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Propagation of short nonlinear second-sound pulses through He-II in one- and three-dimensional geometry

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The results of an experimental study of the evolution of the shape of nonlinear second-sound pulses in superfluid He-II are reported. The pulses propagate in the bulk ($3D$ geometry) and along a cryoacoustic waveguide filled with liquid helium (quasi-$1D$ geometry) at temperatures corresponding to the negative, positive, or zero nonlinearity coefficient. A strong dependence of the shape of the propagating pulse on the dimensionality of the wave was observed. The finite size of the heater (generator of a sound) affects the profile of a short $3D$ pulse even at distances many times greater than the heater size, which restricts the minimal width of the excited pulse. The experimental data are compared with the results of numerical simulations.

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Introduction

The waves of the entropy (second sound) are macroscopic quantum effects which may be observed in a superfluid liquid and perfect crystals [1,2]. The properties of the second sound in a superfluid ^{4}He (He-II) were studied extensively, both experimentally and theoretically. More recently, the attention has been focused on the study of nonlinear acoustic properties of superfluid He-II [3–7].

We report here on experimental and theoretical investigations of the peculiarities of nonlinear evolution of a solitary second-sound pulse in He-II under different geometrical conditions of excitation and propagation of the pulse.

As can be seen from further comparison of the shape of the recorded pulse with the results of computer simulations, one must take into account not only the strong nonlinearity of He-II, but also finite dimensions of the heater (generator of the second sound) and receiver (bolometer).

Evolution with time of the propagating second-sound pulse is defined by dispersive and nonlinear properties of the superfluid and geometrical conditions of the propagation. From the analytical point of view, the partial differential equation that describes the evolution of the shape of a wave traveling through an unrestricted medium can be reconstructed unambiguously from two important properties: the dispersion law of the wave $\omega(k)$ and the vertex function of nonlinear self-interaction $\Gamma(k, k_1, k_2)$, which is the amplitude characterizing the strength of three-wave interaction [8].

For the second sound in superfluid ^{4}He at temperatures $T > 0.9$ K (roton second sound) but not very close to T_λ (so that $\omega < c_2/\xi$, when ξ is correlation length) and at frequencies lower than the inverse time which characterizes phonon-roton interaction, $\omega < c_2/c_1\tau_{r\text{–ph}}$, the spectrum $\omega(k)$ is a linear function of the wave vector k, $\omega(k) = c_2 k$ (here c_1 and c_2 are velocities of the first- and second-sound waves of infinitely small amplitude) [2]. The vertex Γ can be evaluated from the Landau equations of two-fluid hydrodynamics using the method similar to [9], which gives $\Gamma(k, k_1, k_2) = \text{const}\ \alpha(kk_1k_2)^{1/2}$, where α is the nonlinearity coefficient of the second sound,

$$\alpha = c_2 \frac{\partial}{\partial T} \ln\left(c_2^3 \frac{\partial \sigma}{\partial T}\right).$$

Here σ is the entropy per unit of mass. The sign of the nonlinearity coefficient α depends on temperature of the helium bath [2]: $\alpha > 0$ at $T < T_\alpha = 1.88$ K as in ordinary matter, and $\alpha < 0$ at $T_\alpha < T < T_\lambda$.

At temperatures of the bath close to T_α the three-wave interaction is small [because $\Gamma(k, k_1, k_2) \to 0$] and one must account for the next nonlinear term in the expansion of the evolution equation over the amplitudes of the sound. This term is proportional to the vertex of four-waves interaction $\Gamma^{(4)}(k, k_1, k_2, k_3)$ of the second sound.

In real experiments the amplitude of the excited second-sound waves, δT_{max}, is restricted from above by the conditions of film boiling of the superfluid near the generator (heater): from our measurements of the heat pulses with duration $\tau_e \approx 10$ μs the critical value of the heat flux density is $Q_0 \approx \approx 25$ W/cm^2, which corresponds to $\delta T_{max} < 10$ mK in the temperature range of the measurements. At such amplitudes of the second sound the four-wave interaction is sufficiently weak and the condition $|\Gamma^{(4)}|\delta T^2_{max} << c_2/L$ holds. Accordingly, at $T = T_\alpha$ no changes in the shape of a traveling pulse have been observed at distances from the heater to the bolometer up to $L \approx 10$ cm. One can therefore observe a «ballistic» travel of the pulse from the generator to the detector without any evolution of its shape at bath temperatures $T \approx T_\alpha$, in contrast to the case $T \neq T_\alpha$, when the nonlinear transformation of the pulse is significant and the shock second-sound wave is formed at small distances from the heater.

