Low Temperature Physics/Fizika Nizkikh Temperatur, 2019, v. 45, No. 2, pp. 204–209

Scaling laws under quantum Hall effect for a smooth disorder potential

S.V. Gudina, A.S. Klepikova, V.N. Neverov, N.G. Shelushinina, and M.V. Yakunin

M.N. Miheev Institute of Metal Physics of Ural Branch of Russian Academy of Sciences,
18 S. Kovalevskaya Str., Ekaterinburg 620990, Russia
E-mail: klepikova@imp.uran.ru

Received October 3, 2018, published online December 20, 2018

We carried out the analysis of discovered experimental values of the critical parameter κ for the quantum Hall plateau-plateau transitions in modulation-doped GaAs/AlGaAs heterostructures. It turned out that these values are in the main concentrated at the range of 0.5–0.7. We argue that within the theoretical concepts for the large-scale disorder potential, it corresponds to a borderland between quantum tunnelling processes and classical percolation regime. Just, the critical exponent value for the bandwidth of delocalized states, $\kappa = 0.54 \pm 0.01$, obtained by us for HgTe-based heterostructure with inverted band spectrum, can be associated with a smooth character of impurity potential in our system.

Keywords: quantum Hall effect, scaling hypothesis, quantum wells, semiconductors, disorder potential.

1. Introduction

The plateau-plateau transition (between neighboring quantum Hall liquids through an intermediate metal phase) was considered as an electron localization-delocalization-localization quantum phase transition already in the first papers on quantum Hall effect (QHE) interpretation [1,2] and is widely treated at present within the framework of a scaling hypothesis (see, e.g., the reviews [3–7]).

The scaling hypothesis is based on a concept that at the absolute zero of temperature the localization length diverges at the critical energy E_c of the phase transition at the center of the broadened Landau level with a universal exponent γ (the critical exponent of the localization length) [4,8]:

$$\xi(E) = \xi_N \frac{\hbar\omega_c/2}{|E - E_c|^{\gamma}}, \qquad (1)$$

where ω_c is the cyclotron frequency and the constant ξ_N depends on microscopic details of the randomness and on the Landau band index N. For a short-range random potential ξ_N is of the order of cyclotron radius R_c [9].

At finite temperatures, the region of delocalized states at the Landau level center can be described by an energy range where the localization length $\xi(E)$ increases to a characteristic length $\xi(E) > L_\varphi$. Here $L_\varphi \sim T^{-p/2}$ is the phase coherence length and the dynamical exponent p depends on the inelastic scattering mechanism. At $\xi(E) < L_\varphi$ electronic states remain localized and the bandwidth, $\delta\nu$, of delocalized states is determined from the condition $\xi(E) \cong L_\varphi$ [1–4]. Thus the width of the transition between neighboring QHE plateaus, as well as the width of the corresponding peak in the magnetic-field dependence $\sigma_{xx}(B)$ should tend to zero by the power-law dependence T^{κ}, where $\kappa = p/2\gamma$.

2. The current concepts of scaling in the QHE regime

2.1. Short-range random potential

The theoretical investigations of the critical behavior of noninteracting electrons in the quantum Hall system with the short-ranged disorder potentials led to the conclusion about a single diverging length scale and the results of extensive efforts on numerical simulations for the critical exponent gave the value $\gamma = 2.35 \pm 0.03$ (see, for example, reviews [3,4] and the detailed table in the review [5]).

In terms of the dimensionless filling numbers, ν (= n/n_B, n_B being the degeneracy of the Landau level and n is the carrier concentration), Eq. (1) takes the form

$$\xi(\nu) = \xi_N \left|\nu - \nu_c\right|^{-\gamma}. \qquad (2)$$

A schematic representation of localization length divergences with $|\nu - \nu_c|$, where the critical filling factor ν_c is a half-integer value of ν, is provided in Fig. 1(a) for a short-range impurity potential according to theoretical considerations described above.

