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Collapse of a transport current in hard superconductors

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A phenomenon of compression of a transport current I by an external ac magnetic field in hard superconductors being in the critical state is studied both theoretically and experimentally. This phenomenon occurs in a forceless configuration when the ac field and the transport current are parallel. The collapse takes place owing to forcing out the transport current by shielding currents flowing inside a surface region of a sample. Using the Hall probe we have directly observed such stratification of the currents with different orientations. The transport current tube contracts to the sample axis as the ac amplitude increases and occupies the whole sample core at some threshold amplitude H_t. As a result, the superconductor goes to the resistive state. A dependence of H_t on I is analyzed both theoretically and experimentally. It is shown, that the collapse in superconducting rings is accompanied by complex relaxation processes. The mechanism of decay of the circular current in rings under the action of an ac signal is investigated in detail. An application of the collapse to test the homogeneity of the critical current capability of superconductors is discussed.

1. Introduction

The study of the electrodynamics of hard superconductors is the focus of attention for many research groups. Electromagnetic properties of these materials in magnetic fields exceeding the lower critical field H_c are well described by the critical state model [1,2]. This model was proposed by Bean to calculate static hysteresis magnetization curves of hard superconductors. According to this model, the distribution of the magnetic induction B in a sample bulk should be determined by the following equation:

$$\text{rot}\,\mathbf{B} = \frac{4\pi}{c} j_c(B) \frac{\mathbf{E}}{E} . \qquad (1)$$

Here j_c is the critical current density depending on the magnetic induction B; E is the electric field; c is the velocity of light. The vectors B, E and the quantity j_c are averaged over a spatial scale, which exceeds the London penetration depth λ. According to the critical state model, the current density j is everywhere directed along the electric field E and its absolute value is j_c. Without electric field, the current j is parallel to the last direction of E just before it disappeared. The current density j is equal to zero inside a sample bulk where the electric field never penetrated.

The critical state model deals with a single phenomenological function $j_c(B)$ that is defined by the pinning of the Abrikosov vortices. Therefore, it describes adequately the electrodynamic properties of hard superconductors in a low-frequency range up to several kHz [3] only, where such processes as flux creep, flux flow, quasiparticle absorption, etc. (see, for example, Ref. 4, and references cited therein) do not play an essential role.

The interest in the critical state model increased after the discovery of high-T_c superconductivity. Dersch and Blatter [5] generalized this model to apply it to Josephson HTS systems placed in an external magnetic field lower than the first critical field H_{clg} of grains. The form of their equation differs from Eq. (1) in the additional factor μ on the right-hand side. The effective magnetic permeability μ is connected with intergranular currents in ceramics. It has been the critical state model which formed the basis of the contactless method for study of critical current capability in HTS samples [6,7].

The simple consideration shows Eq. (1) to be essentially nonlinear and this nonlinearity has analogues in neither conducting media. Therefore, its physical consequences call for special theoretical investigations. The present paper is devoted to a study of an unusual nonlinear electromagnetic phenomenon that results directly from the critical state model and occurs in hard superconductors.

The critical state model is usually used for the analysis of a one-component situation, when the current density is directed along a single coordinate line inside the whole sample bulk. For example, a radial distribution of the transport current density in a cylindrical sample was studied in Ref. 5 (the current density was directed everywhere along the sample axis). A nonlinear response of a superconducting cylinder to an external ac field was investigated in Refs. 8–10 (the induced current density contained an

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azimuth component only). A more complex situation, when the currents with the different directions coexist, is considered in the present paper. Such a situation is realized in a superconducting cylinder with the transport current I placed in the external ac magnetic field **H** parallel to the cylinder axis. The action of the ac field turns out to cause the phenomenon that we shall call the collapse of the transport current. This effect consists in the following.

In the absence of an ac field the transport current, whose value is less than the critical one, is known to flow in a surface region of the sample, i.e., it occupies a surface tube. The thickness of this tube depends on the value of I. This picture is schematically shown in Fig. 1,*a*. If the ac field has been switched on, the azimuth shielding ac current exists near the sample surface. The transport current turns out to vanish inside the space where the ac field penetrates, and the transport current tube is displaced into a deeper region of the superconductor. In other words, both the tube and its core, which is free of the current, are compressed by the ac field (see Fig. 1,*b*). As the ac field amplitude increases, the diameter of the core decreases and vanishes at some threshold amplitude H_t (see Fig. 1,*c*). Upon further increase in the ac amplitude, the superconductor goes over to the resistive state. It should be particularly emphasized that the collapse of the transport current described above occurs in a forceless configuration, when the ac magnetic field is parallel to the vector of the transport current density. At first sight, it would seem that the ac magnetic field cannot essentially affect the spatial distribution of the longitudinal current. Nevertheless, a simple analysis of Eq. (1) leads to the prediction of this phenomenon. The preliminary reports concerning the collapse of the transport current were published in brief [Refs. 11–13].

The origin of the collapse is elucidated in Sec. 2. The dependence of the threshold amplitude H_t on the value of the transport current I in the designated current regime is analyzed theoretically therein. The experimental study of the collapse in HTS cylinders and the dependence $H_t(I)$ is presented in Sec. 3. The direct observation of the stratification of the screening and transport currents using the Hall probe is described in the same section. Then, the study of the collapse of a circular current and accompanying relaxation processes in a superconducting ring without an external current source is carried out experimentally (Sec. 4) and theoretically (Sec. 5). Finally, the application of this phenomenon to test the homogeneity of the critical current capability of superconducting samples along their length is discussed in Sec. 6.

