MATHEMATICAL MODEL OF AN EXCITATION BY ELECTRON BEAM OF "WHISPERING GALLERY" MODES IN CYLINDRICAL DIELECTRIC RESONATOR

K.V. Galaydych[1], Yu.F. Lonin[1], A.G. Ponomarev[1], Yu.V. Prokopenko[2], G.V. Sotnikov[1]

[1] National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine;
[2]Usikov Institute of Radiophysics and Electronics, Kharkov, Ukraine

For generation of oscillations of millimeter and submillimeter wave lengths can be used "whispering gallery" modes excited in a dielectric resonator by an azimuthally-modulated electron beam. The mathematical model of excitation of these modes is constructed. The dispersion equation for determination of eigen frequencies of "whispering gallery" modes is obtained, eigen waves and their norms are found. Using them, the integro-differential equations for eigen wave amplitudes are derived.

PACS: 07.57.–c; 41.60 Bq; 94.05.Pt

1. INTRODUCTION

A development of sources of electromagnetic radiation of a millimeter and submillimeter range of wavelength and, especially, a range above 1 THz is a perspective and actively investigated direction. Now a generating of oscillation of millimeter wavelength range is provided with classical sources: magnetrons, klystrons and back wave oscillator. However a level of radiation power of these sources with moving towards submillimeter wavelengths dramatically decreases [1,2].

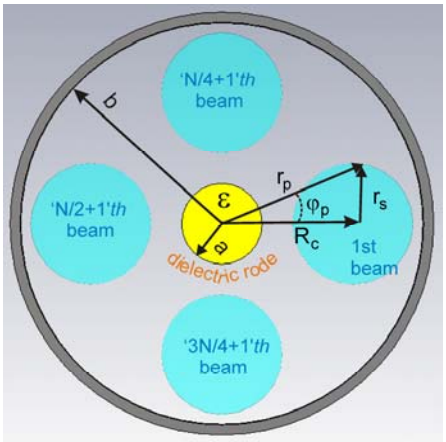
A possibility of obtaining of powerful terahertz radiation because of classical mechanisms transition and Cherenkov radiations actively develops. It becames possible due to considerable progress in obtaining short (some tens micron and more low) high current (with a charge to several tens nC) relativistic electron bunches. Recently [3] it has experimentally been shown that high levels of electromagnetic fields of the THz frequency range it is possible to reach in a dielectric waveguide, because of Cherenkov radiation of a powerful electron bunch. Dielectric structures are necessary for obtaining of such radiation with the cross-section sizes of an order of hundreds micrometers.

As the alternative of obtaining of high frequency radiation in paper [4] it is proposed to use the cylindrical dielectric resonator excited by the modulated relativistic electron bunch. The electron bunch excites high numbers of "whispering gallery" mode with frequencies of several tens of GHz. Thus structures with the millimeter and submillimeter dimensions are not required.

The mathematical model of excitation of "whispering gallery" modes by the azimuthally-modulated electron bunch is constructed in the present work.

2. EXCITATION EQUATIONS

The investigated dielectric structure is the metal resonator on which axis the dielectric rode is placed. The cross-section of the dielectric resonator is shown in figure. Resonator radius is b, its length is L, radius of a dielectric rode is a, its dielectric permittivity is ε. The resonator is excited by the multibeam consisting of the N of cylindrical beams, located evenly spaced on an azimuthal angle at distance R_c from a resonator axis. Radius of each beam is r_s.



Cross section of dielectric structure excited by N azimuthally located bunches. In this sketch are shown only 4 bunches

Considering the geometry of the problem, a current of a multibeam we will write down in the form:

$$\mathbf{j}_e = \sum_{k=1}^{N}\sum_{p\in V_R}\frac{1}{r}\mathbf{v}_p q_p \delta\left[r - r_p(t)\right]\delta\left[z - z_p(t)\right] \delta\left[\varphi - \varphi_p(t) - (k-1)\Delta\varphi_b\right], \quad (1)$$

where q_p is a macroparticle charge, r_p, φ_p, z_p and $\mathbf{v}_p$ are its coordinates and velocity, time-dependent, N is number of bunches. Internal summation in eq. (1) is carried over particles of the first (k=1) bunch which are in resonator volume V_R.

