = \alpha\frac{d^2V_n^2}{dt^2} - \beta\frac{d^2V_n^3}{dt^2} \tag{2.2}$$

wherein $n = 1, 2, \ldots, N$ with N being the number of cells considered. In Eq. (2.2), we have set

$$u_0^2 = \frac{1}{L_1C_0}, \quad \omega_0^2 = \frac{1}{L_2C_0} \quad \text{and} \quad \lambda = \frac{C_s}{C_0}. \tag{2.3}$$

During the computations the following values of the network's parameters are used [9]:

$$L_1 = 200 \pm 5\mu H, \quad L_2 = 470 \pm 10\mu H, \quad V_0 = 2V, \quad C_0 = 370 \pm 10pF, \quad \alpha = 0,21V^{-1}, \quad \beta = 0,0197V^{-2}, \quad \text{and} \quad C_s = 1850 \pm 10pF. \tag{2.4}$$

3. Oscillatory solutions. Now, our attention is focused on the propagation of modulated waves in the system. For this aim, we employ the semi discrete approximation [5, 6] to obtain short wavelength envelope solitons. This approach allows us to treat properly the carrier with its discrete character and to describe the envelope in the continuum approximation. Therefore, slow variables (ξ, τ) are introduced as follows: $\xi = \varepsilon(x - V_g t)$, $\tau = \varepsilon^2 t$, where ε is a small parameter and V_g denotes the group velocity of the packet wave.

Leaning on the idea developed by Taniuti and Yajima [10], the solution $V_n(t)$ of Eq. (2.2) can be taken in the form [11]:

$$V_n(t) = \varepsilon^{1/2}V_{11}(n,t)e^{i\theta} + \varepsilon\left[V_{20}(n,t) + V_{22}(n,t)e^{2i\theta}\right] + + \varepsilon^{3/2}\left[V_{30}(n,t) + V_{33}(n,t)e^{3i\theta}\right] + \varepsilon^2\left[V_{40}(n,t) + V_{42}(n,t)e^{2i\theta} + V_{44}(n,t)e^{4i\theta}\right] + + \varepsilon^{5/2}\left[V_{50}(n,t) + V_{53}(n,t)e^{3i\theta} + V_{55}(n,t)e^{5i\theta}\right] + C.C + o\left(\varepsilon^{7/2}\right), \tag{3.1}$$

in which θ is the phase given by $\theta = kn - \omega t$; $C.C$ stands for the complex conjugation and ε is the smallness parameter that measures the size of the amplitude of the perturbation. During the computations, there are nonzero voltages $V_{lm}(n \pm 1, t)$ which are expanded in the continuum limit around $V_{lm}(x,t)$ with $n = x$. So the fast changes of the phase θ in Eq. (3.1) are correctly taken into account by considering differences in the phase for the discrete variable n. We have also scaled the time and space derivatives as $\partial/\partial t \sim o(\varepsilon)$ and $\partial/\partial x \sim o(\varepsilon)$ respectively and neglected consistently high order in ε terms. Then we keep up to the second order derivative terms of $V_n(t)$ to balance dispersion and nonlinearity. Introduction of $V_n(t)$ and its derivatives into Eq. (2.2) yields series of equations distinguished by the power of ε.

From the equations of $(\varepsilon^{1/2}, e^{i\theta})$ that is the term of $o(\varepsilon^{1/2})$ for the first harmonic, we derive the following linear dispersion relation:

$$\left(1 + 4\lambda\cos^2(k/2)\right)\omega^2 = \omega_0^2 + 4u_0^2\sin^2(k/2), \tag{3.2}$$