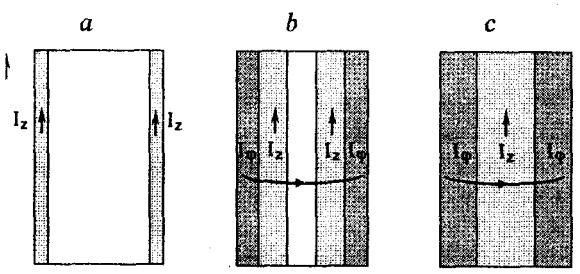

Fig. 1. Schematic picture of current distribution in cylindrical hard superconductor at various amplitudes of the external ac magnetic field: $H_m = 0$ (*a*), $0 < H_m < H_t$ (*b*); $H = H_t$ (*c*).

2. Collapse of the transport current in a cylindrical sample

Let us consider a cylindrical sample of a hard superconductor, the designated nondissipative transport current I is flowing along its axis. We are interested here in the case when the value of I is less than the critical current I_c. Therefore, this current flows in the cylinder surface region (with a thickness specified by the value of I) and has the critical density j_c. The limiting value I_c of the current I corresponds to a situation in which the current density j is equal to j_c over the entire cross section of the sample. The picture of the current flow in the sample at $I < I_c$ is shown schematically in Fig. 1,*a*. Let the sample be placed in the external ac magnetic field $H(t)$,

$$H(t) = H_m \cos(\omega t) \ . \qquad (2)$$

This field is directed along the cylinder axis z. This means that we deal with the forceless configuration: the external magnetic field is parallel to the transport current.

The external magnetic field $H(t)$ induces a longitudinal ac magnetic induction $B(r, t)$ and an azimuth electric field $E_\varphi(r, t)$ inside the sample bulk. They are connected by the Faraday law:

$$\text{rot } E(r, t) = -\frac{1}{c}\frac{\partial B(r, t)}{\partial t} \ . \qquad (3)$$

The field $E_\varphi(r, t)$ leads to the appearance of the current j_φ shielding the field $H(t)$. It is extremely important that only the φ-component of the electric field is present in the sample. The field E_z equals zero as long as the transport current density nowhere exceeds j_c. Therefore, according to critical state equation (1), the azimuth current j_φ exists alone wherever the ac magnetic field has penetrated and the field E_φ is present. This implies that the transport current flowing in this area in the absence of $H(t)$ is displaced into deeper layers of the superconductor (see Fig. 1,*b*). The compression of the transport current occurs until the amplitude H_m of the external field

reaches the threshold value H_t at which the current density j_z at the sample axis becomes nonzero (Fig. 1,*c*). At $H_m > H_t$ the transport current density exceeds the critical value j_c and the sample goes to the resistive state. The arising longitudinal electrical field E_z results in the transport current flowing through the whole cross section of the sample.

Using the critical state model one can calculate the dependence of the threshold field H_t on the transport current value I. It is suitable to rewrite Eq. (1) in the cylindrical coordinates (r, φ, z). This equation has the following form in the region $r < r_0$ where the ac magnetic field B_z does not penetrate and only the dc magnetic field B_φ of the transport current exists:

$$B'_\varphi + \frac{B_\varphi}{r} = \frac{4\pi}{c} j_c(B_\varphi) \ . \qquad (4)$$

Here the prime denotes the derivative with respect to the radial coordinate r. The function $B_\varphi(r)$ should satisfy the following boundary condition at the point $r = r_0$:

$$B_\varphi(r_0) = \frac{2I}{r_0 c} \ . \qquad (5)$$

If the amplitude H_m of the external magnetic field is equal to the threshold value H_t, Eq. (4) is valid inside the whole interval of r running from zero up to r_0. In this case we should use the boundary condition for $B_\varphi(r)$ at $r = 0$

$$B_\varphi(0) = 0 \ . \qquad (6)$$

Outside this region, at $r_0 < r < R$ (R is the radius of the cylinder), where the transport current does not exist ($j_z = 0$), the azimuth component $B_\varphi(r)$ of the magnetic field decreases in inverse proportion to r:

$$B_\varphi(r) = \frac{2I}{rc} \ , \qquad r_0 < r < R \ . \qquad (7)$$

Along with the azimuth component $B_\varphi(r)$, the longitudinal ac component B_z of the magnetic field is also present in this region. Its spatial distribution at the moment $t = 0$ is described by the following equation:

$$B'_z = \frac{4\pi}{c} j_c \left(B_z^2 + (2\mu I/rc)^2 \right)^{1/2} \qquad (8)$$

combined with the boundary conditions

$$B_z(R) = H_t \ , \ B_z(r_0) = 0 \ . \qquad (9)$$

Set (4)–(9) of equations and boundary conditions gives the unambiguous connection between the threshold amplitude H_t and the value of the transport current I.

One cannot solve this system analytically in the general case for an arbitrary function $j_c(B)$. Asymptotical expressions for $H_t(I)$ can be derived at small and large values of the current I:

$$H_t = \begin{cases} H_p - \dfrac{4\pi}{c} \left(\dfrac{I}{\pi j_c(0)} \right)^{1/2} j_c(H_p) \ , & I \ll I_c \ , \\ \dfrac{2}{R_c} (I - I_c) \ , & I - I_c \ll I_c \ . \end{cases} \qquad (10)$$

The values H_p and I_c represent the amplitude of the external field H_m, at which the ac signal reaches the sample axis at $I = 0$, and the critical transport current, respectively. Both of them are the functionals of $j_c(B)$. The amplitude H_p is defined by the equation

$$\int_0^{H_p} \frac{dB}{j_c(B)} = \frac{4\pi}{c} R \ , \qquad (11)$$

whereas the critical current I_c is determined by the following expression:

$$I_c = 2\pi \int_0^R j_c B_\varphi(r) r \, dr \ . \qquad (12)$$

The function $B_\varphi(r)$ in Eq. (12) should be found from Eq. (4) combined with boundary conditions (5) and (6) with $r_0 = R$.