The critical exponent $\kappa = 0.42$ experimentally determined for the first time in the classical study [10] for

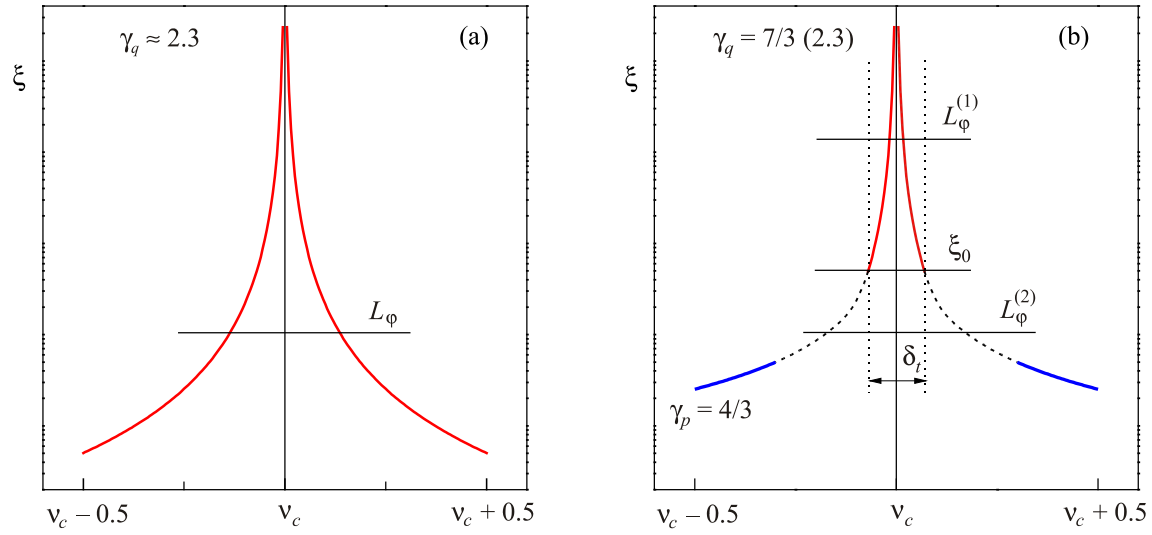

Fig. 1. (Color online) Localization length, ξ, dependences on the filling factor, ν, in the vicinity of critical value $\nu = \nu_c$ within a modern theoretical conception for a short-range (a) or a large-scale (b) impurity potential in QHE regime.

InGaAs/InP systems ($\kappa = 0.42 \pm 0.04$) is in excellent agreement with the conclusions of new unique studies of $Al_xGa_{1-x}As/Al_{0.33}Ga_{0.67}As$ systems in the region of alloy scattering ($\kappa = 0.42 \pm 0.01$) [11] and with the results of recent studies of the first and second Landau levels (both for electrons and holes) in single layer graphene [12,13]. The observable exponent $\kappa = 0.42$ is compatible with a numerical short-ranged potential value $\gamma \approx 2.3$ for the Fermi-liquid dynamical exponent $p = 2$ as it is believed to be the case by Li *et al*. [11] along with the pioneering work of Wei *et al*. [10]. Although the value of the parameter κ is currently the subject of discussion, there is a consensus that $\kappa = 0.42$ indeed describes transitions in the QHE regime (when they are not masked by macroscopic inhomogeneities) for systems with short-range scattering potentials [14,15].

2.2. Large-scale random potential

However, in sharp contrast to the short range alloy potential scattering in InGaAs/InP samples [10], the most perfect and the most studied AlGaAs/GaAs heterostructure has long range Coulomb scattering on remote (by a spacer) ionized impurities which results in nonuniversality of the temperature exponent κ (see both the early [16–19] and the recent works [20–26]).

In modulation-doped GaAs/AlGaAs heterostructures the values $\kappa > 0.42 \pm 0.01$ are regularly observed (see Table 1 in Appendix). In the Table 1 results for critical exponent (κ) values in modulation-doped GaAs/AlGaAs heterostructures from the works of the years 1991–2016 have been collected, and the "nonuniversal" values of parameter κ in the range of 0.5−0.75 come to light.