At temperatures very close to T_λ the nonlinearity coefficient tends to infinity according to the power law $\alpha \sim \varepsilon^{-1}$, where $\varepsilon = (T_\lambda - T)/T_\lambda$ is the reduced temperature [7]. Near the lambda point the nonlinear properties of the second sound therefore play the crucial role even for a wave with a small amplitude [6].

Since the second sound has linear dispersion law and a square root-like dependence of the vertex Γ on the wave vectors, the evolution of the shape of one-dimensional second-sound pulse $\delta T(x, t)$ is governed by the Burgers equation [7]

$$\frac{\partial \delta T}{\partial t} + \alpha \delta T \frac{\partial \delta T}{\partial x} = \mu \frac{\partial^2 \delta T}{\partial x^2} . \qquad (1)$$

The last term describes the dissipation of the wave and is introduced to preserve the turnover of the wave front. Here μ is the damping coefficient.

If $T \neq T_\alpha$ $(\alpha \neq 0)$, the nonlinear evolution leads to the creation of a shock at the profile of the traveling pulse. The width of the shock front d_f is defined by the nonlinear term and the dissipative term, $d_f = \mu/\alpha\Delta T$. The velocity of shock propagation is $v_f = c_2 + \alpha\Delta T/2$ (Refs. 1 and 2). At large distances L from the heater the profile of a 1D shock wave acquires the final form presented by a triangle. The dependence of the length of the triangle (duration of the pulse τ) and its height (the temperature jump at the shock front ΔT) on the distance can be described by a universal power law,

$$\tau = \text{const } (\alpha L)^{1/2} , \quad \Delta T = \text{const } (\alpha L)^{-1/2} , \qquad (2)$$

where the constants depend on the former shape of the second-sound wave. It is important to note that the evolution of the developed shock wave (i.e., the dependence of the parameters of the triangle on distance) is governed only by the value of the nonlinearity coefficient and does not depend on the value of the dissipation μ. The entropy production rate dS/dt at the shock front due to dissipation processes remains finite as dissipation coefficient μ tends to zero, because the small value of μ is compensated for by a big gradient, $\delta T' \sim \Delta T/d_f$. Because of this circumstance, the coefficient μ does not enter in the expressions (2).

2. Results of the investigations

The changes in shape of a solitary second-sound pulse have been studied as a function of duration of the electric pulse τ_e applied to a heater, as a function of emitted heat power Q and the distance L between the source and the superconducting bolometer at three temperatures which correspond to positive (at $T = 1.5$ K), negative ($T = 2.02$ K) and zero ($T = 1.88$ K) values of the nonlinearity coefficient α. The experiments were performed under three different geometrical conditions: a) one dimensional (1D) geometry, when the second-sound pulse travels along the cryoacoustic waveguide — a long capillary with inner diameter $d = 3$ mm filled with superfluid helium; b) «an open» (3D) geometry, when a spherical pulse propagates through a bulk of the liquid, and c) combined geometry, when a formerly excited 3D spherical pulse enters into a capillary placed at some distance from the source and after that propagates along the waveguide as a quasi-1D wave.

a) Experiments on the propagation of 1D second-sound pulse have been divided by two parts. At the beginning, we studied the evolution of the pulse shape with increasing distance L at a constant power Q and duration of a rectangular electrical pulse $\tau_e = 10$ μs. The distance L was increased by a step from 0.7 cm to 8.5 cm.

The typical dependence of the shape on the distance L measured at the bath temperature $T = = 2.02$ K $(\alpha < 0)$ is shown in Fig. 1. It can be seen that for the pulse with initial amplitude $\delta T \sim 3$ mK the shock is formed at the back side of the pulse at a distance $L < 0.7$ cm. The dependence of the amplitude of the wave on the distance L is described well by expressions (2). At $\alpha > 0$ the shock appears at the front of the pulse (which is a general situation in classic nonlinear acoustics).

