DIFFUSION IN VELOCITY OF CHARGED PARTICLES SCATTERING THE ELECTROMAGNETIC WAVE

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The scattering of an external monochromatic electromagnetic wave by identical charged particles is considered, the influence of the scattered radiation on particles motion being taken into account. The diffusion in velocity of particles caused by their collisions and fields of the scattered radiation is investigated. The mean-square spread in the velocity of particles is determined as a function on time, parameters of external electromagnetic wave and the system of the charged particles.
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1. INTRODUCTION

The scattering of electromagnetic waves by the charged particles is of interest for heating [1] and diagnostics [2] of plasma, acceleration of the charged particles [3], and for applications of short wavelength electromagnetic radiation by relativistic beams of electrons in external periodic fields [4]. The emitted spontaneous radiation is incoherent for spatially homogeneous system consisting of identical charged particles that occur in an external electromagnetic field. The influence of the scattering radiation on the charged particle motion leads to the increase in the mean square spread in the velocity of particles [5]. The most actual increase in the mean square spread in the velocity of particles is observed for sources of ultrashort wavelength coherent radiation of the free-electron-lasers class, based on the selfamplified spontaneous emission [6]. The change of the mean square value of particle velocity is found in neglect by its initial thermal motion [5].

In the given work the diffusion of particles in velocity space in view of the initial velocity spread of particles is considered. Besides the diffusion in the velocity of particles caused both by fields of the scattered radiation and Coulomb interaction of particles is considered. The analytical expressions for the mean-square value of the velocity caused by radiation effects are found for a small interval of time when the distance passed by a particle due to thermal motion is much less than the wavelength of radiation, and for a great interval of time when this distance is much greater than the wavelength.

2. FORMULATION OF THE PROBLEM

Let's consider the system consisting of N identical charged particles, each with charge q and mass m, occupying the volume V and occuring in the field of the external monochromatic electromagnetic wave (EMW) $\mathbf{E}_{ext} = \mathbf{e}_x E_0 \cos(\omega t - kz)$, where E_0 is the amplitude of the wave, ω and k are its frequency and the wave number, $\mathbf{e_x}$ is the unit vector along X axis of the Cartesian system of coordinates.

We assume that the particle motion is nonrelativistic with the amplitude of the charge oscillation in the field of the external wave ($r_\perp \equiv qE/m\omega^2$) that is small in comparison with the wavelength λ (λ=2π/*k*): $a = qE_0/mc\omega << 1$. The position of each of the particles will characterize the radius vector $\mathbf{r_s}$ of the Cartesian system of coordinates

$$\mathbf{r}_s = \mathbf{r}_s^{(0)} - \mathbf{e}_x (a/k)\cos\varphi - \mathbf{e}_z \Delta_s(t), \qquad (1)$$

where $\varphi=\omega t-kz_s(t)$, $\mathbf{r}_s^{(0)}=\mathbf{r}_{0s}+\mathbf{v}_{0s}(t-t_0)$, Δ_s is the displacement on z trajectories of a particle relative to equilibrium one, $\mathbf{r}_{0s}$, $\mathbf{v}_{0s}$ are coordinate and velocity of a particle at the initial moment of time t_0.

The charges, moving in the external field, create their own fields. Electric and magnetic fields produced by an individual particle, moving in the trajectory (1), are found from Lienard-Wiechert potentials [7]. They become [5]

$$\mathrm{E}_{zs} = \frac{(\delta z)_s}{R_s^3}, \; H_{ys} = -qk_0 a \frac{(\delta z)_s}{R_{*s}^2}\left(\cos\varphi'_s + \frac{\sin\varphi'_s}{k_0 R_s}\right), \qquad (2)$$

when *a* is a small parameter. Here $(\delta z)_s=\mathbf{e}_z\mathbf{R}_s$,
$\mathbf{R}_s = \mathbf{r} - \mathbf{r}_s^{(0)}(t)$, $\varphi'_s = \omega t - k_0 R_s - kz_s(t)$, $k_0 = \omega/c$.

The equations of longitudinal motion of the test particle in the external electromagnetic field and the fields created by other charges of the system studied are written in the form

$$\frac{d}{dt} p_z = q\sum_s \left(E_{zs} + \frac{\mathrm{v}_x}{c} H_{ys} \right) = \sum_s F_z^s(t). \qquad (3)$$

Let's consider the particle motion in the interval of time greater than the period of the wave, but considerably smaller then the time of relaxation due to its collective interaction. The expression for the force of pair interaction of particles, according to the equations (2) and (3) becomes

$$F_z^{(s)} = q^2 k_0^2 \left[G_Q(x,t;x_s) + G_R(x,t;x_s) \right], \qquad (4)$$

$$G_Q = \frac{(\delta z)_s}{k_0^2 R_s^3}, \; G_R = -a_0^2 \frac{(\delta z)_s}{k_0 R_s^2}\left(\cos\psi_s + \frac{\sin\psi_s}{k_0 R_s}\right), \qquad (5)$$

where $\psi_s = k_0 R_s - kz_s(t)$, $x = (\mathbf{r},\mathbf{p})$, $x_s = (\mathbf{r}_s, \mathbf{p}_s)$,
$\mathbf{p}=m\mathbf{v}$, $a_0 = a/\sqrt{2}$.