The fact that a slowly varying potential turned out to be the generic type of disorder in the standard AlGaAs/GaAs heterostructure has led historically to semiclassical considerations (percolation picture) of delocalization near the Landau band center. The ideas, which relate localization to the classical percolation in the context of the integer quantum Hall effect, have been developed intensively by a number of authors (see the article of Prange [27] for exhaustive information).

In the theoretical calculations, an exponent $\gamma = 4/3$ was obtained within a model of classical percolation [28,29]. On the other hand, after including the effect of quantum tunneling, the universal critical exponent $\gamma = 7/3$ results from a model of quantum percolation [29,30] (see a clear exposition of arguments in a number of reviews [3,31,32]).

The percolation model for QHE supplemented by the quantum effects [29,30] provides a physical background for the Chalker–Coddington network model [33] — a generic model, which is assumed to describe the universal quantum mechanical properties of noninteracting electrons in two dimensions in the presence of a random potential subject to a strong perpendicular magnetic field. An overview of the random network model, invented by Chalker and Coddington, and its generalizations is provided, for example, in [3].

In a seminal paper on percolation and quantum tunneling in the integer quantum Hall effect [33] a network model for localization in the QHE regime has been introduced that made it possible to numerically simulate a system where the disorder potential varies slowly on the magnetic length scale. Using the simplifying features of a slowly varying potential in the model the quantum tunneling and interference effects were incorporated. It turned out that the network model contains the features necessary for a qualitative understanding of the integer quantum Hall effect: localized states in the Landau band tails and extended

states in the band center, existing only at one energy. To this extent, the classical picture survives the introduction of quantum tunneling.

There are, however, quantitative changes. In the classical picture, as was shown earlier [28,29], the localization length diverges with an exponent $\gamma = 4/3$. For the network model [33] the value $\gamma = 2.5 \pm 0.5$ was found in a reasonable agreement with estimates for a rapidly varying potential.

The modern theoretical network models for the large-scale impurity potential with the quantum tunneling give a numerical value of the critical exponent $\gamma \approx 2.3$ in the immediate vicinity of the critical energy $E = E_c$ ($\nu = \nu_c$) (see [3,4] and references therein) in accordance with the findings of Ref. 33. On the other hand, far from the critical energy, dependence of ξ on $|E - E_c|$ (on $|\nu - \nu_c|$) is determined by the model of classical percolation with $\gamma = 4/3$ (see Fig. 1(b)).

Let's turn our attention on the results of Wei *et al*. [16] who have found that the T dependence of $(d\rho_{xy}/dB)_{\max}$ behaves like $T^{-0.42}$ in two low-mobility GaAs/$Al_xGa_{1-x}As$ heterostructures from the experiments down to T = 200 mK (see Table 1 in Appendix). It is similar to their earlier reported result for the $In_xGa_{1-x}As$/InP heterostructure [10] but at more lower temperatures. The 2DEG in the $In_xGa_{1-x}As$/InP heterostructure is in the alloy $In_xGa_{1-x}As$ layer and the potential fluctuations are therefore short ranged compared to the cyclotron radius (typically 100 Å). On the other hand, the 2DEG in the GaAs/$Al_xGa_{1-x}As$ heterostructures is in the GaAs layer, and the dominant scattering mechanism at low T is the remote ionized impurities away from the 2DEG layer. One should then expect smooth, long-range potential fluctuations [34,35]. The necessity to lower the temperature for detecting the "universal" scaling in GaAs/$Al_xGa_{1-x}As$ is attributed just to the dominance of the long-range random potential.

Recently Li *et al*. [11] studied the dependence of the exponent κ on x for $Al_xGa_{1-x}As$/$Al_{0.33}Ga_{0.67}As$ heterostructures in a wide Al concentration range and have distinguished three regimes.