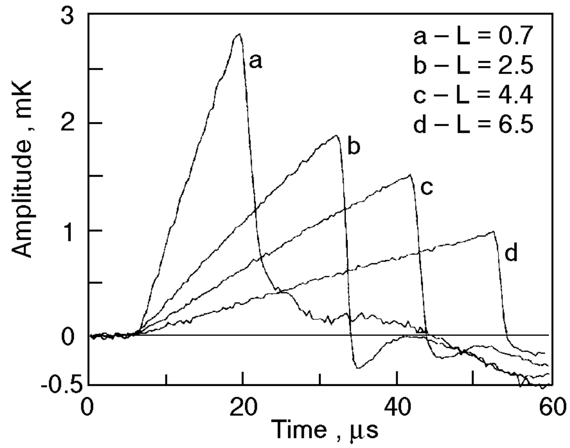

Fig. 1. Evolution with the distance L of the one-dimensional second-sound pulse propagating in a long capillary. The bath temperature is $T = 2.02$ K ($\alpha < 0$). The emitted heat power is $Q = 20$ W/cm^2, $\tau_e = 10$ µs. The distance L if measured in cm.

Figure 2 shows the evolution of the second-sound pulse profile at $T = 1.5$ K with increasing heat power from $Q = 2.4$ W/cm^2 to $Q = 20.2$ W/cm^2 at a constant distance $L = 2.5$ cm and heat pulse duration $\tau_e = 10$ µs. We see that the slope of the profiles $a = \Delta T/\tau$ does not depend on Q: in accordance with the general relations (2), the value $a = c_2/\alpha L$.

We see from Figs. 1 and 2 that the pulses traveling in a capillary are followed by oscillating tails with the amplitudes of the order less than the amplitude of the pulse. The appearance of these tails could be attributed to the nonideality of the experimental cell: the heater consists of a rectangular thin metal film 1.2×2 mm; its dimensions are therefore less than the inner diameter of a capillary. The tails appear because of the multiple reflection of the heater-irradiated nonplanar wave from the walls of the capillary. The subsequent computer simulation revealed the influence of the geometrical nonideality of the cell in the region near the heater on the shape of the recorded pulse.

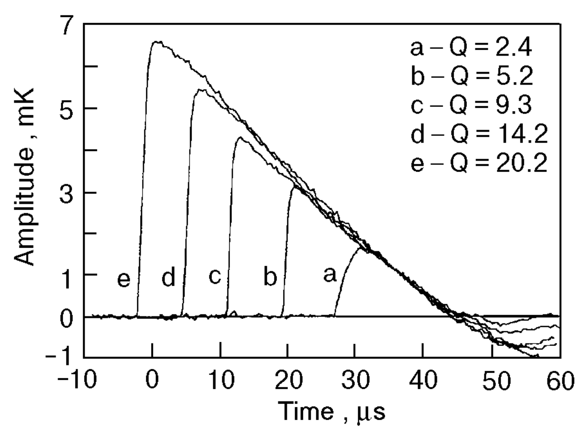

Fig. 2. Change in the shape of the one-dimensional pulse with increasing emitted heat power Q from 2.4 W/cm^2 to 20.2 W/cm^2 at a fixed distance from the heater $L = 2.5$ cm and $\tau_e = 10$ µs. The bath temperature is $T = 1.50$ K ($\alpha > 0$).

b) Open ($3D$) geometry.

An evolution of shape of the spherical pulse propagating in the bulk of liquid helium with increasing power and electric pulse duration has been studied at three different temperatures $T = 1.50$ K ($L = = 4.7$ cm), 1.88 K and 2.02 K (in two latter cases $L = 5.8$ cm). The pulse was generated by the same heater immersed in superfluid helium.

It is known [5] that the second-sound wave generated in He-II by a point source heated by a rectangular electric pulse consists of a heating pulse (compression wave in the roton gas) followed by a cooling wave (rarefaction wave in the gas). In the linear approximation the amplitude of the traveling spherical pulse decreases as $\delta T_{max} \sim 1/R$ with increasing distance R. Allowance for nonlinearity leads to slow (logarithmic) corrections to this dependence [1]. In the case of a shock wave we have $\Delta T \sim (1 + \text{const}\,(\ln R))^m/R$, where the exponent m depends on the asymptotic shape of the wave (and, hence, on the sign of the nonlinearity coefficient), and the constant in the latter relation is defined by initial conditions.