To obtain the dependence $H_t(I)$ in the whole range of the transport current change ($0 < I < I_c$, $H_p > H_t > 0$) one should specify a particular form of $j_c(B)$. Below we use the following relation:

$$j_c(B) = \frac{j_0}{1 + (B/B^*)^{3/2}} \ . \qquad (13)$$

This relation adequately describes $j_c(B)$ for HTS ceramic samples [6,7] which represent a suitable object for the experimental study of the described phenomenon. The parameter B^* is the characteristic scale of $j_c(B)$ decrease and j_0 is the critical current density at $B = 0$. Set (4)–(9) can not be solved analytically. However, we note that, by using new dimensionless variables

$$h = H_t/H_p \ , \ i = I/I_c \ , \ b_\varphi = B_\varphi/H_p \ , \\ b_z = B_z/H_p \ , \ \xi = r/R \ , \ \xi_0 = r_0/R \qquad (14)$$

one can transform this system to the form containing the single parameter α, which specifies the sensitivity of the investigated effect to the magnetic field dependence of the critical current density:

$$\alpha = \frac{4\pi}{cB^*} j_0 R \ . \qquad (15)$$

The transformed system was resolved by numerical methods for different values of α. The result of $h(i)$ calculations is presented in Fig. 2 for $\alpha = 100$. We call attention that the curve in Fig. 2 has points of inflection where $d^2h/di^2 = 0$. The appearance of such points is stipulated by a competition of two circumstances affecting the form of the curve $h(i)$. The first circumstance consists in the sensitivity of the critical current density to the magnetic field, and the second one is connected to the nonlinear decrease (quadratic at $I << I_c$) in the transport current flowing area with the increase in H_m. Owing to the second (geometrical) factor alone (at $j_c(B) = \text{const}$) the relation $h(i)$ takes the universal form:

$$h = (1 - i)^2 \ . \qquad (16)$$

In this situation the second derivative d^2h/di^2 is positive everywhere. If we take into account the dependence $j_c(B)$ but not the geometrical factor (for example, if we are interesting in the collapse of the transport current in a slab), we shall obtain the curve $h(i)$ with the curvature of an alternating sign. In a cylindrical geometry the existence of points, where d^2h/di^2 equals zero, depends on the value of the parameter α. In our model (13) these points appear on the curve $h(i)$ at $\alpha \geq 18$. The calculations show that the plot $h(i)$ is insensitive practically to the value of α at $\alpha \geq 50$.

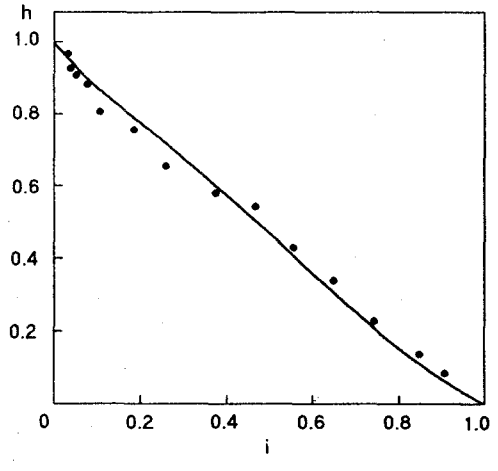

Fig. 2. Dependence of the dimensionless critical current $i = I/I_c$ on the threshold amplitude $h = H_1/H_p$. A solid line is the result of numerical calculations at $\alpha = 100$, closed circles present the data of our experiment. The criterion for definition of I_c was equal to 0.1 μV/cm.

3. Observation of the transport current collapse in cylindrical samples

The experimental research of the transport current collapse in HTS ceramic cylinders was carried out by two methods.

1. The first method consists in the measurement of the threshold amplitude H_t and in the study of its dependence on the value of the transport current I.

The experiments were conducted on cylindrical samples of yttrium HTS ceramics of about 0.52 cm across and 4–5 cm in length, cooled to liquid nitrogen temperature. The values of the critical current I_c at $H_m = 0$ and the penetration field H_p at $I = 0$ were $I_c = 20$ A and $H_p = 33.5$ Oe, respectively. The sample was placed inside a long solenoid which induced ac field (2). We measured the ac amplitude H_m, at which the voltage drop appeared, for each designated value of the current I. The appearance of the voltage drop testified that the amplitude had reached the threshold value, $H_m = H_t$. Points in Fig. 2 represent the results of our determination of the dimensionless amplitude $h = H_t/H_p$ as a function of the dimensionless transport current $i = I/I_c$. These results were practically insensitive to the value of ac frequency within a wide range from 10 up to 10^4 Hz. The calculated data are presented in this figure by a solid line. The experimental points are seen to be located rather close to the theoretical curve.

We would like to pay attention that there exists one more mechanism of the sample transition to the resistive state under the action of the external magnetic field. As mentioned above, the critical current density in HTS ceramics is sensitive to the magnetic field. Therefore, for any transport current there exists such a value of the external magnetic field, at which $j_c(B)$ becomes less than the transport current density, and the sample goes over to the resistive state. Such a mechanism may be realized in a superconductor placed in a static magnetic field H_0. In this case the transport current occupies the whole sample cross section as the sample transiting to the resistive state. If the transition to the resistive state is stipulated by the ac field, the transport current, owing to the collapse, flows through the central area only where the external field does not penetrate. Therefore, the dependence of the critical current on the external field $I_c(H_0)$ under the static conditions differs from the dependence $I(H_t)$ in the ac field. To demonstrate this difference the experiment was performed not only in the ac field (2) but also under the static conditions.

