THE REGULAR AND CHAOTIC DYNAMICS AT WEAK-NONLINEAR INTERACTION OF WAVES

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It is shown that different regimes at weak-nonlinear interaction of waves are possible: regular regimes with the growth of degree of coherence of some waves which take part in interaction and also chaotic regimes. It is shown that in some cases the increasing number of waves which take part in interaction leads to significant simplification of dynamics of interacting waves and not to complication. Possible electrodynamics structures in which the regimes with dynamical chaos can realize are discussed in this paper. In particular the properties of plasma waveguides filled with rare plasma are investigated. It is shown that in such waveguides the regimes with chaotic dynamics may be realized.
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1. THE INCREASING OF THE DEGREE OF COHERENCE AT THREE-WAVE INTERACTION

Let us consider the process of decay of a high-frequency wave on high-frequency and low-frequency one. The regular process of such decay is well enough investigated [1, 2]. We shall consider that during the initial moment of time in nonlinear medium the nonmonochromatic high-frequency wave is propagated. The spectrum width of this wave is equal $\delta\omega$. Besides for simplicity of the further analysis, we shall consider that the source of a nonregularity of this wave is random diffusion of a phase. In such model this fact is important and the equation for random phase can be written down in the form $\dot{\varphi} = \xi(t)$, where $\xi(t)$ - delta-correlated function with a zero average: $\langle \xi \rangle = 0$, $\langle \xi(t) \cdot \xi(t') \rangle = \delta\omega \cdot \delta(t - t')$.

Function $\xi(t)$ is the instantaneous frequency of a signal. The diffusion coefficient $\delta\omega$ is the width of the spectrum of signal. Besides high-frequency wave in nonlinear medium there are small coherent high-frequency perturbations and also small low-frequency perturbations. We shall consider also that the amplitude of the basic signal is great enough, so the increment of decay instability is more than the width of a spectrum of this signal ($\Gamma > \delta\omega$). In this case the equations that describe dynamics of complex amplitudes of interacting waves can be presented in such form:

$$\dot{A}_1 = -A_2 \cdot A_3 \; \dot{A}_2 = A_1 \cdot A_3^* \; \dot{A}_3 = A_1 \cdot A_2^* . \qquad (1)$$

When we obtain these equations the laws of conservation of energy and an impulse have been considered:

$$\sum_{i=1}^{3} \omega_i N_i = const , \; \sum_{i=1}^{3} \vec{k}_i N_i = const . \qquad (2)$$

Besides that the normalization has been used: $|A_k|^2 = N_k$ - number of quanta in k -wave; $\dot{A} = dA / d\tau$, $\tau = V \cdot t$, V - matrix element of waves interaction.

At the initial stage of process of decay it is possible to consider that the amplitude of decaying wave does not vary. In this case it is necessary to analyze the last two equations of system (1). Thus it is convenient to introduce new function $B = A_1 \cdot A_3^*$. Then the last two equations of system (1) can be rewrite in the form of:

$$\dot{A}_2 = B , \; \dot{B} = \xi \cdot B + |A_1|^2 \cdot A_2 . \qquad (3)$$

From system (3) using Furutsu-Novikov equation [3] it is possible to obtain the following equations for the first moments (average quantity):

$$\langle \dot{A}_2 \rangle = \langle B \rangle , \; \langle \dot{B} \rangle = |A_1|^2 \cdot \langle A_2 \rangle + \delta\omega / 2 \cdot \langle B \rangle . \qquad (4)$$

It is seen from (4), that the process of decay for average quantities goes the same way (practically with the same increment $\Gamma = \sqrt{|A_1|^2 + (\delta\omega)^2 / 16} + \delta\omega / 16 \approx |A_1|$) as well as at decay of the regular wave ($|A_1|^2 >> \delta\omega$). Thus from the function $B(\tau)$ it is possible to make the inference, that the fluctuations of the phase of a decaying wave are compensated by the fluctuations of a phase of a low-frequency wave. As a result, the fluctuations of complex amplitude A_2 and function $B(\tau)$ are small. For the analysis of nonlinear dynamics it is useful to rewrite the set of equations (1) in such form:

$$\frac{d\left(|A_1|^2\right)}{d\tau} = -\left[A_2 B^* + A_2^* B \right], \; \dot{A}_2 = B, \qquad \dot{B} = A_2 \left[2|A_1|^2 - |A_1(0)|^2 \right]. \qquad (5)$$

Considering that the fluctuations of amplitude A_2 and the function $B(\tau)$ are small, it is easy to make averaging of the set of equations (5). One can see, that in this case these average equations will coincide with the equations (5). In turn, the equations (5) do not differ from the equations which describe the regular dynamics of decay process. Thus, as it is known, during the order of reciprocal increment ($T \sim \Gamma^{-1}$) practically all the energy of the decaying wave transfers into the energy of high-frequency coherent components. The insignificant part of the energy, according to Manley-Rowe's relation, transfers to a low-frequency wave ($\delta E \sim (\Omega / \omega_1) E << E$).