For the theoretical description of excitation of azimuthal modes with high number, "whispering gallery» modes, we will start with the general theory of excitation of resonators [5]. Its application to not monochromatic excitation of the dielectric resonator by streams of the charged particles is implemented in the paper [6]. According to [5] we will present required electric and a magnetic field in form of the sums of solenoidal and potential parts [6]:

$$\mathbf{E} = \mathbf{E}^{\mathbf{t}} + \mathbf{E}^{\mathbf{l}},\ \ \mathbf{H} = \mathbf{H}^{\mathbf{t}},$$

where $\mathbf{E}^{\mathbf{t}}$ and $\mathbf{H}^{\mathbf{t}}$ are solenoidal components of an electromagnetic field, and $\mathbf{E}^{\mathbf{l}}$ is potential electric field.

For solenoidal components of an electromagnetic field we search in the form of decomposition on eigen solenoidal fields of the empty dielectric resonator:

$$\mathbf{E^t} = \sum_s A_s(t)\mathbf{E}_s,\ \mathbf{H^t} = -i\sum_s B_s(t)\mathbf{H}_s\,, \qquad (2)$$

where $\mathbf{E}_s$, $\mathbf{H}_s$ are eigen solenoidal fields of the resonator without an excitation source (here, for brevity, the index s replaces three indexes: n (radial), m (azimuthal) and ℓ (axial)). Then for amplitudes of fields A_s and B_s we have the equations [6]:

$$\frac{d^2A_s}{dt^2} + \omega_s^2 A_s = -\frac{dR_s}{dt}, \qquad \frac{d^2B_s}{dt^2} + \omega_s^2 B_s = -\omega_s R_s. \qquad (3)$$

The quantity R_s in the equations (3) makes sense a coupling resistancen of a beam with eigenn wave of frequency ω_s. It is defined as:

$$R_s = \frac{1}{P_s}\int \mathbf{j_e}\cdot\mathbf{E_s}dV\,, \qquad (4)$$

$$4\pi P_s = \int_{V_R} \varepsilon|\mathbf{E}_s|^2 dV = \int_{V_R} |\mathbf{H}_s|^2 dV\,. \qquad (5)$$

Eigen frequencies ω_s are determined from the equation:

$$k_\perp^4\kappa^2a^2\left[\frac{k\varepsilon}{k_\perp}\frac{J_m'(k_\perp a)}{J_m(k_\perp a)} - \frac{k}{\kappa}\frac{F_m'(\kappa a,\kappa b)}{F_m(\kappa a,\kappa b)}\right]\left[\frac{k}{k_\perp}\frac{J_m'(k_\perp a)}{J_m(k_\perp a)}\right. \qquad (6)$$
$$\left.-\frac{k}{\kappa}\frac{\Phi_m'(\kappa a,\kappa b)}{\Phi_m(\kappa a,\kappa b)}\right] - m^2k_\ell^2(\kappa^2-k_\perp^2)^2 = 0,$$

$$F_m(x,y) = J_m(x)Y_m(y) - Y_m(x)J_m(y)\,,$$
$$\Phi_m(x,y) = J_m(x)Y_m'(y) - Y_m(x)J_m'(y)\,,$$

$$k_\perp^2 = \omega^2\varepsilon/c^2 - k_\ell^2\,,\ \kappa_\perp^2 = \omega^2/c^2 - k_\ell^2\,,\ k_\ell = \pi\ell/L\,,$$
$$\Phi_m'(x,y) \equiv d\Phi_m(x,y)/dx\,,\ F_m'(x,y) \equiv dF_m(x,y)/dx\,,$$

$$J_m'(x) \equiv dJ_m(x)/dx\,,\ Y_m'(x) \equiv dY_m(x)/dx\,,$$

$J_m(x)$ and $Y_m(x)$ are Bessel and Veber function of m*th*-order, axial index $\ell = 0,1\ldots$, azimuthal index $m = 0,\pm1\ldots$.

Having solved the Maxwell equations together with boundary conditions, for solenoidal fields we obtain:

$$\begin{aligned}
&E_{sz} = e_{zm}(r)\cos(k_\ell z)\exp(im\varphi),\ e_{z,-m}(r) = e_{zm}(r),\\
&E_{sr} = e_{rm}(r)\sin(k_\ell z)\exp(im\varphi),\ e_{r,-m}(r) = e_{rm}(r),\\
&E_{s\varphi} = ie_{\varphi m}(r)\sin(k_\ell z)\exp(im\varphi),\ e_{\varphi,-m}(r) = -e_{\varphi m}(r),\\
&H_{sz} = h_{zm}(r)\sin(k_\ell z)\exp(im\varphi),\ h_{z,-m}(r) = -h_{zm}(r),\\
&H_{sr} = h_{rm}(r)\cos(k_\ell z)\exp(im\varphi),\ h_{r,-m}(r) = -h_{rm}(r),\\
&H_{s\varphi} = ih_{\varphi m}(r)\cos(k_\ell z)\exp(im\varphi),\ h_{\varphi,-m}(r) = h_{\varphi m}(r),
\end{aligned} \qquad (7)$$