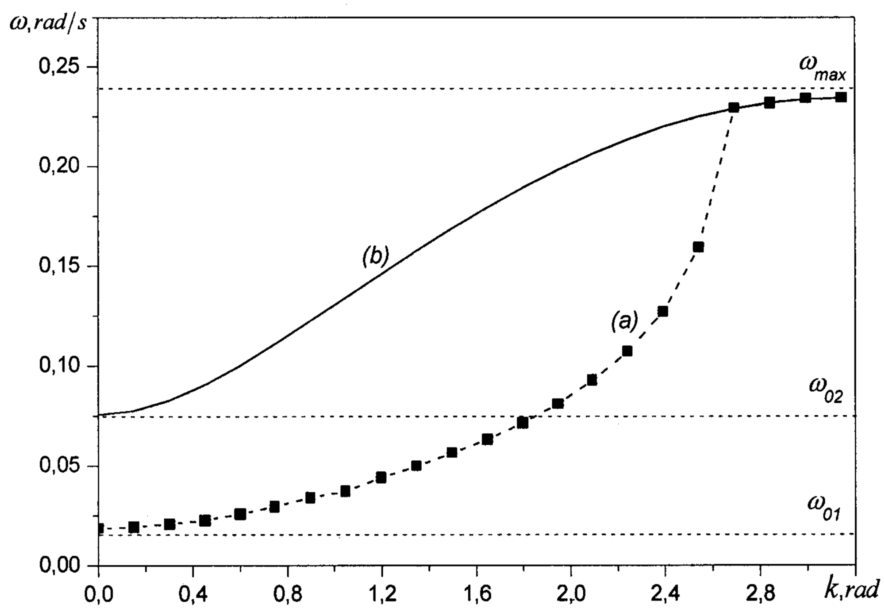


Fig. 2. Theoretical linear dispersion curves defined by relation (3.2). Diverse constants used here are given by expression (2.2).(a): case where $\lambda \neq 0$; (b): case where $\lambda = 0$. While comparing these two plots, we note that the gap zone is larger in the absence of the linear capacitance C_s.

where the wavenumber k is taken in the Brillouin zone. The linear dispersion curve that deals with expression (3.2) is shown on Fig. 2(a) and represents a bandpass filter. As displayed on this figure, the corresponding linear spectrum has a gap $f_{01} = \omega_{01}/(2\pi)$ which is the lower cutoff frequency introduced by the parallel inductance L_2 and it is limited by the cut-off frequency

$$f_{\max} = \frac{\omega_{\max}}{2\pi} = \frac{1}{2\pi}\left(\omega_0^2 + 4u_0^2\right)^{1/2} \tag{3.3}$$

due to the lattice effects. On the other hand, Fig. 2(b) presents the plot of (3.2) in the case where $\lambda = 0$ that is for $C_s = 0$.While comparing the graphs of Fig. 2(a) and Fig. 2(b), we remark that the upper cut-off frequency $f_{\max}$ has not changed in the presence of the linear dispersion capacitance C_s in the system. But its existence reduces the value of the lower cutoff frequency since $f_{01} < f_{02}$ with $f_{02} = \omega_{02}/(2\pi)$. The direct physical consequence of such result is that the extension of the gap zone in the system is highly reduced. Because the width of the interval $[f_{01}, f_{\max}]$ is bigger than that of $[f_{02}, f_{\max}]$, introduction of C_s in the circuit has increased the propagation domain of the signal. On the other hand, relation (3.2) is very different from the result established in [12] that corresponds to the figure $\lambda = 0$ with no parallel linear inductance $(L_2 = 0)$.

From the equations of $(\varepsilon^{3/2}, e^{i\theta})$, we obtain the expression

$$i\left[V_g \frac{\partial V_{11}}{\partial \xi} + \frac{\partial V_{11}}{\partial \tau}\right] = -Q(k)\left|V_{11}\right|^2 V_{11} \tag{3.4}$$

in which

$$Q(k) = \frac{\omega}{2}\left[2\alpha\left(N_{20}+N_{22}\right)-3\beta\right] = \frac{\omega}{2}\left\{4\alpha^2\left(\frac{2\omega^2}{D}+\frac{V_g^2}{V_g^2-u_0^2}\right)-3\beta\right\}$$

with $D = 4\omega^2(1+4\lambda\cos^2 k)-4u_0^2\sin^2 k-\omega_0^2$. Here we follow Kakutani and Michihiro idea [13] and assume that $\triangle k = k-k_c$ is of $o(\varepsilon)$ and write $Q = \varepsilon Q_1$ where Q_1 is of $o(1)$ and is given approximately by

$$Q_1(k) = \frac{\triangle k}{\varepsilon}\left(\frac{dQ(k)}{dk}\right)_{k=k_c} = \frac{V_g}{\omega}Q(k)-\left(\frac{\omega u_0^2}{V_g^2}\right)N_{20}^2\left[\frac{6\lambda\omega}{\chi}\sin k-\frac{V_g}{\omega}+\frac{\cos k}{\sin k}\right]+$$

$$+2\alpha N_{22}V_g-\frac{N_{22}^2}{\omega}\left[2\omega V_g\left(1+4\lambda\cos^2 k\right)-\left(u_0^2+4\lambda\omega^2\right)\sin 2k\right]. \tag{3.5}$$