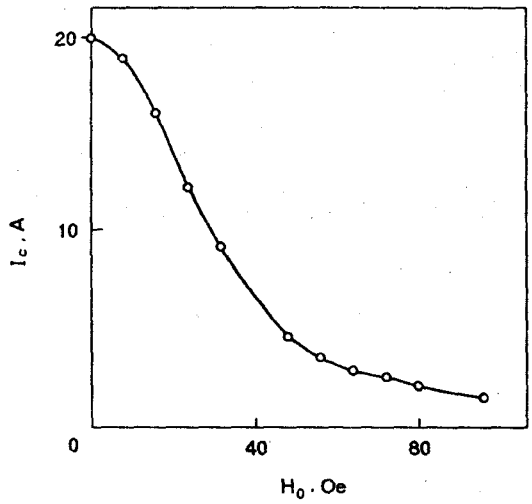

Fig. 3. Dependence of the critical current I_c on the dc magnetic field H_0. Points are results of the experiment.

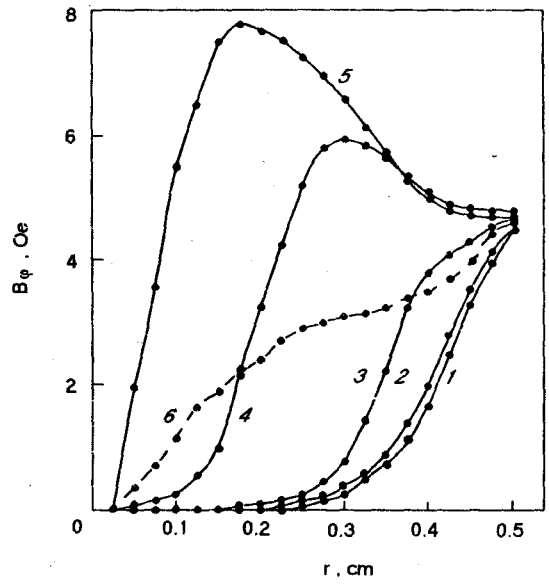

Fig. 4. Radial distribution of the intrinsic magnetic field of the transport current $I = 10$ A at different amplitudes of ac magnetic field H_m, Oe: 0 (*1*); 10.2 (*2*); 15 (*3*); 20 (*4*); 25 (*5*); 3000 (*6*).

The dependence $I_c(H_0)$ obtained in the wide range of the dc magnetic field H_0 is shown in Fig. 3. The comparison of the results in Figs. 2 and 3 shows that the main distinction between curves $I_c(H_0)$ and $I(H_t)$ is observed at small currents. Actually, the flow of the undissipative transport current is impossible at $H_t > H_p \approx 50$ Oe, while at the static conditions there exists the finite undissipative transport current in the field $H_0 > H_p$. Its value decreases monotonously with the increase in H_0.

2. In the experiment described above, we registered the final stage of the collapse, when the sample went over to the resistive state due to the compression of the transport current. However, in this experiment it was impossible to analyze the current density distribution inside the sample and to confirm the predicted phenomenon of the stratification of the transport and azimuthal currents when collapsing. To confirm this fact we executed the direct observation of the collapse in cylindrical HTS samples using the miniature Hall gauge.

We drilled an opening of 0.8 mm in diameter perpendicular to the axis of a cylindrical sample 10 mm in diameter. The Hall gauge was placed into this opening in such a way to allow us to move it with a micrometric screw. The device permitted us to measure the spatial distribution of the azimuth components of the dc magnetic field B_φ associated with the transport current. The experiment was performed at a temperature of 77 K over the frequency range 30–1000 Hz and the designated transport current of 10 A.

The results of $B_\varphi(r)$ measurements at various amplitudes H_m are indicated in Fig. 4. In the absence of the external ac magnetic field (curve *1*) and at the small amplitude H_m (curve *2*), the maximum value of the azimuth field B_φ is reached near the sample surface ($r = R$). At the same time, this field is close to zero within the wide region near the cylinder axis. This means the transport current flows in the surface layer of the superconductor. The distribution $B_\varphi(r)$ qualitatively changes at reasonably strong ac fields (curves *4* and *5*). The distinct maximum appears on the curves and its position displaces deep inside the sample as H_m increases. In the strong external ac field $H_m = 25$ Oe (curve *5*) the magnetic induction B_φ becomes nonzero at $r = 0$. Just under these conditions the voltage drop arises along the sample (this drop was registered on potential contacts). This suggests that at the amplitude $H_m = H_t = 25$ Oe the density of the collapsed current reaches critical value and the sample goes over to the resistive state. Starting from this value of H_m the maximum of the function $B_\varphi(r)$ degenerates (curve *6*), i.e., the distribution of the current becomes more and more uniform with the increase in H_m.

The described results directly prove the phenomenon of transport current collapse in the external ac magnetic field. Actually, the penetration of ac field into the sample bulk causes the azimuth shielding current of the critical density to flow near the surface. The transport current is accordingly displaced from this region. The existence of the maximum on the curve $B_\varphi(r)$ implies that the tube containing the transport current and having the initial external radius

close to the sample radius is compressed (about twice at $H_m = 20$ Oe) by the external ac field. The distribution of the ac magnetic induction $B_z(r)$ near the surface, measured by the same Hall gauge revolved on 90 °, correlates well with the prediction of the critical state model. The position of the maximum of $B_\varphi(r)$ coincides with the border separating the transport current from the azimuth one. Mixing the azimuth and axial currents occurs if the flow of the transport current becomes dissipative.