where the functions describing radial dependence of electromagnetic fields, in partial areas I ($r \le a$) and II ($a \le r < b$) look like:

$$\begin{aligned}
&e_{zm}^I = \frac{J_m(k_\perp r)}{J_m(k_\perp a)},\\
&e_{rm}^I = -\frac{k_\ell}{k_\perp}\left[\frac{J_m'(k_\perp r)}{J_m(k_\perp a)} + \frac{m^2k^3(\varepsilon-1)}{ra\kappa^2k_\perp^3D_m^e}\frac{J_m(k_\perp r)}{J_m(k_\perp a)}\right],\\
&e_{\varphi m}^I = -\frac{mk_\ell}{k_\perp^2}\left[\frac{1}{r}\frac{J_m(k_\perp r)}{J_m(k_\perp a)} + \frac{k^3(\varepsilon-1)}{a\kappa^2k_\perp D_m^e}\frac{J_m'(k_\perp r)}{J_m(k_\perp a)}\right],\\
&h_{zm}^I = \frac{mk_\ell k^2(\varepsilon-1)}{a\kappa^2k_\perp^2D_m^e}\frac{J_m(k_\perp r)}{J_m(k_\perp a)},\\
&h_{rm}^I = \frac{mk}{\kappa^2}\left[\frac{\varepsilon}{r}\frac{J_m(k_\perp r)}{J_m(k_\perp a)} + \frac{mk_\ell^2k(\varepsilon-1)}{a\kappa^2k_\perp D_m^e}\frac{J_m'(k_\perp r)}{J_m(k_\perp a)}\right],\\
&h_{\varphi m}^I = \frac{k}{k_\perp}\left[\varepsilon\frac{J_m'(k_\perp r)}{J_m(k_\perp a)} + \frac{m^2k_\ell^2k(\varepsilon-1)}{ra\kappa^2k_\perp^3D_m^e}\frac{J_m(k_\perp r)}{J_m(k_\perp a)}\right],
\end{aligned} \qquad (8)$$

$$\begin{aligned}
&e_{zm}^{II} = \frac{F_m(\kappa r,\kappa b)}{F_m(\kappa a,\kappa b)},\\
&e_{rm}^{II} = -\frac{k_\ell}{\kappa}\left[\frac{F_m'(\kappa r,\kappa b)}{F_m(\kappa a,\kappa b)} + \frac{m^2k^3(\varepsilon-1)}{ra\kappa^3k_\perp^2D_m^e}\frac{\Phi_m(\kappa r,\kappa b)}{\Phi_m(\kappa a,\kappa b)}\right],\\
&e_{\varphi m}^{II} = -\frac{mk_\ell}{\kappa^2}\left[\frac{1}{r}\frac{F_m(\kappa r,\kappa b)}{F_m(\kappa a,\kappa b)} + \frac{k^3(\varepsilon-1)}{a\kappa k_\perp^2D_m^e}\frac{\Phi_m'(\kappa r,\kappa b)}{\Phi_m(\kappa a,\kappa b)}\right],\\
&h_{zm}^{II} = \frac{mk_\ell k^2(\varepsilon-1)}{a\kappa^2k_\perp^2D_m^e}\frac{\Phi_m(\kappa r,\kappa b)}{\Phi_m(\kappa a,\kappa b)},\\
&h_{rm}^{II} = \frac{mk}{\kappa^2}\left[\frac{1}{r}\frac{F_m(\kappa r,\kappa b)}{F_m(\kappa a,\kappa b)} + \frac{mk_\ell^2k(\varepsilon-1)}{a\kappa k_\perp^2D_m^e}\frac{\Phi_m'(\kappa r,\kappa b)}{\Phi_m(\kappa a,\kappa b)}\right],\\
&h_{\varphi m}^{II} = \frac{k}{\kappa}\left[\frac{F_m'(\kappa r,\kappa b)}{F_m(\kappa a,\kappa b)} + \frac{m^2k_\ell^2k(\varepsilon-1)}{ra\kappa^3k_\perp^2D_m^e}\frac{\Phi_m(\kappa r,\kappa b)}{\Phi_m(\kappa a,\kappa b)}\right],
\end{aligned} \qquad (9)$$

where: $D_m^e = \dfrac{k}{k_\perp}\dfrac{J_m'(k_\perp r)}{J_m(k_\perp a)} - \dfrac{k}{\kappa}\dfrac{\Phi_m'(\kappa a,\kappa b)}{\Phi_m(\kappa a,\kappa b)}$.