In relation (3.5), the group velocity is expressed as

$$V_g = \frac{\partial\omega}{\partial k} = \frac{2}{\chi}(u_0^2+\lambda\omega^2)\sin k, \qquad \chi = \omega(1+4\lambda\cos^2(k/2)) \tag{3.6}$$

and the real k_c designates the critical value of the wavenumber of the signal. Therefore at $o\left(\varepsilon^{3/2}\right)$, Eq. (3.4) becomest

$$V_g\frac{\partial V_{11}}{\partial\xi}+\frac{\partial V_{11}}{\partial\tau} = 0. \tag{3.7}$$

This result means that in the reference frame moving with the group velocity V_g, the complex amplitude V_{11} of the signal remains constant to the concerned scale [14]. Hence, the right-hand side of Eq. (3.4) is shifted to the corresponding nonsecular condition at $o(\varepsilon^{5/2})$.

From the equations of $(\varepsilon^{5/2}, e^{i\theta})$, we establish that the resulting equation that describes the dynamics of a packet wave in the discrete nonlinear transmission line (Fig. 1) is the HONLS equation:

$$i\frac{\partial V_{11}}{\partial\tau}+P\frac{\partial^2 V_{11}}{\partial\xi^2} = Q_1|V_{11}|^2V_{11}+Q_2|V_{11}|^4V_{11}+iQ_3V_{11}^2\frac{\partial V_{11}^*}{\partial\xi}+iQ_4|V_{11}|^2\frac{\partial V_{11}}{\partial\xi} \tag{3.8}$$

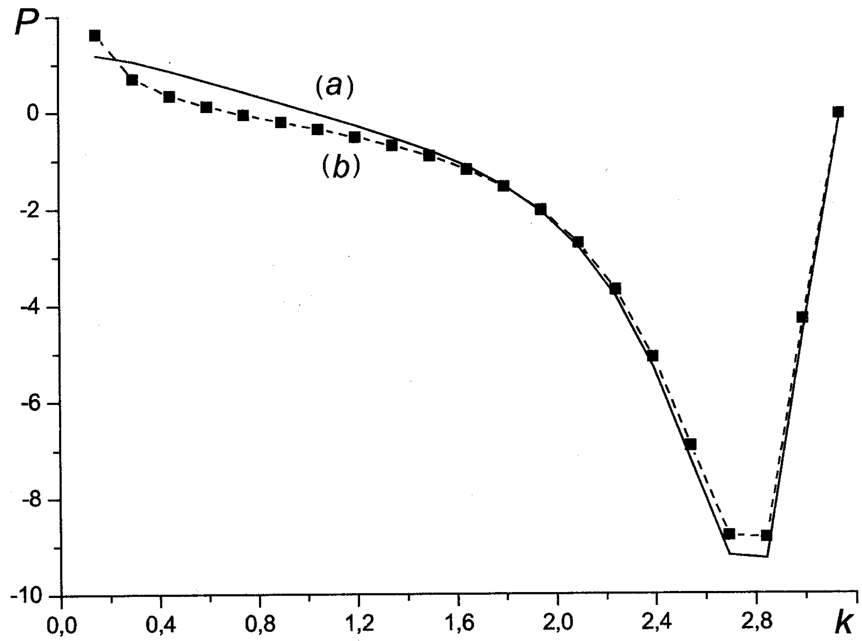
in which

$$P = \frac{1}{2\omega}\left[(u_0^2+\lambda\omega_0^2)\cos k-4\lambda V_g^2\sin^2(k/2)-4\omega V_g\sin k\right], \tag{3.9}$$

$$Q_2 = \omega\left[\alpha(N_{40}+N_{42}+N_{22}N_{33})-\frac{3\beta}{2}(2N_{22}^2+N_{33}+N_{20}^2+2N_{20}N_{22})\right], \tag{3.10}$$