For samples in the first regime (x < 0.0065), where the long-range potential for scattering on remote ionized impurities is the main one, κ reaches 0.56−0.58. For the second regime (0.0065 < x < 0.016), the probability of short-range alloy scattering becomes significantly higher, the transport has a quantum nature, and $\kappa = 0.42$ for all samples. Finally, at x > 0.016, κ again increases to 0.57−0.59 because of Al-atom clusterization resulting in a change in the character of disorder in the system (macroscopic inhomogeneities), thus breaking the universal scaling.

It is assumed in Ref. 11 that quantum tunnelling processes (for the short-range impurity potential) are followed by classical processes (for the large-scale potential) with increasing disorder range. Due to the quantum-classical crossover effect the exponent κ increases from 0.42 towards the classical value of 0.75. The fact that the κ values obtained in the first and third regimes, which are still well below 0.75, show that the system is still away from an ideal classical percolation regime. In their subsequent work [25], extending temperature range from 1.2 K down to 1 mK for $Al_xGa_{1-x}As$/$Al_{0.32}Ga_{0.68}As$ heterostructures in a region of long-range disorder (for x = 0 and 0.0021) Li *et al*. have observed a crossover behavior from the high-temperature nonuniversal scaling regime to the low-temperature universal scaling regime with the temperature exponent κ changing from $\kappa = 0.58$ to 0.42, respectively (see Table 1 in Appendix).

3. Diagrams of scaling

As first pointed out by Chalker and Coddington [33] (see above), the tunneling through saddle points should be taken into account for a disorder potential, smooth on the scale of magnetic length $l_B = (\hbar/eB)^{1/2}$. The tunneling becomes determinative in an energy band of the width Δ_t around the critical energy E_c. By assuming a quadratic form for the potential near saddle points, the estimate for the band of tunneling has been obtained [36] (see also [37] or [38]):

$$\Delta_t \approx (l_B/a)^2 \Gamma . \tag{3}$$

Here a is the correlation length of the random potential, which one has taken to be much larger than l_B, Γ equals the disorder-induced width of the Landau level. As $a \gg l_B$, we have from (3) that $\Delta_t \ll \Gamma$.

In dimensionless units Eq. (3) takes the form

$$\delta_t = (l_B/a)^2 (\omega_c \tau)^{-1}, \tag{4}$$

where $\delta_t = \Delta_t/\hbar\omega_c$, and a simple relation $\Gamma = \hbar/\tau$ (τ is the elastic relaxation time) is used to estimate the broadening of the Landau level due to disorder.

Figure 1 schematically shows the diagrams of scaling (2): theoretically assumed dependences of QHE localization length on the filling factor, $\xi(\nu)$, when counting from the critical value of $\nu = \nu_c$ at the center of a Landau level, both for a short-range (a) and for a large-scale impurity potential (b) (see a description in the text).

In Fig. 1(a) it is displayed that for a short-range potential the critical exponent of localization length should be equal to $\gamma_q \approx 2.3$ for the entire interval $|\nu - \nu_c| \le 0.5$.

In Fig. 1(b) the solid lines are: the divergence law (2) with $\gamma = \gamma_p = 4/3$ in regions of classical percolation (thick blue lines) and with $\gamma = \gamma_q$ in regions of the quantum tunnelling processes (thin red lines). Here $\gamma_q = 7/3$ within a modified percolation model [30] and $\gamma_q \approx 2.3$ within the modern network models [3,4]. The dash lines in Fig. 1(b) show an intermediate region of Eq. (2) with $4/3 < \gamma < 7/3$ (or 2.3) that gives $0.42 < \kappa < 0.75$ (if the exponent $p = 2$) in the interspace of crossover from a

Table 1. The critical exponent κ values for modulation-doped GaAs/AlGaAs heterostructures (1991–2016 years)