4. Collapse of the circular current in HTS ceramic rings. Experiment

In the previous sections we studied the collapse of the transport current, when the value of I is kept constant by the external source. Hence, the transport current did not disappear at any value of the ac amplitude. The final stage of the collapse consisted in the transition of the sample into the resistive state.

It is interesting to consider the collapse development in a physical situation that differs in principal — without source of the transport current. In this case the compression of a current should be accompanied by irreversible dissipative processes whose role is not obvious beforehand. To analyze this situation we used superconducting rings as subjects of inquiry. In this section we shall, at first, discuss the results of our experiments and give their theoretical interpretation in the next section.

The typical sizes of rings prepared from yttrium ceramics were: $a \approx 0.4$ cm, $R \approx 0.06$ cm, $R \ll a$ (Fig. 5). The circular current was induced by switching off an external dc magnetic field. For this purpose a long solenoid was placed coaxially inside the ring. The electric current flowing through the solenoid was switched on, when the ring temperature was above T_c. After cooling the ring down to the liquid nitrogen temperature this current was switched off. As a result, the circular current arose in the ring. Its value is specified by the condition of the conservation of the magnetic flux through the ring. It is necessary to remark that under such an excitation method the induced current has a non-uniform distribution over the cross section of the torus. The circular current of the critical density j_c occupies the surface region of the sample (the shaded region in Fig. 5). Outside this region $j = 0$. The size of the shaded region is obviously determined by the full circular current I.

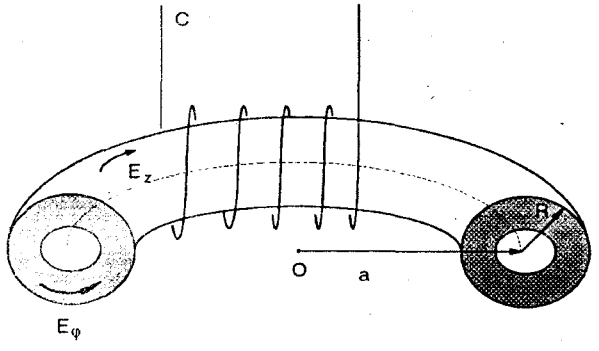

Fig. 5.

The ac magnetic field (2) was generated along the ring by the coil C (Fig. 5). The collapse of the circular current with the increase in the ac amplitude H_m was studied by measuring the static magnetic moment M of a ring vs. H_m. Obviously, the value of M for the thin ring with $R \ll a$ is proportional to the circular current I:

$$M = \pi a^2 I / c \ . \qquad (17)$$

The record of the dependence $M(H_m)$ under the conditions, when the maximum allowable circular undissipative current $I = I_c$ has been previously induced in the ring, is presented by curve *1* in Fig. 6. Owing to the collapse, the circular current disappears from the region where the ac magnetic field penetrates. Since the whole cross section of the ring is occupied initially by the circular current with the extreme allowable density j_c, the part of this current which was forced out by the ac field should completely dissipate. Our experiment shows that such a process lasts for a rather extended time interval of about several hundred seconds. It is remarkable that the curve $M(H_m)$ is identical to the plot of the inverse function $I(H_t)$ in Fig. 2. This result reflects the fact that under the dis-

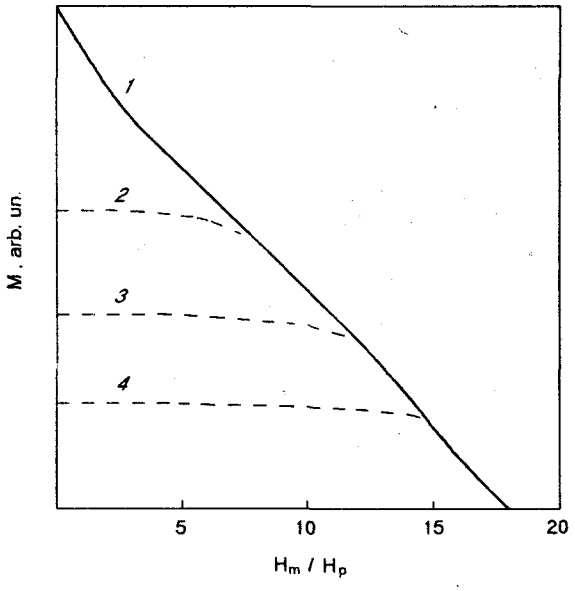

Fig. 6. Dependences of the magnetic moment M of a ring on the dimensionless amplitude H_m/H_p of the ac magnetic field. Curve *1* corresponds to an initial value of the circular current $I = I_c$, curves *2–4* were obtained at initial currents $I < I_c$, $\omega/2\pi = 100$ Hz.

cussed conditions the current distribution at any value of H_m is similar to the picture presented in Fig. 1,c for the case $H_m = H_t$.

