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The collective mode spectrum in polar phase of superfluid ^{3}He

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Using the path integration technique, developed by Brusov and Popov earlier [1], we calculate the collective mode spectrum at zero momentum of excitations for polar phase of superfluid ^{3}He. We found six modes with energy $E = (1.20-1.75i)\Delta_0$ which damp strongly and three Goldstone modes with zero energy at zero momentum of excitations.

1. Introduction

The polar phase with order parameter $\sim \begin{pmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&1 \end{pmatrix}$ and gap in the single-particle spectrum $\Delta^2 = \Delta_0^2 \cos^2\theta$ is the only phase in superfluid ^{3}He that has a line of gap nodes. It is very interesting, by making use of this fact, to calculate the collective mode spectrum in this phase and compare it with the spectrum of isotropic *B* phase or with one of the axial *A* phase or the planar 2*D* phase with points of gap nodes. It is clear that because of the disappearence of the gap along the equator and the possibility of decay of the collective excitations into initial fermions, the energies of all high-frequency modes (with $E \neq 0$ at $k = 0$) should be complex. The imaginary part of this energy determines the damping of these collective modes.

The plan of this paper is as follows. In Section 2 we describe very briefly the method of calculations based on the hydrodynamic action functional. In Section 3 we obtain the equations for the collective mode spectrum and calculate it numerically. In the Conclusion we discuss the obtained results and compare them with ones obtained by kinetic equation method.

2. The method of calculation

The physical properties of the superfluid phases of ^{3}He are determined by the hydrodynamic action functional S_h obtained by the functional integration over the «fast» and «slow» Fermi-fields:

$$S_h = \frac{1}{g} \sum_{p,i,a} c_{ia}^{+}(p) c_{ia}(p) +$$

$$+ \frac{1}{2} \ln \det \left[\hat{M}(c^{+}, c) / \hat{M}(c^{+(0)}, c^{(0)}) \right] .$$

Here $c_{ia}(p)$ is the Fourier transform of Bose-fields $c_{ia}(\mathrm{x}, \tau)$ describing Cooper pairs of the quasi-fermions on the Fermi surface with condensate values $c_{ia}^{(0)}$, and the operator $\hat{M}$ is given by

$$\hat{M} = \begin{pmatrix} Z^{-1}(i\omega - \xi + \mu H \sigma_3)\delta_{p_1 p_2} & \dfrac{(n_{1i} - n_{2i})}{\sqrt{\beta V}} \sigma_a c_{ia}(p_1 + p_2) \\ -\dfrac{(n_{1i} - n_{2i})}{\sqrt{\beta V}} \sigma_a c_{ia}^{+}(p_1 + p_2) & Z^{-1}(-i\omega - \xi + \mu H \sigma_3)\delta_{p_1 p_2} \end{pmatrix},$$

where $\xi = c_F(k - k_F)$; $n_i = k_i / k_F$; H is magnetic field; μ is magnetic moment of quasi-particle; $Z = \mathrm{const}$; $\beta = T^{-1}$; V is the volume of the system, σ_a $(a = 1, 2, 3)$ are two-by-two Pauli matrices, and $\omega = (1 + 2n)\pi T$ are the Fermi frequencies. The negative constant g is proportional to the scattering amplitude of two quasi-fermions near the Fermi sphere under the assumption that the amplitude is equal to $g(\mathbf{k}_1 - \mathbf{k}_2, \mathbf{k}_3 - \mathbf{k}_4)$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of the incident fermions and $\mathbf{k}_3$ and $\mathbf{k}_4$ are those of the outgoing ones. At $T \to 0$, the functional S_h must be expanded in terms of fluctuations of the fields $c_{ia}(p)$

above the condensate values $c_{ia}^{(0)}(p)$. In this temperature region the Bose spectrum of the system is determined in first order approximation by the quadratic part of the functional S_h obtained by the shift $c_{ia}(p) \to c_{ia}^{(0)}(p) + c_{ia}(p)$ where for polar phase $c_{ia}^{(0)}(p) = c(\beta V)^{1/2}\delta_{p0}\delta_{i3}\delta_{a3}$ and is obtained from equation $\det Q = 0$, where Q is the matrix of the quadratic form.

3. The collective mode spectrum

To calculate the quadratic part of S_h one needs to put $\hat{M}(c^{+(0)}, c^{(0)}) = \hat{G}^{-1}$, $\hat{M}(c^{+}, c) = \hat{G} + \hat{u}$ and to keep the first two terms ($n = 1, 2$) of the expansion

$$\ln\det[\hat{M}(c^{+}, c)/\hat{M}(c^{+(0)}c^{(0)}] = \mathrm{Sp}\ln(1 + \hat{G}\hat{u}) = -\sum \frac{1}{2n}\mathrm{Sp}\,(\hat{G}\hat{u})^{2n}.$$

In case of the polar phase

$$\hat{G}^{-1} = Z^{-1}\begin{pmatrix} (i\omega - \xi)\delta_{p_1p_2} & \Delta\delta_{p0} \\ -\Delta\delta_{p0} & (-i\omega + \xi)\delta_{p_1p_2} \end{pmatrix};$$

$$\hat{u} = \begin{pmatrix} 0 & \dfrac{(n_{1i} - n_{2i})}{\sqrt{\beta V}}\sigma_a c_{ia}(p_1 + p_2) \\ -\dfrac{(n_{2i} - n_{1i})}{\sqrt{\beta V}}\sigma_a c_{ia}^{+}(p_1 + p_2) & 0 \end{pmatrix},$$

here $\Delta = \Delta_0\cos\theta$, $\Delta_0 = 2cZ$.