$$Q_3 = 2V_g\left[\beta-\alpha(N_{20}+N_{22}\right], \tag{3.11}$$

$$Q_4 = \alpha\omega\widetilde{N}_{42}+2Q_3, \tag{3.12}$$



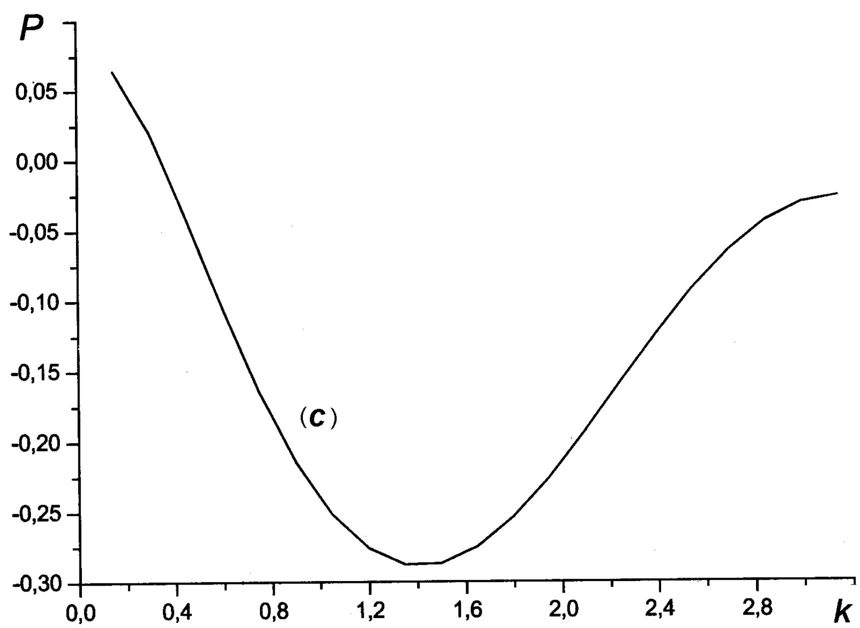


Fig. 3. Dispersive coefficient in terms the wavenumber k taken in the Brillouin zone for the line parameters given by (2.2). (a): case where $\lambda \neq 0$; and $\omega_0 \neq 0$; (b):case where $\lambda \neq 0$ and $\omega_0 = 0$; (c): case where $\lambda = 0$ and $\omega_0 \neq 0$. These plots show that P admits both positive and negatives values and inform that the range values of the dispersion coefficient P increases with the introduction of C_s in the system.

and Q_1 is given by relation (3.5). The diverse parameters and voltages that intervene in Eq. (3.8) are determined in the Appendix.

The HONLS equation (3.8) is known to govern the modulations of weakly nonlinear ions in acoustic plasma waves [5], ferromagnetic chains [15] and the modulations of Stokes waves [13, 16]. This equation is also used as envelope equation for describing a weakly subcritical bifurcation to counter propagating waves [17] and also accounts for the slows modulations of an oscillatory mode close to a subcritical bifurcation [18] when $Q_3 = Q_4 = 0$. In Eq. (3.8), the first two terms of the right-hand side are the nonlinear terms while the others represent the nonlinear dispersion.

Using the line parameters (2.2), the coefficient of the spatial dispersion (3.9) is plotted as function of the wavenumber k taken in the Brillouin zone (Fig 3). We could remark from these graphs that the coefficient P has both positive and negative values whether λ is null or not. When comparing Fig. 3 (a, b) and Fig. 3 (c), we note that the presence of C_s (i.e., for $\lambda \neq 0$) increases the interval of the values of P. In other words, introduction of C_s adds the dispersion in the system. This result will be useful to predict the stability of modulated waves in the network.

4. Modulational instability. In this section, we research under which conditions a uniform wave train moving along the discrete nonlinear electrical transmission line of Fig. 1 will become unstable to a small perturbation. For this purpose, we use the HONLS equation (3.8) derived from the exact equations (2.2) describing the wave propagation in the network. First, we look for solutions of Eq. (3.8) in the form:

$$V_{11}(\xi,\tau) = E_0 \exp\left[i(k_n\xi - \omega_n\tau)\right], \tag{4.1}$$

where E_0 is a complex constant amplitude. Report of (4.1) into Eq. (3.8) liberates the nonlinear dispersion relation

$$\omega_n = \omega_n\left(k_n, |E_0|^2\right) = Pk_n^2 + Q_1|E_0|^2 + Q_2|E_0|^4 - k_n(Q_3 - Q_4)|E_0|^2 \tag{4.2}$$

in which k_n and ω_n are respectively the wavenumber and the angular frequency of the carrier wave. From relation (4.2), the sinusoidal wave is nonlinear and the principle of superposition is invalid. To investigate the modulational instability [19] of the carrier wave, a small perturbation of the solution (4.1) is taken as follows [5, 20–22]:

$$V_{11}(\xi,\tau) = \big[1 + A(\xi,\tau)\big] E_0 \exp\big[i(k_n\xi - \omega_n\tau)\big] \tag{4.3}$$

where $A(\xi,\tau)$ is a complex quantity. Substituting this solution into the HONLS equation and linearizing the result with respect to $A(\xi,\tau)$ give the following differential equations:

$$\begin{aligned} iA_\tau + \omega_n(1+A) + P\big(A_{\xi\xi} + 2ik_nA_\xi - k_n^2(1+A)\big) = \\ = Q_1(1+2A+A^*)|E_0|^2 + Q_2(1+3A+2A^*)|E_0|^4 + \\ + i(Q_3A_\xi^* + Q_4A_\xi)|E_0|^2 + k_n(Q_3-Q_4)(1+2A+A^*)|E_0|^2 \end{aligned} \tag{4.4}$$

in which the asterisk denotes the complex conjugation. The use of relation (4.2) permits to simplify Eq. (4.4) as:

$$iA_\tau + P(A_{\xi\xi} + 2ik_n A_\xi) = Q_1(A + A^*)|E_0|^2 + 2Q_2(A + A^*)|E_0|^4 +$$

$$+ i(Q_3 A_\xi^* + Q_4 A_\xi)|E_0|^2 + k_n(Q_3 - Q_4)(A + A^*)|E_0|^2. \tag{4.5}$$

Solutions of Eq. (4.5) could be found in the form

$$A(\xi, \tau) = A_1 \exp\big[i(l\xi + \Omega\tau)\big] + A_2^* \exp\big[-i(l\xi + \Omega^*\tau)\big], \tag{4.6}$$

where l and Ω indicate respectively the wavenumber and the angular frequency of the perturbation. Parameters A_1 and A_2 are complex constants. Substitution of relation (4.6) into Eq. (4.5) yields a linear homogeneous system for A_1 and A_2, i.e.,

$$\begin{gathered}(\Omega + Pl^2 + b + c)A_1 + \left(c - lQ_3|E_0|^2\right) A_2 = 0,\\ -\left(c + lQ_3|E_0|^2\right) A_1 + (\Omega - Pl^2 + b - c)A_2 = 0\end{gathered} \tag{4.7}$$

wherein

$$b = 2k_n lP - lQ_4|E_0|^2 \quad \text{and} \quad c = Q_1|E_0|^2 + 2Q_2|E_0|^4 + k_4(Q_3 - Q_4)|E_0|^2. \tag{4.8}$$

The condition of nonrtrivial solutions of the system (4.7) determines the dispersion relation for the perturbation wave:

$$\Omega^2 + 2b\Omega + \left(b^2 - P^2 l^4 - l^2 Q_3^2 |E_0|^4 - 2cPl^2\right) = 0 \tag{4.9}$$

which is a second order equation for Ω with real coefficients. If its discriminant (Δ) is negative, then the frequency Ω will be complex and the perturbations will grow in the system which becomes unstable. This situation occurs when $\Delta < 0$, i.e.,

$$2l^2 PQ_1|E_0|^2 + 4l^2 PQ_2|E_0|^4 + l^4 P^2 + l^2 Q_3^2|E_0|^4 + 2l^2 k_n P(Q_3 - Q_4)|E_0|^2 < 0. \tag{4.10}$$

Some simple arrangements of expression (4.10) lead to

$$PQ_1 - r < -\left(\frac{l^2 P^2 + Q_3^2|E_0|^4}{2|E_0|^2}\right) < 0 \tag{4.11}$$

and necessarily

$$PQ_1 - r < 0 \tag{4.12}$$

with

$$r = k_n(Q_3 - Q_4)|E_0|^2 - 2PQ_2|E_0|^2. \tag{4.13}$$

Relation (4.12) together with (4.13) represent the modulational instability criterion associated to the HONLS equation in the electrical transmission line of Fig. 1. This criterion is a function of the nonlinear and dispersive parameters β and λ since it depends on the coefficients of Eq. (3.8) which are related to β and λ. The result (4.12) is more general than the one obtained by Ketchakeu et al. [11] during the study of ϕ^4 models. It also generalizes the family criterion for the standard nonlinear Schrödinger (NLS) equations [19, 23]. However, this criterion is similar to that established by Kakutani and Michihiro [13] when they examine the motion of water waves near the marginal state of instability.