Structure	PPT	Value of κ	Method	Ref. (year)	Authors
GaAs/$Al_xGa_{1-x}As$	$2 \to 1$ $3 \to 2$ $4 \to 3$	0.42 [$T < 2$ K] 0.72 ± 0.2 [$T > 0.75$ K]	$\left\lvert\frac{d\rho_{xy}}{dB}\right\rvert_{max}$ [0.02–5 K]	[16] (1992)	Wei *et al.*
GaAs/$Al_xGa_{1-x}As$	$3 \to 2$ $4 \to 3$ $5 \to 4$	0.68 ± 0.04 0.72 ± 0.05 0.67 ± 0.06	$\Delta B \sim T^{\kappa}$ $\left\lvert\frac{d\rho_{xy}}{dB}\right\rvert_{max}$ [0.025–1 K]	[17] (1991)	Koch *et al.*
GaAs/$Al_xGa_{1-x}As$	$3 \to 2$ $4 \to 3$ $5 \to 4$ $6 \to 5$	0.5 ± 0.03 0.5 ± 0.03	$\Delta B \sim T^{\kappa}$ [0.3–1.2 K] $\left\lvert\frac{d\rho_{xy}}{dB}\right\rvert_{max}$	[18] (1994)	Yoo *et al.*
GaAs/$Al_xGa_{1-x}As$	$4 \to 3$	0.62 ± 0.04 0.59 ± 0.04	$\Delta B \sim T^{\kappa}$ [0.05–1 K] $\Delta B \sim J^{\kappa/2}$	[19] (1995)	Koch *et al.*
GaAs/$Al_xGa_{1-x}As$	$2 \to 1$	0.66 ± 0.02 S1 0.60 ± 0.02 S2 0.62 ± 0.02 S3	$\Delta B \sim T^{\kappa}$ [0.05–1 K]	[20] (2002)	Hohls *et al.*
GaAs/$Al_xGa_{1-x}As$	$2 \to 1$	0.64 ± 0.09	$\left\lvert\frac{d\rho_{xy}}{dB}\right\rvert_{max}$ [0.3–1 K]	[21] (2004)	Huang *et al.*
$Al_xGa_{1-x}As/Al_{0.32}Ga_{0.68}As$ $x < 0.0085$	$6 \to 5$ $5 \to 4$ $4 \to 3$	0.58–0.49 0.58–0.50 0.57–0.49	$\Delta B \sim T^{\kappa}$ [0.03–1 K] $\left\lvert\frac{d\rho_{xy}}{dB}\right\rvert_{max}$	[11] (2005)	Li *et al.*
GaAs/$Al_{0.35}Ga_{0.65}As$	$3 \to 2$ $4 \to 3$	0.66–0.77	$\Delta B \sim T^{\kappa}$ [1.7–4 K] $\left\lvert\frac{d\rho_{xy}}{dB}\right\rvert_{max}$	[22] (2007)	Tao Tu *et al.*
GaAs/$Al_xGa_{1-x}As$	$6 \to 5$ $7 \to 6$ $8 \to 7$ $10 \to 8$	0.72 (0.74) 0.72 (0.80) 0.75 ± 0.05	$\Delta\nu \sim T^{\kappa}$ [0.05–1.2 K] $\left\lvert\frac{d\rho_{xy}}{dB}\right\rvert_{max}$	[23] (2008)	Zhao *et al.*
GaAs/$Al_xGa_{1-x}As$ mesoscopic system	$1 \to 0$ $3 \to 2$	0.79 0.54	$\Delta\nu \sim T^{\kappa}$ [0.05–5 K]	[24] (2007)	Nakajima *et al.*
$Al_xGa_{1-x}As/Al_{0.32}Ga_{0.68}As$ $x = 0$	$4 \to 3$	0.42 [$T < 120$ mK] 0.58 [$T > 150$ mK]	$\Delta B \sim T^{\kappa}$ [0.03–1.2 K] $\left\lvert\frac{d\rho_{xy}}{dB}\right\rvert_{max}$	[25] (2010)	Li *et al.*
$Al_xGa_{1-x}As/Al_{0.32}Ga_{0.68}As$ $x = 0.0021$	$4 \to 3$	0.42 [$T < 250$ mK] 0.58 [$T > 250$ mK]	$\Delta B \sim T^{\kappa}$ [0.03–1.2 K] $\left\lvert\frac{d\rho_{xy}}{dB}\right\rvert_{max}$	[25] (2010)	Li *et al.*
p-GaAs/$Al_xGa_{1-x}As$	$3 \to 2$ $4 \to 3$ $5 \to 4$	0.52 ± 0.01 0.52 ± 0.02 0.53 ± 0.02	$\Delta\nu \sim T^{\kappa}$ [0.05–1 K]	[26] (2016)	Wang *et al.*

classical percolation to the quantum tunneling as pointed out by Li *et al.* [11].