For record simplification everywhere in (7)-(9) indexes n,ℓ are skipped.

Using found eigen functions (8) and (9), and also expression for current density (1), for the quantity R_s, after simple transformations, we obtain:

$$R_s \equiv R_{nm\ell} = R_{nN\ell}\delta_{m,N} + R_{nN\ell}^*\delta_{m,-N}\,, \qquad (10)$$

$$\begin{aligned}
R_{nN\ell} = \frac{N}{P_N}\sum_{p\in V_R} q_p\Big\{\Big[\big[&v_{pz}e_{zN}(r_p(t))\cos k_\ell z_p(t)\\
&+v_{pr}e_{rN}(r_p(t))\sin k_\ell z_p(t)\big]\cos N\varphi_p(t)\\
&-v_{p\varphi}e_{\varphi N}(r_p(t))\sin k_\ell z_p(t)\sin N\varphi_p(t)\Big]\\
&-i\Big[\big[v_{pz}e_{zN}(r_p(t))\cos k_\ell z_p(t)\\
&+v_{pr}e_{rN}(r_p(t))\sin k_\ell z_p(t)\big]\sin N\varphi_p(t)\\
&+v_{p\varphi}e_{\varphi N}(r_p(t))\sin k_\ell z_p(t)\cos N\varphi_p(t)\Big].
\end{aligned} \qquad (11)$$

From eq. (3) and eq. (10) follows that amplitudes of fields A_s and B_s satisfy to relations:

$$\begin{aligned}
&A_{nm\ell} = B_{nm\ell} = 0,\ \text{ for } m \ne N\\
&A_{n,-N,\ell} = A_{n,N,\ell}^*,\ B_{n,-N,\ell} = B_{n,N,\ell}^*.
\end{aligned} \qquad (12)$$

Using the property (12) for a components of a solenoiddal electromagnetic field we obtain the expressions:

$$E_z^t = 2\sum_{n,\ell}|A_{nN\ell}|e_{zN}(r)\cos(k_\ell z)\cos(N\varphi+\alpha_{nN\ell}),$$
$$E_r^t = 2\sum_{n,\ell}|A_{nN\ell}|e_{rN}(r)\sin(k_\ell z)\cos(N\varphi+\alpha_{nN\ell}),$$
$$E_\varphi^t = -2\sum_{n,\ell}|A_{nN\ell}|e_{\varphi N}(r)\sin(k_\ell z)\sin(N\varphi+\alpha_{nN\ell}),$$
$$H_z^t = 2\sum_{n,\ell}|B_{nN\ell}|h_{zN}(r)\sin(k_\ell z)\sin(N\varphi+\beta_{nN\ell}),$$
$$H_r^t = \sum_{n,\ell}|B_{nN\ell}|h_{rN}(r)\cos(k_\ell z)\sin(N\varphi+\beta_{nN\ell}), \qquad (13)$$
$$H_\varphi^t = -2\sum_{n,\ell}|B_{nN\ell}|h_{\varphi N}(r)\cos(k_\ell z)\cos(N\varphi+\beta_{nN\ell}),$$

where: $\tan\alpha_{nN\ell} = \frac{\operatorname{Im} A_{nN\ell}}{\operatorname{Re} A_{nN\ell}}$, $\tan\beta_{nN\ell} = \frac{\operatorname{Im} B_{nN\ell}}{\operatorname{Re} B_{nN\ell}}$.