5. Exact solitary solutions. The main purpose of this section is to check whether the discrete nonlinear transmission line under study can support solitary waves. Hence, we follow Pathria and Morris [24] idea and set $\tau = Pt$. Then, the amplitude wave equation (3.8) takes the form:

$$iV_{11,\tau} + V_{11,\xi\xi} = q_c|V_{11}|^2V_{11} + q_q|V_{11}|^4V_{11} + iq_mV_{11}^2V_{11,\xi}^* + iq_u|V_{11}|^2V_{11,\xi} \tag{5.1}$$

with $q_c = -Q_1/P$, $q_q = -Q_2/P$, $q_m = -Q_3/P$, and $q_u = -Q_4/P$. At this level, we introduce the following notations:

$$Q_c = q_c, \quad Q_q = q_q + \frac{1}{8}(q_u + 2q_m)\left(\frac{3}{2}q_u - q_m\right),$$
$$Q_m = q_m - \frac{1}{2}(q_u + 2q_m) = -\frac{1}{2}q_u \quad \text{and} \quad Q_u = q_u. \tag{5.2}$$

The solutions of Eq. (5.1) strongly depend on the sign of the coefficient Q_q [24]. Two cases can be distinguished. When $Q_q < 0$, the solution of Eq. (5.1) is given by

$$V_{11}(\xi,t) = \left(\frac{r_1r_2}{r_1 + (r_1 - r_2)\sin h^2(\chi_0)}\right)^{1/2} \exp\left[i\phi(\xi,t)\right] \tag{5.3}$$

in which

$$\phi(\xi,t) = -\left(\frac{q_u + 2q_m}{4}\right)\sqrt{-3/Q_q}\tan h^{-1}\left(\sqrt{\frac{r_1}{r_2}}\tan h(\chi_0)\right) + \frac{\eta}{2}(\xi - \mu t) + \vartheta_0,$$

with $\chi_0 = (-r_1r_2Q_q/3)^{1/2}(\xi - \eta t) + \vartheta_1$.

In expressions (5.3), the quantities μ and η denote the speeds of the carrier and envelope waves of V_{11} respectively; ϑ_0 and ϑ_1, are arbitrary constants. The values of r_1 and r_2 determine the form of this solution. Solitary waves arise in the system if r_1 and r_2 are real, with $r_1 > r_2 > 0$ [24].

On the other hand, if $Q_q > 0$, r_1 and r_2 are real with $r_1 > 0 > r_2$, a solitary wave also exists and the corresponding solution for Eq. (5.1) is [24]:

$$V_{11}(\xi,t) = \left(\frac{r_1r_2}{r_2 + (r_2 - r_1)\sin h^2(\chi_0)}\right)^{1/2} \exp\left[i\phi(\xi,t)\right]$$

which

$$\phi(\xi,t) = -\left(\frac{q_u+2q_m}{4}\right)\sqrt{3/Q_q}\tan^{-1}\left(\sqrt{-\frac{r_1}{r_2}}\tan h(\chi_0)\right)+\frac{\eta}{2}(\xi-\mu t)+\vartheta_0,$$

and $\chi_0 = (-r_1 r_2 Q_q/3)^{1/2}(\xi-\eta t)+\vartheta_1$.

Furthermore, if r_1 and r_2 are real with $r_1 > r_2 > 0$, then the solution is oscillatory and has the following form:

$$V_{11}(\xi,t) = \left(\frac{r_1 r_2}{r_1+(r_2-r_1)\cos^2(\chi_1)}\right)^{1/2}\exp\left[i\phi(\xi,t)\right]$$

where

$$\phi(\xi,t) = -\left(\frac{q_u+2q_m}{4}\right)\sqrt{3/Q_q}\tan^{-1}\left(\sqrt{\frac{r_2}{r_1}}\tan h(\chi_1)\right)+\frac{\eta}{2}(\xi-\mu t)+\vartheta_0,$$

and $\chi_1 = (r_1 r_2 Q_q/3)^{1/2}(\xi-\eta t)+\vartheta_1$.

From this investigation, we note that the HONLS equation possesses solitary wave solutions for both positive and negative values of Q_q.

6. Conclusion. In this paper, we have considered a discrete nonlinear electrical transmission line and examined dynamics of modulated waves in the system. Exploiting the reductive perturbation method, it has been shown in the semi discrete limit that propagations of modulated wave trains are governed by a modified form of the NLS equation that involves higher orders nonlinearities, i.e., the HONLS equation. Through our investigation, it has been obtained that the capacitance C_s adds the linear dispersive effects in the circuit with the consequence that the gap zone is greatly reduced and the range of frequencies for the propagation of the signal has substantially increased.

Based on the obtained amplitude wave equation, we have utilized the Stokes wave analysis to construct the criterion for the modulational instability of a plane wave introduced in the electrical line. It has appeared that the new criterion generalizes the family criterion for NLS equations and depends both on the amplitude and wavenumber of the propagating signal.

Besides this study of the asymptotic behaviour of the signal in the network, the Pathria and Morris's method has been exploited to show that the discrete nonlinear transmission can support solitary waves. This last result is of higher importance since it is known that solitons are good waves for the transport of information in some physical systems.