The dimensionless width of the tunneling band, δ_t, as well as the value of $\xi = \xi_0$, corresponding to a condition

$|\nu - \nu_c| = \delta_t/2$ are also marked on Fig. 1(b). For $|\nu - \nu_c| < \delta_t/2$ (i.e., for $|E - E_c| < \Delta_t/2$) the bandwidth of delocalized states, $\delta\nu$, becomes less than the width of tunneling strip. In these and only in these circumstances the genuine "universal" scaling

$$\xi(\nu) \sim |\nu - \nu_c|^{-\gamma_q}, \qquad (5)$$

corresponding to the quantum tunneling, should be observed. In reality, it is a condition of sufficiently low temperatures, $L_\varphi > \xi_0$ (for example, $L_\varphi = L_\varphi^{(1)}$ in Fig. 1(b)).

We believe that the critical exponent value for the bandwidth of delocalized states, $\kappa = 0.54 \pm 0.01$, obtained by us in [39] for HgTe-based heterostructure with inverted band spectrum, as well as a number of results with κ = 0.5–0.75 for systems with large-scale impurity potentials (see Table 1) are driven by a situation schematically represented on Fig. 1(b) with $L_\varphi = L_\varphi^{(2)} < \xi_0$: the line $L_\varphi^{(2)} = \text{const}$ crosses the curves $\xi(\nu)$ just at the intermediate region of γ values that (for $p = 2$) gives $0.42 < \kappa < 0.75$. This situation, quite possibly, is typical for modulation-doped GaAs/AlGaAs heterostructures [16–26] manifesting itself in supervision of "nonuniversal" values of parameter κ.

Conclusions

Thus, the analysis of the available in the literature a great amount of experimental results on critical exponent κ values, extracted from the temperature dependences of the QHE plateau-plateau transition width in modulation-doped GaAs/AlGaAs heterostructures, led us to the conclusion that in most experiments one is dealing with an intermediate situation between quantum tunnelling processes (genuine scaling, $\kappa = 0.42$) and classical percolation regime ($\kappa = 0.75$).

This work was done in frames of the state task on the topic "Electron" No. AAAA-A18-118020190098-5, project No. 18-10-2-6 of the Ural Branch Program of the Russian Academy of Sciences and partly by the Russian Foundation for Basic Research (project Nos. 18-02-00172 and 18-02-00192).

Appendix

Here is a table of experimental results for critical exponent κ values extracted from the temperature dependences of QHE plateau-plateau transition (PPT) width in modulation-doped GaAs/AlGaA heterostructures [11,16–26].

In the Table 1 the following abbreviations for a method of determination of the critical exponent from the experimental data on the Hall, ρ_{xy}, and the longitudinal, ρ_{xx}, resistivities are used. The values of κ have been found from the temperature dependences both of the slope of the steps between adjacent quantum Hall plateaus:

$$\left|\frac{d\rho_{xy}}{dB}\right|_{B=B_c} \equiv \left|\frac{d\rho_{xy}}{dB}\right|_{\max} \sim T^{-\kappa}, \qquad (11)$$

and of the longitudinal resistance peak width at the PPT:

$$\Delta B \sim T^{\kappa}. \qquad (12)$$

In Ref. 19 a scaling analysis of the current (J) dependence of the resistance peak width was also carried out:

$$\Delta B \sim J^{-\kappa/2}. \qquad (13)$$

It is seen from the Table 1 that the discovered values of parameter κ are in the main concentrated at the range of 0.5–0.75. Within the theoretical concepts for the large-scale impurity potential (see the text) it corresponds to a borderland between quantum tunnelling processes (genuine scaling, $\kappa = 0.42$) and classical percolation regime ($\kappa = 0.75$).