Thus, we have shown that despite that fact, that the phase of the decaying wave undergoes random fluctuations, this wave can practically transmit all the energy to a monochromatic coherent high-frequency wave and the part of the energy will transfer to a low-frequency wave. Now if we interrupt nonlinear interaction of waves after the complete transmitting total energy from pumping wave pass into new high-frequency one and in the field of low-frequency wave. The process appear as transformation of the energy of incoherent wave into the energy of coherent wave. It can seem that such process proceeds with breaking of the second law of thermodynamics. However, as it is shown in paper [4], such processes proceed with the growth of entropy in the complete accordance with the second law of thermodynamics. The matter is that all the entropy from the high-frequency wave transfers into the entropy of low-frequency wave.

2. DYNAMICS OF CASCADES ALLOW FOR PROCESSES OF FUSION

In the theory of weak nonlinear interactions of waves it is known the model which is widely used at the analysis of the processes which arise at the action of powerful laser radiation on plasma. In this model we shall consider, that the number of high-frequency waves can be infinitely large and interaction between them is carried out through one low-frequency wave. The mathematical model of such processes, apparently, for the first time has been obtained in papers [5,6] and analyzed by many authors:

$$i\frac{da_n}{dt} = ba_{n-1}e^{i\delta t} + b^* a_{n+1}e^{-i\delta t},$$
$$i\frac{db}{dt} = \sum_{n=-n_1}^{n_2} a_{n-1}^* a_n e^{-i\delta t}. \quad (6)$$

Here a_n, b - amplitude of a HF and LF wave, $\delta = \omega_n - \omega_{n-1} - \Omega$ - detuning, ω_n - frequency of a HF wave, Ω -frequency of a LF wave.

In real systems the number of high-frequency interacting waves is great enough but finite. Therefore, we shall assume – the amplitudes of waves $a_n = 0$ if n is outside the range of values $-n_1 \le n \le n_2$ (n_1 -number of "red" satellites, n_2 - number of "blue" satellites). It is possible to show, that from (6) except the conservation laws of type (2) at $n_1 = n_2$ there is an integral of a motion $\sum_{n=-n_1}^{n_2} a_{n-1}^* a_n = const$ and the equation of system (6) becomes free. As it is seen from (6) excitation of more and more high numbers of harmonics eventually occurs because of interaction with LF wave. Analytically (at $n \to \infty$) it is possible to show [5] that if the quantity $B(t) = \int_0^t b(t')\exp(-i\delta t')dt'$ is restricted, the maximal number of excited waves on an order of magnitude is equal $n_{\max} \sim |B_{\max}|$. The amplitudes of higher harmonics will be negligibly small. In case when $n_{\max} > n_1, n_2$, the number of excited waves is defined from the requirement $-n_1 \le n \le n_2$. Depending on the initial state the various modes of the dynamics of wave interaction can be realized: with constant amplitude of a LF-wave, with periodic changes of this amplitude, with frequency detuning δ and at $\delta \to 0$ the amplitude of a LF wave linearly grows in time [5, 7].

For the case with finite values n the set of equations (6) is solved numerically for various initial values of the amplitudes of HF and LF fields, and also for the parameter of detuning δ [7]. Dynamics of LF and HF fields is in strict correspondence with analytical consideration up to the moment of time $t = t^*$ of excitation of boundary harmonics (harmonic with numbers n_1 or n_2). At $t > t^*$ dynamics of amplitudes essentially differs from the case $n \to \infty$ though equality $|a_{-i}| = |a_i|$ is carried out. In the symmetrical case ($n_1 = n_2$) dynamics of waves remains enough regular. In the asymmetrical case $n_1 \ne n_2$ dynamics of waves gets the nonregular character. The spectrum of power of HF waves is wide with slow diminution of the field of high frequencies. The correlation function promptly decreases up to zero and has the nonregular oscillations close to zero level what is characteristic for chaotic processes. Presence of parameter of detuning δ conducts to additional complication of dynamics of interaction of waves. In case with the broken symmetry ($n_1 \ne n_2$) and values of detuning $\delta = 0.05$ the dynamics of waves becomes chaotic.