Appendix. The equations of $(\varepsilon^3, e^{0i\theta})$ lead to the potential

$$V_{20} = N_{20}|V_{11}|^2 \quad \text{with} \quad N_{20} = \frac{2\alpha V_g^2}{V_g^2-u_0^2}. \tag{A.1}$$

From the equations of $(\varepsilon^3, e^{2i\theta})$, we get

$$V_{22} = N_{22}(V_{11})^2 \quad \text{where} \quad N_{22} = \frac{4\alpha\omega^2}{D}. \tag{A.2}$$

The equations of $(\varepsilon^{5/2}, e^{2i\theta})$ and $(\varepsilon^3, e^{i\theta})$ yield respectively the voltages

$$V_{30} = 0 \quad \text{and} \quad V_{50} = 0. \tag{A.3}$$

From the equations of $(\varepsilon^{3/2}, e^{3i\theta})$, we obtain the potential

$$V_{33} = N_{33}(V_{11})^3 \quad \text{in which} \quad N_{33} = \frac{1}{D_1}(18\alpha\omega^2 N_{22} - 9\beta\omega^2) \tag{A.4}$$

with $D_1 = 9\omega^2(1 + 4\lambda\cos^2(3k/2)) - 4u_0^2\sin^2(3k/2) - \omega_0^2$.

The equations of $(\varepsilon^3, e^{0i\theta})$ liberate

$$V_{40} = N_{40}|V_{11}|^4 \quad \text{with} \quad N_{40} = \frac{1}{2}N_{20}(N_{20}^2 + 2N_{22}^2) - \frac{\beta}{\alpha}N_{20}(2N_{20} + 3N_{22}). \tag{A.5}$$

From the equations of $(\varepsilon^2, e^{2i\theta})$, we determine the voltage

$$V_{42} = N_{42}|V_{11}|^2 V_{11}^2 + i\widetilde{N}_{42}(V_{11}^2)_\xi \tag{A.6}$$

in which the divers coefficients are defined by

$$N_{42} = \frac{4\omega^2}{D}\left[2\alpha(N_{33} + N_{20}N_{22}) - 3\beta(N_{20} + 2N_{22})\right]$$

and

$$\widetilde{N}_{42} = \frac{2}{D}\left[2\omega V_g N_{22}(1 + 4\lambda\cos^2 k) - (u_0^2 + 4\lambda\omega^2)N_{22}\sin k - 2\alpha\omega V_g\right].$$

Remark. The other coefficients are nonzero but do not intervene in the derivation of the amplitude wave equation that characterizes the motion of the signal in the system. Nevertheless, we have found their expressions which are listed below.

From the equations of $(\varepsilon^2, e^{4i\theta})$, we deduce

$$V_{44} = N_{44}(V_{11})^4 \quad \text{wherein} \quad N_{44} = \frac{1}{D_2}\left[16\alpha\omega^2(N_{22}^2 + 2N_{33}) - 48\beta\omega^2 N_{22}\right] \tag{A.7}$$

with $D_2 = 16\omega^2(1 + 4\lambda\cos^2(2k)) - 4u_0^2\sin^2(2k) - \omega_0^2$.

The equations of $(\varepsilon^{5/2}, e^{2i\theta})$ help to determine the voltage

$$V_{53} = N_{53}|V_{11}|^2 V_{11}^3 + iN'_{53}(V_{11}^2)_\xi V_{11} + iN''_{53}(V_{11}^3)_\xi \tag{A.8}$$

in which the different parameters are given by

$$N_{53} = \frac{18\omega^2}{D_1}\left[\alpha(N_{42} + N_{44} + N_{20}N_{33}) - \beta(2N_{33} + 2N_{20}N_{22} + 2N_{22}^2)\right],$$

$$N'_{53} = \frac{6\alpha\omega}{D_1}(3\omega\widetilde{N}_{42} - 2V_g N_{22})$$

and

$$N''_{53} = \frac{1}{D_1}\left[6\omega V_g N_{33}(1+4\lambda\cos^2(3k/2)) - 2(u_0^2 + 9\lambda\omega^2)N_{33}\sin 3k + 6\beta\omega V_g\right].$$

Form the equations of $(\varepsilon^{5/2}, e^{5i\theta})$, we get

$$V_{55} = N_{55}(V_{11})^5 \quad \text{in which} \quad N_{44} = \frac{25\omega^2}{D_3}\left[2\alpha(N_{44} + N_{22}N_{33}) - 3\beta(N_{22}^2 + N_{33})\right]$$

where $D_3 = 25\omega^2(1+4\lambda\cos^2(5k/2)) - 4u_0^2\sin^2(5k/2) - \omega_0^2$.