If the circular current I previously induced in the ring is less than its critical value I_c, the magnetic moment of the ring decreases with the increase in the ac amplitude H_m as it is shown in Fig. 6 (curves *2–4*). The magnetic moment of the ring is practically unchanged at low amplitudes H_m. When the curve $M(H_m)$ reaches curve *1*, the magnetic moment starts to decrease noticeably. The curves $M(H_m)$ pass together with curve *1* independently of the initial value of the circular current at this stage. These facts permit us to propose the following picture of the collapse of the circular current. At the beginning, the circular current changes the region of its flowing, displacing itself to the axial line of the ring under the influence of the external ac field. The current value is practically unchanged at this stage of the ac amplitude increase. Just at this stage we observe a plateau on the plots $M(H_m)$ (see curves *2–4*). After the circular current occupies the whole core of the sample [at $H_m = H_t(I)$], the second stage of the collapse starts. At this stage the strong dissipation of a current energy arises because the whole core of the torus is filled with the current. It is clear that only the beginning of the second stage, when the plot $M(H_m)$ reaches curve *1* in Fig. 6, depends on the initial value of the circular current I.

Thus, the process of the compression of the circular current in the ring with increase in the ac amplitude is divided into two stages. At the first stage [at $H_m < H_t(I)$] the circular current is displaced into deep layers of the sample under the influence of the ac field. The interaction between the current and the ac field leads to the strong dissipation of I at the second stage, when $H_m = H_t(I)$. We should note that some dissipation of the current also occurs at the first stage of the collapse. It is completely obvious that the displacement of the circular current into the deep layers of the superconductor is accompanied by the appearance of the longitudinal (directed along the axial line of the ring) electrical field E_z. Accordingly, the Joule losses arise

$$\dot{Q} = \mathrm{jE} = j_c E_z \ . \qquad (18)$$

The following section is devoted to the analysis of a role of the dissipative processes at each of two stages of the collapse.

5. Collapse of the circular current in HTS ceramic rings. Theory

1. To describe the first stage of the collapse of the circular current we propose the following simple theoretical model. We accept the moment of switching on the ac field as the time origin ($t = 0$). The bulk of the ring is naturally divided into three parts at this moment. The first part represents the surface closed tube where the ac signal penetrates and where a fraction $I_1(0)$ of the circular current is flowing. The other fraction $I_2(0) = I(0) - I_1(0)$ of the circular current occupies the second region. Since the total circular current $I(0) < I_c$, the sample core (the third region) is free of any current. In other words, the amplitude $H_m < H_t(I)$ at the first stage of the collapse. As time passes, the circular current $I_1(t)$ flowing through region 1 should disappear owing to the collapse. A part of this current is dissipated. The other part of I_1 is displaced deep inside the sample bulk. Therefore, region 2 expands at the expense of the reduction of region 3. As a result, the current $I_2(t) = I(t) - I_1(t)$ in region 2 increases.

The process of the circular current displacement under the action of the ac field accompanied by its partial dissipation may be described by the following set of equations:

$$2\pi a E_z(I_1) + \frac{L_1}{c^2}\frac{dI_1}{dt} + \frac{L_{12}}{c^2}\frac{dI_2}{dt} = 0 \ ,$$
$$\frac{L_2}{c^2}\frac{dI_2}{dt} + \frac{L_{21}}{c^2}\frac{dI_1}{dt} = 0 \ . \qquad (19)$$

Here L_1 and L_2 are the self-inductions of regions 1 and 2, respectively, $L_{12} = L_{21}$ is the mutual induction between regions 1 and 2:

$$L_1 \approx L_2 \approx L_{12} \approx 2\pi a \ln (a/R) \ ; \qquad (20)$$

E_z is the longitudinal (directed along the axial line of the torus) electric field existing in region 1 only. It arises owing to the penetration of ac magnetic field (2).

The field E_z can be evaluated as follows. According to critical state Eq. (1), the current density j is

$$\mathrm{j} = j_c \mathrm{E}/E \ . \qquad (21)$$

Since $I_1(0) \propto j_c$, we obtain from Eq. (21) the following relation:

$$I_1(t) = I_1(0) E_z(I_1)/E \ . \qquad (22)$$

Except for the desired component E_z, the vector E contains an azimuthal component in region 1 as well:

$$E_\varphi \approx H_m \omega R/c \ . \qquad (23)$$

Therefore, the absolute value of E in Eq. (22) equals

$$E = \left[(H_m \omega R/c)^2 + E_z^2(I_1)\right]^{1/2} \ . \qquad (24)$$

Using Eqs. (22) and (24) we find

$$E_z(I_1) = \frac{H_m \omega R}{c} \frac{I_1}{(I_1^2(0) - I_1^2)^{1/2}} . \qquad (25)$$

Thus, the displacement of the circular current from region 1 into region 2 is described by the set of Eqs. (19) and (25). A physical picture of this process is following. Under the influence of the external ac field an effective resistance $2\pi a E_z(I_1)/I_1$ arises in region 1. It results in the reduction of the current I_1. Owing to the mutual induction [see the second equation in (19)] the current I_2 increases because region 2 expands. Using Eqs. (19) and (25) we obtain the following equation for the current $I_1(t)$:

$$2\pi a \frac{H_m \omega R}{c} \frac{I_1}{[I_1^2(0) - I_1^2]^{1/2}} + \frac{1}{c^2}\left(L_1 - \frac{L_{12}^2}{L_2}\right)\frac{dI_1}{dt} = 0 . \qquad (26)$$

Since the induction coefficients L_1, L_2, and L_{12} are equal to $2\pi a \ln(a/R)$ with the accuracy to small terms of the order of $2\pi R \ln(a/R)$, the effective induction of region 1 can be evaluated as follows:

$$L_1 - L_{12}^2/L_2 \sim 2\pi R \ln(a/R) . \qquad (27)$$

Equation (26) has the exact solution. However, it is very cumbersome and we do not present it here, but formulate some important conclusions following from this solution.