It is necessary to calculate the longitudinal diffusion coefficient describing the change of the mean-square velocity along the direction of external EMW propagation [5]:

$$D = \frac{d}{2dt}\left\langle (\Delta \mathrm{v}_z)^2 \right\rangle = \qquad (6)$$
$$\frac{1}{m^2}\int_{\Omega_x} F_z^{(1)}\left[x,t;x_1^{(0)}(t,x_{01})\right] F_z^{(1)}\left[x,t';x_1^{(0)}(t',x_{01})\right] f(x_{01})dx_{01},$$

where $\Delta v_z = v_z(t) - \langle v_z \rangle$, $x^{(0)} = (\mathbf{r}^{(0)}, \mathbf{p}^{(0)})$, f is the one-particle distribution function, angular brackets denote ensemble averaging.

Deriving the equation (6) we neglected the correlation of particles at the initial moment of time.

3. DIFFUSION COEFFICIENT

Substituting expressions for the pair interaction force of charges (4) in the formula (6) the expression for diffusion coefficient takes the form

$$D = D_Q + D_R , \tag{7}$$

$$D_\alpha = \frac{q^4}{m^2}\int_{-\infty}^{\infty} d\mathbf{v}_s f(\mathbf{v}_s) \int_0^\tau J_\alpha(\tau', \mathbf{v}-\mathbf{v}_s)\, d\tau' ,\quad \alpha = Q, R, \tag{8}$$

$$J_Q = \frac{\partial^2}{\partial z\, \partial z'} I(\mathbf{w}, 0), \quad J_R = J_{(+)} + J_{(-)}, \tag{9}$$

$$J_\pm = \frac{a_0^4}{2}\,\mathrm{Re}\left(\frac{\partial}{\partial z} + ik\right)\left(\frac{\partial}{\partial z'} \pm ik\right) I(\mathbf{w}, \psi_\pm), \tag{10}$$

$$I(\mathbf{w}, g) = \int_V \frac{\exp(ig)}{R_s R_s'}\, d\mathbf{r}_{0s} , \tag{10a}$$

where $\mathbf{w} = \mathbf{r} - \mathbf{r}' - \mathbf{v}_s(t - t')$, $\psi_\pm = \psi_s \pm \psi_s'$, $\psi_s' = \psi_s(t')$, $\mathbf{R}_s' = \mathbf{R}_s(t')$.

For calculation of integral $I(\mathbf{w},g)$ it is expedient to replace integration for $\mathbf{r}_{0s}$ by integration for $\boldsymbol{\rho} = \mathbf{r}_s(t, x_{0s}) - \mathbf{r}$. In these variables function ψ_- depends on the angle between vectors $\boldsymbol{\rho}$ and $\mathbf{w}$ but does not depend on ρ at greater values of the latter. In its turn ψ_+ is proportional to ρ, and hence, subintegral expression $I(\mathbf{w}, \psi_+)$ is an oscillating function of ρ. Therefore the second term $J_{(-)}$ introduces the main contribution to J_R. We shall assume, that the linear sizes of area R occupied by particles are supposed to be greater than the wavelength ($R \gg \lambda$), being greater than the distance the charge passes owing to the thermal motion during the considered process ($R \gg v_T\tau$). Besides we study the diffusion in velocity of particles which are near to the center of the considered system. Integrating for $\boldsymbol{\rho}$ in Eq. (10a) and denoting

$I_{(-)}(\mathbf{w}) \equiv I(\mathbf{w}, \psi_-)$, we find

$$I_{(-)}(\mathbf{w}) = \frac{2\pi}{k_0}\left[(2k_0 R - i)\frac{\sin k_0 w}{k_0 w} + i\exp(ik_0 w)\right]\exp(-ikw_z). \tag{11}$$

It is possible to obtain the equation for $I(\mathbf{w},0)$ tending k to zero in expression for $I(\mathbf{w}, \psi_-)$ and neglecting constants in the obtained equation: $I(w,0) = 2\pi w$.