3. THE DISPERSION PROPERTIES OF THE CYLINDRICAL WAVEGUIDE FILLED WITH RARE PLASMA

It is known, that for excitation of broadband noise radiation making special type of generators is necessary. This problem is complex enough and not well investigated at present time. On the other hand, the generators of regular intensive oscillations are well investigated. It is enough to mention such devices as klystrons, magnetrons, TWT and the others; however on output they give narrow spectral lines. In many cases (for example, for the purposes of broadband detection) wide spectrums are necessary to us. Therefore it is possible to imagine, that radiation with the narrow spectral line obtained from magnetron, for example, gets in nonlinear medium in which the regimes with dynamic chaos are possible. On an output from such medium the spectrum of radiation can have the necessary form. However, as numerical estimations show, in usual requirements the necessary intensity for embodying chaotic regimes becomes significantly greater (see [8-10]). So, for example, for the unbounded plasma this intensity of fields make is more then 20 kV/cm. Is of interest to find such electrodynamic structures with nonlinear devices in which the regime with dynamic chaos would develop at much smaller intensities of the fields. We know (from the previous investigations), that the less the distance on frequency between eigen waves of electrodynamic structure are, the smaller the intensities of the fields are, witch are necessary for transition into regimes with dynamic chaos. Therefore our problem is to find such mediums and structures in which the distance on frequency between eigen waves could be minimal. The possible candidate for such structure is the cylindrical metal waveguide filled with rare plasma and held in external magnetic field.

Such electrodynamic structures were investigated but consideration in them was restricted by studying slow waves [11]. It is related with the fact that such structures, first of all, were considered for the purposes of acceleration of charged particles - especially heavy particles. The dispersion of fast waves remained insufficiently investigated [12].

We investigated the dispersion of cylindrical waveguide partially filled with cylindrical magnetoactive plasma, coaxial with a wave guide. For this system the dispersion equation which was explored analytically and numerically was obtained. It is shown analytically and numerically, that in the field of frequencies between electronic cyclotron and upper hybrid there is the infinite number of branches of eigenmodes. In this area such frequencies can be easily chosen and the distance between them will correspond to the necessary requirements for realization of the regimes with dynamic chaos.

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РЕГУЛЯРНАЯ И ХАОТИЧЕСКАЯ ДИНАМИКА ПРИ СЛАБОНЕЛИНЕЙНОМ ВЗАИМОДЕЙСТВИИ ВОЛН

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Показано, что при слабонелинейном взаимодействии волн возможны разнообразные режимы: регулярные режимы с ростом степени когерентности отдельных, участвующих во взаимодействии волн, а также хаотические режимы. Показано, что в некоторых случаях рост участвующих во взаимодействии волн (числа степеней свободы) приводит не к усложнению динамики взаимодействия волн, а к существенному упрощению этой динамики. Обсуждаются возможные электродинамические структуры, в которых могут реализоваться режимы с динамическим хаосом. В частности, изучены дисперсионные свойства плазменных волноводов с редкой плазмой. Показано, что в таких волноводах легко реализовать режимы с динамическим хаосом.

РЕГУЛЯРНА Й ХАОТИЧНА ДИНАМІКА ПРИ СЛАБКОНЕЛІНІЙНІЙ ВЗАЄМОДІЇ ХВИЛЬ

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Показано, що при слабконелінійній взаємодії хвиль можливі різноманітні режими: регулярні режими з ростом ступеня когерентності окремих, що беруть участь у взаємодії, хвиль, а також хаотичні режими. Показано, що в деяких випадках ріст хвиль, що беруть участь у взаємодії (числа ступенів волі), приводить не до ускладнення динаміки взаємодії хвиль, а до істотного спрощення цієї динаміки. Обговорюються можливі електродинамічні структури, у яких можуть реалізуватися режими з динамічним хаосом. Зокрема, вивчені дисперсійні властивості плазмових хвилеводів з рідкою плазмою. Показано, що в таких хвилеводах легко реалізувати режими з динамічним хаосом.