The equations (3),(5),(6),(11),(13), together with relativistic movement equations completely define self-consistent dynamics of excitation of "whispering gallery" modes by azimuthally modulated electron bunch at any law of beam particle movement. In the case, when transverse movement of beam particles is negligibly small, the formula for the coupling of a beam with a wave can be considerable simplified. Really, taking in the eq. (11) $v_{p\varphi} = v_{pr} = 0$ and $r_p = \text{const}$, $\varphi_p = \text{const}$, we obtain:

$$R_{nN\ell} = \frac{N}{P_N}\sum_{p\in V_R} q_p v_{pz} e_{zN}(r_p)e^{-iN\varphi_p}\cos k_\ell z_p(t) \quad . \quad (14)$$

In last formula we can make summation over transverse distribution of particles. For this purpose it is necessary to express radius of particle r_p through radius of the centre of a separate bunch and its position relative to this centre (see figure) and use the addition theorem of cylindrical functions. Having made integration by transverse particle positions for homogeneous distribution of particle density we finally obtain:

$$R_{nN\ell} = \frac{2NQ_s}{P_N \kappa r_s}\frac{F_N(\kappa R_c,\kappa b)}{F_N(\kappa a,\kappa b)}J_1(\kappa r_s)\sum_{s\in Z_R} v_{sz}(t)\cos k_\ell z_s(t) \qquad (15)$$

where: Q_s is a total charge of all macroparticles in a plane which is perpendicular to the movement direction of particles, and summation is carried out over all such charged sheets, being in the resonator.

Potential electric field can be found at solving of the Poisson equation by well-known electrostatic methods. For example, it can be found by the method applied for excitation of the multizone resonator by bunches [6]. This field is localized in beam volume, and effect weeakly on a dynamics of relativistic electron bunches. Therefore, in the first approximation, we can neglect the numerical calculations of the potential field.

1.CONCLUSIONS

The obtained self-consistent system of the excitation equations of the dielectric resonator by electronic multibeam allows to investigate process of growth of electromagnetic fields, a spectrum of oscillations and saturation power levels.

REFERENCES

1. R.K. Parker, R.H. Abrams, B.G. Danly, B. Levush. Vacuum electronics// *IEEE Trans. on Microwave theory and techniques*. 2002, v.50, N 3, p. 835-845.
2. J.H. Booske. Plasma physics and related challenges of millimeter-wave-to-terahertz and high power microwave generation// *Physics of Plasmas*. 2008, v.15, p. 055502.
3. M.C. Tompson, H. Badakov, A.M. Cook, et al. Breakdown Limits on Gigavolt-per-Meter Electron-Beam-DrivenWakefields in Dielectric Structures// *Phys. Rev. Lett.* 2008, v.100, N 21, p. 214801.
4. A.Ya. Kirichenko, Yu.F Lonin, V.G. Papkovich, et al. Microwave oscillator with "whispering gallery" resonator // *Problems of Atomic Science and Technology. Series "Nuclear Physics Investigations" (53)*. 2010. N 2, p.135-139.
5. L.A. Vainstein. *Electromagnetic waves*. 2-nd ed. M.: "Radio i svyaz", 1988.
6. K.V. Galaydych, G.V. Sotnikov. The nonlinear theory of wakefield excitation by electron bunches in a rectangular multilayered dielectric resonator// *Problems of Atomic Science and Technology. Series "Nuclear Physics Investigations" (54)*. 2010. N3, p. 98-102.

Article received 13.09.10

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ВОЗБУЖДЕНИЯ ЭЛЕКТРОННЫМ ПУЧКОМ МОД "ШЕПЧУЩЕЙ ГАЛЕРЕИ" В ЦИЛИНДРИЧЕСКОМ ДИЭЛЕКТРИЧЕСКОМ РЕЗОНАТОРЕ

К.В. Галайдыч, Ю.Ф. Лонин, А.Г. Пономарёв, Ю.В. Прокопенко, Г.В. Сотников

Построена математическая модель возбуждения мод «шепчущей галереи». Получено дисперсионное уравнение, определяющее собственные частоты этих мод, найдены собственные волны и их нормы. Используя их, получено интегро-дифференциальное уравнение для амплитуд возбуждаемых волн.

МАТЕМАТИЧНА МОДЕЛЬ ЗБУДЖЕННЯ ЕЛЕКТРОННИМ ПУЧКОМ МОД "ШЕПОЧУЧОЇ ГАЛЕРЕЇ" У ЦИЛІНДРИЧНОМУ ДІЕЛЕКТРИЧНОМУ РЕЗОНАТОРІ

К.В. Галайдич, Ю.Ф. Лонін, А.Г. Пономарьов, Ю.В. Прокопенко, Г.В. Сотніков

Побудована математична модель збудження мод «шепочучої галереї». Отримано дисперсійне рівняння, що визначає власні частоти цих мод, знайдені власні хвилі і їх норми. Використовуючи їх, отримане интегро-диференційне рівняння для амплітуд збуджуваних хвиль.