The shape of the bipolar pulse that was generated in He-II at $T = 1.88$ K by the heat pulse of the same duration $\tau_e = 10$ µs and the evolution of the shape with changing nonlinearity coefficient are illustrated in Fig. 3. The distance L is fixed $L = = 5.8$ cm, and the heat density changes slightly from $Q = 23$ W/cm^2 at $T = 2.02$ K (upper graph), to $Q = 21$ W/cm^2 at $T = 1.88$ K (middle graph), and to $Q = 16$ W/cm^2 at $T = 1.50$ K (bottom graph). We see that the final shape of the bipolar nonlinear pulse depends significantly on the sign of the nonlinearity coefficient. If $\alpha > 0$, two shocks appear at the profile of the pulse (at the front of the compression wave and at the back side of the rarefaction wave). If $\alpha < 0$, the shock appears at the middle of the pulse. The asymptotic dependence of the jump ΔT at the shock front at large distances is defined by the final shape of the bipolar pulse: the exponent of the logarithmic factor $m = -1/2$ in the case of the profile with two shocks at the edges of the pulse and $m = -1$ in the case of the profile with a discontinuity in the middle.

In the experiments with spherical $3D$ pulses we have observed an unexpected dependence of the shape of the recorded pulse on the duration τ_e of an electric pulse (in contrast to the one-dimensional case). If τ_e is less than a characteristic time $\tau_0 \approx 20$ µs,

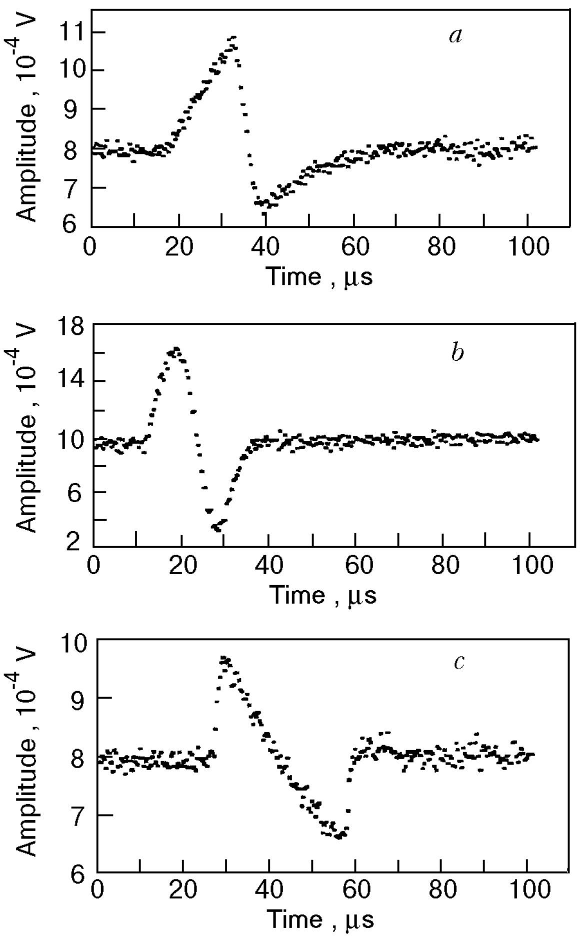

Fig. 3. Detected profile of the $3D$ second-sound pulse at various temperatures. The distance $L = 5.8$ cm, $\tau_e = 10$ μs. *a*) $T = 2.02$ K ($\alpha < 0$) $Q = 23$ W/cm^2; *b*) $T = 1.88$ K ($\alpha = 0$), $Q = 21$ W/cm^2; *c*) $T = 1.50$ K ($\alpha > 0$), $Q = 16$ W/cm^2.

the detected bipolar pulse is presented by coupled heating and cooling pulses. But if τ_e exceeds τ_0 one could observe a shell between the heating and cooling waves (a region with a zero deviation of the temperature from the temperature of the bath). The duration of electric pulse τ_0 at which the shell appears depends only slightly on the temperature of the bath.

In order to explain the appearance of the shell we considered a simple theoretical model which describes the generation of linear second-sound pulse ($\alpha = 0$) by a spherical heater with a radius b. We assumed that a rectangular electric pulse is applied to the heater. The problem involved can be mapped to a well-known problem of the wave equation for a scalar field φ by introducing the hydrodynamic potential using the relation $\nabla\varphi = \mathbf{p}/S$. Here $\mathbf{p}$ is a momentum per unit of mass of relative motion of the normal and superfluid components of He-II, and S is the entropy of a unit volume. The boundary conditions for the potential φ express the continuity of the heat flux at the surface of the heater.