1. H. Aoki and T. Ando, *Phys. Rev. Lett.* **54**, 831 (1985).
2. A.M.M. Pruisken, *Phys. Rev. Lett.* **61**, 1297 (1988).
3. B. Kramer, T. Ohtsuki, and S. Kettemann, *Phys. Rep.* **417**, 211 (2005).
4. B. Huckestein, *Rev. Mod. Phys.* **67**, 367 (1995).
5. B. Kramer, S. Kettemann, and T. Ohtsuki, *Physica E* **20**, 172 (2003).
6. V.F. Gantmakher and V.T. Dolgopolov, *Phys. Usp.* **51**, 3 (2008).
7. F. Evers and A.D. Mirlin, *Rev. Mod. Phys.* **80**, 1355 (2008).
8. B. Huckestein and B. Kramer, *Phys. Rev. Lett.* **64**, 1437 (1990).
9. M.M. Fogler, A.Yu. Dobin, and B.I. Shklovski, *Phys. Rev. B* **57**, 4614 (1998).
10. H.P. Wei, D.C. Tsui, M.A. Paalanen, and A.M.M. Pruisken, *Phys. Rev. Lett.* **61**, 1294 (1988).
11. W. Li, G.A. Csathy, D.C. Tsui, L.N. Pfeiffer, and K.W. West, *Phys. Rev. Lett.* **94**, 206807 (2005).
12. K. Bennaceur, P. Jacques, F. Portier, P. Roche, and D.C. Glattli, *Phys. Rev. B* **86**, 085433 (2012).
13. A.J.M. Giesbers, U. Zeitler, L.A. Ponomarenko, R. Yang, K.S. Novoselov, A.K. Geim, and J.C. Maan, *Phys. Rev. B* **80**, 241411(R) (2009).
14. A.M.M. Pruisken, *Intern. J. Mod. Phys. B* **24**, 1895 (2010).
15. I.S. Burmistrov, S. Bera, F. Evers, I.V. Gornyi, and A.D. Mirlin, *Ann. Phys.* **326**, 1457 (2011).
16. H.P. Wei, S.Y. Lin, D.C. Tsui, and A.M.M. Pruisken, *Phys. Rev. B* **45**, 3926(R) (1992).
17. S. Koch, R.J. Haug, K. von Klitzing, and K. Ploog, *Phys. Rev. Lett.* **67**, 883 (1991).
18. K.-H. Yoo, H.C. Kwon, and J.C. Park, *Solid. State Commun.* **92**, 821 (1994).
19. S. Koch, R. J. Haug, K. von Klitzing, and K. Ploog, *Semicond. Sci. Technol.* **10**, 209 (1995).
20. F. Hohls, U. Zeitler, and R.J. Haug, *Phys. Rev. Lett.* **88**, 036802 (2002).
21. C.F. Huang, Y.H. Chang,H.H. Cheng, C.-T. Liang, and G.J. Hwang, *Physica E* **22**, 232 (2004).
22. Tao Tu , Yong-Jie Zhao, Guo-Ping Guo, Xiao-Jie Hao, and Guang-Can Guo, *Phys. Lett. A* **368**, 108 (2007).