Acknowledgements. The authors are grateful to Professor Moise Godfroy Njock Kwato for his invaluable technical assistance. They also acknowledge the Cameroonian Government for the financial support given to Dr F. B. Pelap through the "Program of mobility" of the Ministry of Higher Education.

1. *Trullinger S. E., Zakharov V. E., and Pokrovsky V. L.* Solitons. — Amsterdam: North-Holland, 1986.
2. *Eckhaus W.* Studies in the nonlinear stability theory. — Berlin: Springer, 1965.
3. *Kivshar Y. S., Salerno M.* Modulational instability in the discrete deformable nonlinear Schrödinger equation // Phys. Rev. E. — 1994. — **49**. — P. 3543–3546.
4. *Toda M.* Theory of nonlinear lattice // J. Phys. Soc. Jap. — 1967. — **22**. — P. 413.
5. *Parkes J.* The modulation of weakly nonlinear dispersive waves near the marginal state of instability // J. Phys. A.: Math. and Gen. — 1987. — **20**. — P. 2025–2036.
6. *Pelap F. B., Kofané T. C., Flytzanis N., and Remoissenet M.* Waves modulations in the biinductance transmission line // J. Phys. Soc. Jap. — 2001. — **70**. — P. 2568–2577.
7. *Kofané T. C., Michaux B., and Remoissenet M.* Theoretical and experimental studies of diatomic lattice solitons using an electrical transmission line // J. Phys. C. — 1988. — **21**. — P. 1395–1412.
8. *Remoissenet M.* Waves called solitons. — 3rd ed. — Berlin: Springer, 1999.
9. *Marquie P., Bilbault J. M., and Remoissenet M.* Generation of envelope and hole solitons in an experimental transmission line // Phys. Rev. E. — 1994. — **46**. — P. 828–835.
10. *Taniuti T., Yajima N.* Perturbation method for nonlinear equations // J. Math. Phys. — 1969. — **10**. — P. 1369–1372.
11. *Ketchakeu J., Kofané T. C., and Zibi A.* Modulational instability in an extended ϕ^4 model near the marginal state // Phys. scr. — 1991. — **44**. — P. 505–510.
12. *Pelap F. B., Kofané T. C.* The revised Modulational instability criterion: I. The monoinductance transmission line // Ibid. — 1998. — **57**. — P. 410–415.
13. *Kakutani T., Michihiro K.* Marginal state of modulational instability — note of the Benjamin–Feir instability // J. Phys. Soc. Jap. — 1983. — **52**. — P. 4129–4137.
14. *Kawahara T., Sakai J., and Kakutani T.* Nonlinear wave modulation in dispersive media // Ibid. — 1970. — **29**. — P. 1068–1073.
15. *Ndohi R., Kofané T. C.* Solitary waves in ferromagnetic chains near marginal state of instability // Phys. Lett. A. — 1991. — **154**. — P. 377–379.
16. *Johnson R. S.* Proc. Roy. Soc. A. — 1977. — **357**. — P. 131.
17. *Deissler R. J.* J. Stat. Phys. — 1989. — **54**. — P. 1459.
18. *Pomeau Y.* Front motion, metastability and structural bifurcations in hydrodynamics // Physica D. — 1986. — **23**. — P. 3–11.

19. *Benjamin T. B., Feir J. F.* The disintegration of wave trains on deep water // J. Fluid Mech. — 1967. — **27**. — P. 417 – 430.
20. *Stuart J. T., Di Prima R. C.* The Eckhauss and Benjamin – Feir resonance mechanisms // Phys. Roy. Soc. London. Ser. A. — 1978. — **362**. — P. 27 – 41.
21. *Dodd R. K., Eilbeck J. C., Gibbon J. D., and Morris H. C.* Solitons and nonlinear wave equations. — London: Academic, 1987.
22. *Pelap F. B., Kofané T. C.* Modulational instability in some physical systems // Phys. scr. — 2001. — **64**. — P. 410 – 412.
23. *Kengne E.* Modified Ginzburg – Landau equation and Benjamin — Feir instability // Nonlinear Oscillations. — 2003. — **6**, № 3. — P. 346 – 356.
24. *Pathria D., Morris J. Li.* Exact solutions for a generalized nonlinear Schrödinger equation // Phys. scr. — 1989. — **39**. — P. 673 – 679.

Received 08.06.2004