The analyses of the solution show that, in contrast with the one-dimensional case, the shape of generated $3D$ second-sound pulse is defined by an integral relation and, in general, is proportional to a time derivative dQ/dt with the exponentially decaying post-action with characteristic time $\tau_0 \sim$ $\sim b/c_2$. For a rectangular heat pulse the observed profile of the second-sound wave that propagates through a liquid is given by two uncoupled thermal peaks if $\tau_e > \tau_0$. If $\tau_e < \tau_0$, the peaks overlap and the nonlinearity results in an additional broadening of the propagating pulse.

In order to evaluate the broadening τ_0 we made a computer simulation of the generation of linear second-sound pulse, taking into account the real geometry of the heater (flat rectangular film). This gave the value of $\tau_0 \approx 20$ μs, concistent with the measurements results.

The results of such treatment can be used immediately only at $T = T_\alpha$ because the action of nonlinearity is ignored. As follows from previous considerations, the change in the duration of the pulse at $\alpha \neq 0$ is logarithmically small at finite distances, so the measured time τ_0 must depend slowly on the temperature. This conclusion has been confirmed by our observations.

c) The combined geometry.

In these experiments the heater was placed at a distance of 1 cm outside the capillary. The heater generated a nearly spherical bipolar wave that can enter through the open edge of the capillary and propagate along it as a $1D$ wave. The profile of the pulse was measured at the far end of the capillary. This technique makes it possible to create quasi-one-dimensional bipolar second-sound pulse. As in pure spherical case, such a pulse propagates «ballistically» at $T = T_\alpha$ and two shocks appear at the edges of the pulse at $T < T_\alpha$ or the shock is formed in the middle of the pulse at $T > T_\alpha$.

Evolution of the shape of the pulse that propagates through a capillary with increasing heat power ($Q = 2.4$, 9.2 and 20.2 W/cm^2) is shown in Fig. 4. The duration of the electric pulse is $\tau_e =$ $= 10$ μs (short pulse) and the length of the capillary is ~ 5 cm. The temperature of the liquid helium bath is $T = 2.02$ K ($\alpha < 0$).

Computer simulations of the nonlinear evolution of a $1D$ pulse with distance wave performed in the framework of the Burgers equation. Two cases have been studied: a) a rectangular pulse was excited by a plane infinite heater and b) the initial condition

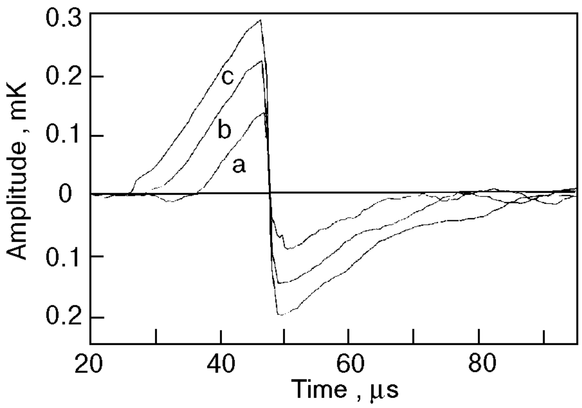

Fig. 4. The shape of the second-sound pulses detected at the far end of the capillary in the experiment with combined geometries. The bath temperature $T = 2.02$ K, the heat pulse duration $\tau_e = 10$ µs. $Q = 24$ W/cm^2 (*a*), $Q = 9.2$ W/cm^2 (*b*), $Q = 20$ W/cm^2 (*c*).

for the wave is given by a sine-like function (a simulation of the combined geometry).

The results of calculations for this case are shown in Fig. 5. In case a) the bath temperature is