The current I_1 turns out to be damped in the characteristic time τ,

$$\tau = \frac{I_1(0)}{H_m} \frac{\ln(a/R)}{\omega a c} . \qquad (28)$$

We note that the ratio $I_1(0)/H_m$ is of the order of Rc [see Eq. (1)]. Therefore, the decay time τ appears to be much shorter than the period $2\pi/\omega$ of the ac field:

$$\tau \approx \frac{R \ln(a/R)}{\omega a} << 2\pi/\omega . \qquad (29)$$

The decrease in $I_1(t)$ is described by the exponential law at $t > \tau$:

$$I_1(t) = I_1(0) \exp(-t/\tau) . \qquad (30)$$

At $t < \tau$ the damping of $I_1(t)$ runs even faster.

The circular current energy Q dissipated during the relaxation time τ can be evaluated as

$$Q \lesssim (2\pi a)(\pi R^2) j E \tau \lesssim (2\pi a)(\pi R^2) j_c \frac{H_m \omega R}{c} \times \frac{R \ln(a/R)}{\omega a} \lesssim \frac{8\pi^3 R^5 j_c^2}{c^2} \ln(a/R) . \qquad (31)$$

This energy is $a/R >> 1$ times less than the total energy F of the circular current $I(0)$ at the initial time moment $t = 0$,

$$F = L_1 I_1^2/c^2 \sim \frac{2\pi^3 a R^4}{c^2} j_c^2 \ln(a/R) \gtrsim Q(a/R) >> Q . \qquad (32)$$

Hence, the main fraction of the current disappearing in region 1 passes into region 2 and only the small part of the total energy is dissipated during the first stage of the collapse.

The formulated above features of the first stage of the collapse in rings agree with our experimental results.

2. To describe the second stage of the collapse we accept the time moment when the wave amplitude H_m has been set to $H_m > H_f[I(0)]$ as the time origin $t = 0$. At this moment, in contrast to the first stage, sample region 3 is absent because the sample core is completely filled with the circular current. As before, the surface region where the ac signal penetrates is said to be region 1, and the other part of the sample is termed as region 2. The current I_1 flowing through region 1 should disappear and its energy will be completely dissipated with time. Since the current density is equal to the critical value j_c over whole region 2, this relaxation process will come to the end when the equality $H_m = H_f(I)$ is established. The dynamics of currents $I_1(t)$ and $I_2(t)$ variations can be described by a set of equations similar to Eq. (19):

$$2\pi a E_z(I_1) + \frac{L_1}{c^2}\frac{dI_1}{dt} + \frac{L_{12}}{c^2}\frac{dI_2}{dt} = 0 ,$$
$$2\pi a E_z(I_2) + \frac{L_2}{c^2}\frac{dI_2}{dt} + \frac{L_{21}}{c^2}\frac{dI_1}{dt} = 0 . \qquad (33)$$

The electric field $E_z(I_1)$ in region 1 is given by preceding formula (25). Since the current density $j = j_c$ at the initial time moment $t = 0$ over whole region 2, the displacement of the current I_1 from region 1 into region 2 results at first in the increase in the current I_2 in region 2, i.e., in the supercritical current ΔI_2,

$$\Delta I_2(t) = I_2(t) - I_2(0) > 0 . \qquad (34)$$

As a result, the resistive state arises, and, contrary to the first stage of the collapse, the electric field $E_z(I_2)$ appears. This field plays the principal role in the relaxation process during the second stage of the collapse. The connection of the field $E_z(I_2)$ with the cur-

rent ΔI_2 can be written in a form consistent with the generally used $I-V$ characteristic for hard superconductors:

$$E_2(I_2) = A\Delta I_2^{\gamma} \ . \qquad (35)$$

Here A and $\gamma > 1$ are phenomenological constants.

Owing to the fact that all four induction coefficients in Eqs. (33) coincide with an accuracy to the small parameter R/a, it is possible to draw an important conclusion with respect to process of supercritical current arising in region 2. During an initial period, while the field $E_z(I_1)$ remains higher than $E_z(I_2)$, the derivatives dI_1/dt and dI_2/dt are equal to each other in absolute value with a relative accuracy of $R/a \ll 1$ and have opposite signs. As a result, the essential redistribution of the currents between regions 1 and 2 occurs in a short time of order τ [see Eq. (28)]. The current I_1 decreases by some value $\Delta I \sim I_1(0)$ while the current I_2 increases by the same value. Thus, the total current is practically unchanged:

$$I(t) = I_1(t) + I_2(t) \approx I(0) \ , \quad t \lesssim \tau \ . \qquad (36)$$

As a result of the current redistribution, the fields $E_z(I_1)$ and $E_z(I_2)$ are equalized:

$$E_z(I_1) = E_z(I_2) \ . \qquad (37)$$

Beginning from the moment of establishment of equality (37), the total current I starts to decline. The current I_1 decreases from $I_1(0) - \Delta I$ down to zero and the current I_2 fall from $I_2(0) + \Delta I$ down to its initial value $I_2(0)$. Equality (37) remains valid during this relaxation process.

Let us substitute Eqs. (25) and (35) into Eq. (35) and then differentiate it with respect to time. Using the obtained relation, we can exclude the derivative dI_1/dt from the second equation of the set (33). Taking Eq. (35) into account, we obtain a following equation for the supercritical current $\Delta I_2(t)$:

$$\frac{d\Delta I_2}{dt} + \frac{Ac^2}{\ln(a/R)} \Delta I_2^{\gamma} \left(1 + \frac{Ac^2\gamma}{4\omega} \Delta I_2^{\gamma-1}\right)^{-1} = 0 \ . \qquad (38)$$

One can easy solve Eq. (38) combined with the initial condition $\Delta I_2 = \Delta I$ at $t \gtrsim \tau$. Since the answer has a cumbersome form, we present only some of its asymptotics here. Contrary to the first stage of the collapse, the relaxation of the circular current can not be characterized by a unique time scale at the second stage.