23. Y.J. Zhao, T. Tu, X.J. Hao, G.C. Guo, H.W. Jiang, and G.P. Guo, *Phys. Rev. B* **78**, 233301 (2008).
24. T. Nakajima, T. Ueda, and S. Komiyama, *J. Phys. Soc. Jpn*. **76**, 094703 (2007).
25. W. Li, J.S. Xia, C. Vicente, N.S. Sullivan, W. Pan, D.C. Tsui, L.N. Pfeiffer, and K.W. West, *Phys. Rev. B* **81**, 033305 (2010).
26. X. Wang, H. Liu, J. Zhu, P. Shan, P. Wang, H. Fu, L. Du, L.N. Pfeiffer, K.W. West, X. C. Xie, R.-R. Du, and X. Lin, *Phys. Rev. B* **93**, 075307 (2016).
27. R.E. Prange and S.M. Girvin (eds.), *The Quantum Hall Effect*, Graduate Texts in Contemporary Physics, Springer, Berlin (1987).
28. S.A. Trugman, *Phys. Rev. B* **27**, 7539 (1983).
29. D.-H. Lee, Z. Wang, and S. Kivelson, *Phys. Rev. Lett.* **70**, 4130 (1993).
30. G.V. Mil'nikov and I.M. Sokolov, *JETP Lett.* **48**, 536 (1989).
31. A. Hansen, E.H. Hauge, J. Hove, and F.A. Maaø, *cond-mat/9703026*.
32. I.S. Burmistrov, *Vvedenie v teoriju tselochislennogo kvantovogo effekta Holla*, *IPHF RAN*, Thernogolovka (2015).
33. J.T. Chalker and P.D. Coddington, *Phys. Rev. Lett*. **21**, 2665 (1988).
34. A.L. Efros, *Solid State Commun*. **65**, 1281 (1988).
35. A. Gold, *Appl. Phys. Lett.* **54**, 2100 (1989).
36. H.A. Fertig and B.I. Halperin, *Phys. Rev. B* **36**, 7969 (1987).
37. D.G. Polyakov and M.E. Raikh, *Phys. Rev. Lett* **75**, 1368 (1995).
38. A.M.M. Pruisken, B. Skoric, and M.A. Baranov, *Phys. Rev. B* **60**, 16838 (1999).
39. Yu.G. Arapov, S.V. Gudina, V.N. Neverov, S.M. Podgornykh, M.R. Popov, G.I. Harus, N.G. Shelushinina, M.V. Yakunin, N.N. Mikhailov, and S.A. Dvoretsky, *Semiconductors* **49**, 1545 (2015).

Скейлінг в режимі квантового ефекту Холла для плавного потенціалу безладу

С.В. Гудина, А.С. Клепікова, В.Н. Неверов, Н.Г. Шелушініна, М.В. Якунін

Проведено аналіз експериментальних значень критичного параметра квантових холлівських переходів к типу «плато-плато» в селективно легованій гетероструктурі GaAs/AlGaAs. Виявилося, що ці значення в основному зосереджені в діапазоні к = 0,5–0,7. Стверджується, що в рамках теоретичних уявлень про великомасштабний потенціал безладу це відповідає межі між процесами квантового тунелювання та класичним режимом перколяції. Так само величина критичної експоненти к = = 0,54 ± 0,01 для ширини смуги делокалізованних станів, отримана для гетероструктури на основі HgTe з інвертованим спектром, може бути пов'язана з плавним характером домішкового потенціалу в дослідженій системі.

Ключові слова: квантовий ефект Холла, скейлінг, квантові ями, напівпровідники, потенціал безладу.

Скейлинг в режиме квантового эффекта Холла для плавного потенциала беспорядка

С.В. Гудина, А.С. Клепикова, В.Н. Неверов, Н.Г. Шелушинина, М.В. Якунин

Проведен анализ экспериментальных значений критического параметра к квантовых холловских переходов типа плато–плато в селективно легированной гетероструктуре GaAs/AlGaAs. Оказалось, что эти значения в основном сосредоточены в диапазоне к = 0,5–0,7. Утверждается, что в рамках теоретических представлений о крупномасштабном потенциале беспорядка это соответствует границе между процессами квантового туннелирования и классическим режимом перколяции. Точно так же величина критической экспоненты к = 0,54 ± 0,01 для ширины полосы делокализованных состояний, полученная для гетероструктуры на основе HgTe с инвертированным спектром, может быть связана с плавным характером примесного потенциала в исследованной системе.

Ключевые слова: квантовый эффект Холла, скейлинг, квантовые ямы, полупроводники, потенциал беспорядка.