Finding $\hat{G}$

$$\hat{G} = \frac{Z}{M}\begin{pmatrix} -(i\omega + \xi)\delta_{p_1p_2} & \Delta\delta_{p0} \\ -\Delta\delta_{p0} & (i\omega + \xi)\delta_{p_1p_2} \end{pmatrix}$$

and calculating $-\sum \frac{1}{2n}\mathrm{Sp}\,(\hat{G}\hat{u})^{2n}$ we get the quadratic form of S_h:

$$\sum_{p,i,a} c_{ia}^{+}(p)c_{ia}(p)\left[\frac{\delta_{ij}}{g} + \frac{4Z^2}{\beta V}\sum_{p_1+p_2=p} n_{1i}\,n_{1j}\,M_1^{-1}M_2^{-1}(i\omega_1 + \xi_1)(i\omega_2 + \xi_2)\right] -$$

$$-\frac{2Z^2}{\beta V}\sum_{p_1+p_2=p}\Delta^2 n_{1i}\,n_{1j}\,M_1^{-1}M_2^{-1}\left(c_{ia}^{+}(p)\,c_{ja}^{+}(-p) + c_{ia}(p)\,c_{ja}(-p)\right),$$

here $M_i = \omega_i^2 + \xi_i^2 + \Delta^2$.

Taking the limit $T \to 0$ we transform from summing to integrating in the vicinity of the Fermi surface by the rule

$$(\beta V)^{-1}\sum_{p_1} \to k_F^2(2\pi)^{-4}c_F^{-1}\int d\omega_1\,d\xi_1\,d\Omega_1,$$

where $d\Omega_1$ is the integral with respect to the angle variables.

After calculation of the coefficients of the quadratic form, one has

$$S_h \sim -\frac{Z^2k_F^2}{2\pi^2 c_F}\int_0^1 dx\left\{(1 - x^2)\left[J(1 - 4\Delta^2/q^2) - 2\right]\left(u_{13}^2 + u_{23}^2 + v_{11}^2 + v_{21}^2 + v_{22}^2 + v_{12}^2\right) +\right.$$

$$+\left(1 - x^2\right)(J - 2)\left(v_{13}^2 + v_{23}^2 + u_{11}^2 + u_{21}^2 + u_{22}^2 + u_{12}^2\right) +$$

$$+ 2x^2J\left(1 + 4\Delta^2/q^2\right)\left(u_{33}^2 + v_{31}^2 + v_{32}^2\right) + 2x^2J\left(v_{33}^2 + u_{31}^2 + u_{32}^2\right)\Big\}.$$

Here $J = b^{-2}\ln\frac{1+b}{1-b}$, $b^2 = 1 + 4\Delta^2/q^2$, $u_{ia} = \operatorname{Re} c_{ia}$, $v_{ia} = \operatorname{Im} c_{ia}$, $q^2 = \omega^2 + c_F^2(\mathrm{kn}_1)^2$.

From this quadratic form we obtain the following set of equations for collective mode spectrum using the equation $\det Q = 0$:

$$\int_0^1 (1 - x^2)[J(1 - 4\Delta^2/q^2) - 2]\,dx = 0\,,\quad (u_{13}, u_{23}, v_{11}, v_{21}, v_{22}, v_{12})\,,$$

$$\int_0^1 (1 - x^2)(J - 2)\,dx = 0\,,\qquad (v_{13}, v_{23}, u_{11}, u_{21}, u_{22}, u_{12})\,,$$

$$\int_0^1 x^2J(1 - 4\Delta^2/q^2)\,dx = 0\,,\quad (u_{33}, v_{31}, v_{32})\,,$$

$$\int_0^1 x^2J\,dx = 0\,,\qquad (v_{33}, u_{31}, u_{32})\,.$$

Putting $k = 0$ and solving these equations we get the root $E = \Delta_0(1.20 - 1.75i)$ for the second equation and $E = 0$ for the third one. We didn't find the roots for the first and the fourth equations.

4. Conclusions

Let us discuss the obtained results briefly. First of all we would like to note that we got six branches of the $E = \Delta_0(1.20 - 1.75i)$ mode which correspond to the variables $v_{13}, v_{23}, u_{11}, u_{21}, u_{22}, u_{12}$ and three branches of Goldstone mode $E = 0$, which corresponds to the variables u_{33}, v_{31}, v_{32}.

It turns out that the high frequency mode $E = 1.20\,\Delta_0$ has a large enough imaginary part, of the same order as the real one. This couples to the fact that, in the polar phase, the gap disappears at the equator line which is opposite to the case of the axial and planar phases, where the gap has point nodes, and the imaginary parts of the mode energies are small as compared with the real parts. In this connection, we recall that the energies of all high frequency modes in the isotropic B phase have real energies at zero momenta of the collective excitations.

We could compare our results to those of Ref. 2. There, the authors solved the same problem by using the kinetic equation method. They got the same energy $E = 1.20\,\Delta_0$ for high frequency mode but they did not obtain the imaginary part of this mode. This is the usual shortcoming of kinetic equation method as compared with the path integration one.

The authors of Ref. 2. showed that this energy $E = 1.20\,\Delta_0$ increases with T reaching $2\Delta_0(T)$ as $T \to T_c$ and this mode becomes high damping at least at $T \sim T_c$. For this temperature region this conclusion is clear from physical reasons without calculations. They also found a few Goldstone modes. They didn't find solutions for some equations and found them only at $T = T_c$, where these modes have energies $E = 2\Delta_0$.

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