In the beginning, the current ΔI_2 is damped in a linear fashion:

$$\Delta I_2 \simeq \Delta I \left(1 - \frac{t}{\tau_1}\right) \ , \quad \tau \ll t \ll \tau_1 \ , \qquad (39)$$

$$\tau_1 = \left(A^{-1}c^{-2}\Delta I^{1-\gamma} + 0.5\gamma\omega^{-1}\right) \ln(a/R) \ . \qquad (40)$$

The rate of the current relaxation is seen to increase within the temporal interval $\tau \ll t \ll \tau_1$ as the ac frequency increases.

As time passes, the law of the $\Delta I_2(t)$ decrease change:s

$$\Delta I_2(t) \simeq \Delta I(\tau_2/t)^{1/(\gamma-1)} \ , \quad \tau_1 < \tau_2 < t \ , \qquad (41)$$

$$\tau_2 = A^{-1}c^{-2}\Delta I^{1-\gamma} \ln(a/R) \ . \qquad (42)$$

According to Eq. (42), the relaxation rate is independent of the ac frequency at $t \gg \tau_2$.

It is important to notice, that the current $I_1(t)$ relaxes considerably faster than $I_2(t)$. This conclusion follows directly from Eqs. (25),(35), and (37). Therefore, the relaxation of the total current $I(t)$ is defined mainly by the decay of the current $I_2(t)$. Since $I_2(t) = I_2(0) + \Delta I_2(t)$, the analysis of the behavior of the supercritical current $\Delta I_2(t)$ carried out above actually concerns the total current $I(t)$ as well. Thus, it might be assumed that the equality $H_m = H_t[I_2(0)]$ is set up as a result of the relaxation process.

The suggested picture of the second stage of the collapse correlates well with our experimental results. The dependences of the magnetic moment of the superconducting ring on time, kept from the moment of ac field switching on, are presented in Fig. 7. The

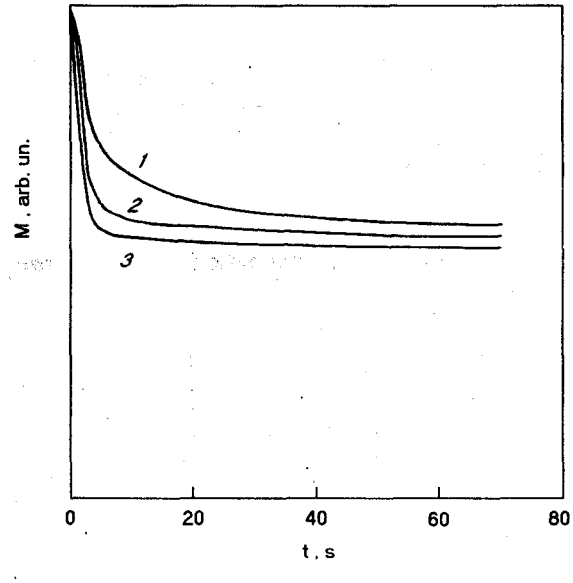

Fig. 7. Relaxation of the magnetic moment of a ring under the action of the external ac magnetic field with $H_m < H_p$ at various frequencies $\omega/2\pi$, Hz: 20 (*1*), 200 (*2*), 2000 (*3*).

induced circular current equal to I_c was adjusted at $t = 0$. Curves *1–3* correspond to three different ac frequencies. It turns out that the relaxation time is very long (about hundreds seconds) in all cases. This time decreases with the increase in the frequency. However, the relaxation rate is independent of the frequency at $t > 50$ s.

6. Summary and Application

Hard superconductors in the critical state exhibit the specific nonlinear electrodynamic properties. The main nonlinearity is related to the characteristic material equation of these media. Unlike ordinary conducting media, where the electric field determines both a value of the current density and its direction, hard superconductors are exemplified by the current density whose value is independent of E. Accordingly, critical state equation (1) contains the factor $j_c E/E$ on the right-hand side. This equation, which might appear to be simple at first glance, leads to interesting unusual consequences. One of them, namely, the collapse of the transport current under the action of the external ac field in the forceless configuration is studied in the present paper.

The discussed phenomenon is typical for any hard superconductor being in the critical state. However, HTS ceramics, owing to the relatively low critical current density, are the most convenient objects for experimentall research on the collapse and on any other physical consequences of the nonlinearity of the critical state equation.

The collapse of the transport current is of interest not only to fundamental physics but to some possible applications as well. For example, this phenomenon can be used (and is used by us) to test the homogeneity of melt-processed samples. These samples are characterized by the high critical current density, up to $10^5 - 10^6$ A/cm^2. However, an induced current in a melt-processing prepared ring is sometimes damped very quickly. The reason is connected with the inhomogeneity of the critical current capability in different parts of the ring. A bad part can be identified with the effect of the circular current compression under the action of an ac magnetic field. Using the experimental geometry described above (see Fig. *5*) one can investigate the influence of ac field applied with the coil C to different parts of the ring on magnetic moment of the circular current. If the bad part of the ring is located inside the coil, the magnetic moment falls off even at low ac amplitude. Otherwise, when the coil C irradiates the ring parts with the high current capability, the magnetic moment remains to be constant over wide intervals of ac amplitude change. It is certainly possible to test the homogeneity of cylindrical samples in similar fashion.

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