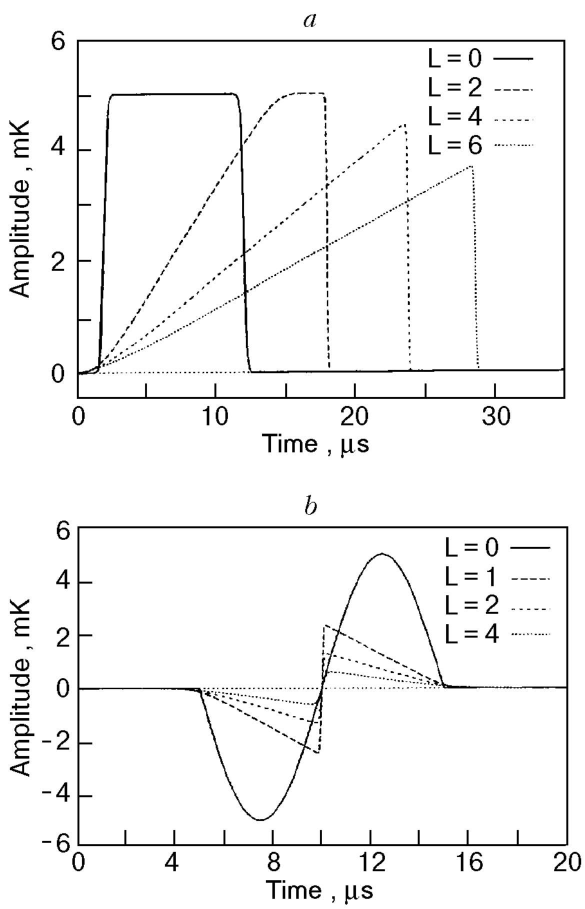

Fig. 5. Results of computer simulation of the evolution with distance of the 1*D* second-sound pulse. The distance *L* is measured in cm. The rectangular pulse, the bath temperature is $T = 1.6$ K ($\alpha > 0$) (*a*). The sine-like initial pulse, the bath temperature is $T = 2.1$ K ($\alpha < 0$) (*b*).

assumed to be $T = 1.6$ K and the initial amplitude of the pulse is 5 mK. We see a formation of the shock at the front of the pulse at a distance from the open edge of the capillary $L \sim 3$ cm.

In case b) the bath temperature is $T = 2.1$ K. The shock appears at the center of the wave (in accordance with our previous considerations). The jump of the temperature ΔT decreases in inverse proportion to the distance $\Delta T \sim 1/L$. (Note that the direction of the x axes on the two last plots is inverse with respect to previous plots; Figs. 1–4 correspond to the signal recorded from the bolometer.) We see that the evolution of the second-sound pulses obtained in numerical simulations coincide qualitatively with the results of observations.

3. Concluding remarks

Thus, the results of experimental and theoretical studies have shown that the shape of excited second-sound waves and their evolution due to nonlinear properties of superfluid helium depend strongly on the dimensionality of the wave and geometrical conditions of the generation and propagation of the wave, as well as on the temperature of the liquid-helium bath. For 3*D* pulses the influence of the finite dimensions of the source of the wave can not be ignored even at large distances.

Despite the relative simplicity of the equation that governs the evolution of the second-sound pulse (the Burgers equation), it exhibits a very interesting behavior. The character of the evolution and the final shape of the pulse can vary significantly under various conditions of the experiment.

It is interesting to note that the dependence of the $\Gamma(k, k_1, k_2)$ function on the wave vectors of the second-sound waves $\Gamma \sim (\Pi_{i=1...3}\, k_i)^{1/2}$ is typical of a wide class of problems of the nonlinear wave propagation, and the Burgers equation appears in many problems in nonlinear physics. In principle, this makes it possible to use the second sound in superfluid helium as an object for model study of many of nonlinear processes in optics and acoustics, in plasma physics, etc.

Interesting applications of the Burgers equation are also found in the theory of turbulence [10]. The experiments with nonlinear second sound could be useful for developing this theory.

Short nonlinear second-sound pulses could be very useful in investigations of dynamic phenomena very close to the lambda point: when a monochromatic second sound is emitted by a heater to which a sinusoidal voltage is applied, a steady heat flux is created in its interior, and the value of this flux is comparable to the oscillating part of the flux within the wave. The existence of such stationary counterflow results in the creation of quantum vortices in the superfluid liquid. This process is most important at temperatures close to T_λ because the threshold of the vortex creation is small. This means that in the case of sinusoidal excitation of the heater, the generated monochromatic second-sound waves propagate through a strongly disturbed liquid, in contrast to studies with short heat pulses. This makes it possible to correctly investigate the dynamic and relaxation processes in the superfluid near the phase transition temperature. In this sense, it would be more correct to use the $1D$ bipolar second-sound pulses, because their nonlinear evolution is relatively simple: the length of the pulse is fixed by the conditions of generation and at the final stage there exists only one parameter ΔT, which changes with distance. This could improve the accuracy of interpretation of the experimental data.

Thus, the experiments with short second-sound pulse performed under microgravity conditions could give, in principle, a more accurate information about the behavior of the superfluid near and far from the phase transition.

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