The differentiation on z′ in Eqs. (7) and (8) may be replaced that of –z because $\mathbf{w}$ depends on a difference of $\mathbf{r}$ and $\mathbf{r}'$. In the right-hand- side of the Eqs. (9) and (10) we take into account the dependence of radius-vector of the test particle on initial velocity and time. Substituting the coordinate of the test particle $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}(t - t_0)$ in expression for $\mathbf{w}$ and replacing differentiation with respect to z by that of v_z in Eqs. (9) and (10), we find

$$J_Q == \frac{2\pi}{\tau'}\frac{u^2 - u_z^2}{u^3}, \tag{12}$$

$$J_R = -\frac{a_0^4}{2}\,\mathrm{Re}\left(\frac{\partial}{\tau' \partial u_z} + ik\right)^2 I_{(-)}(\mathbf{u}\tau'), \tag{13}$$

where $\mathbf{u} = \mathbf{v} - \mathbf{v}_s$, $\tau = t - t'$.

The diffusion coefficient due to Coulomb interaction of particles is obtained by substituting expression (12) in the Eq. (8). The divergence on the bottom limit of integration for τ' in Eq. (8) can be corrected by taking into account, that the minimal distance between two particles isn't equal to zero and is finite being equal to r_{min}. Substituting expression (12) in (8), after integration with respect to τ' in this equation, taking into account, that $r_{min} \ll R$ for times $\tau \gg \tau_{min}$ we obtain (cf. [8])

$$D_Q = 2\pi \frac{q^4}{m^2}\int_{-\infty}^{\infty} \Lambda\, \frac{u^2 - u_z^2}{u^3} f(\mathbf{v}_s)\, d\mathbf{v}_s , \tag{14}$$

where $\Lambda = \ln(r_{max}/r_{min})$, $r_{max} = \min(u\tau, R)$.

Let's find the diffusion coefficient due to the fields of scattered electromagnetic radiation. In general case expression for D_R is clumsy. Therefore we find the asymptotic D_R describing the initial stage of diffusion $\tau \ll \tau_c$ and the opposite limiting case $\tau \gg \tau_c$, where $\tau_c = \lambda / v_T$.

At the initial stage of diffusion at $\tau \ll \tau_c$, expanding the right-hand-side of expression (13) into series on the small parameter $ku\tau'$ up to the terms proportional to this parameter, and then substituting the obtained expression in the Eq. (8), we find

$$D_R = \pi \frac{q^4}{m^2} a_0^4 \left(\frac{2}{3} n_0 k_0^2 R \tau + \int_{-\infty}^{\infty} \Lambda\, \frac{u^2 - u_z^2}{u^3} f(\mathbf{v}_s)\, d\mathbf{v}_s\right). \tag{15}$$

This expression consists of two parts. The first term in the right-hand-side of expression (15) is connected with the interaction of charges through the scattered incoherent electromagnetic radiation. The second term is connected with quasi-static fields of charges-radiators. It can be easily seen that the second term in brackets in (15) is the same as in Eq. (14).

In the limiting case $\tau \gg \tau_c$ the top limit of integration for the time in the right-hand-side of the Eq. (8) may tend to infinity. Retaining the terms proportional to the linear size of the system, which are dominant in this subintegral expression, we obtain:

$$D_R = \pi^2 a_0^4 k_0 R \times \frac{q^4}{m^2}\int_{-\infty}^{\infty}\left[\frac{1}{2}\left(1 - \frac{u_z^2}{u^2}\right)^2 + \left(\frac{u_z}{u}\right)^4\right]\frac{1}{u} f(\mathbf{v}_s)\, d\mathbf{v}_s . \tag{16}$$

Notice, that at $\tau \gg \tau_c$ the expression for D_Q takes the form (14).

4. DISCUSSION

Thus the diffusion coefficient for velocity of particles at scattering of a monochromatic EMW by the identical charged particles is found in this work. The principal analytical tool used in this study was the analytical descriptive model of motion of the point identical charged particles in their scattered radiation field.

It should be noted, that the diffusion coefficient for velocity due to Coulomb interaction of particles (14) agrees with that, which follows from the collisions term derived in [9].

As regards to the diffusion coefficient connected with the influence of incoherent scattered radiation on particle motion the following should be noted. This diffusion coefficient is proportional to the linear size of area occupied by particles. The diffusion coefficient is proportional to τ for the small τ ($\tau \ll \tau_c$). Due to this fact the particles pair interaction force does not depend on time. In this limit case the expression for root-mean-square value of longitudinal velocity can be written in the form:

$$\left\langle (\Delta \mathrm{v}_z)^2 \right\rangle^{1/2} = a_0^2 c r_q \,\omega\tau \sqrt{2\pi R\, n_0/3} ,$$

where $r_q = q^2 / mc^2$.

This formula agrees with the corresponding formula of [5], when the volume occupied by particles represents a plane layer in the width of (2/3) R.

For the great times $\tau \gg \tau_c$, when the motion of particles in the field of external wave is uncorrelated, the diffusion coefficient does not depend on time. In this case uncorrelated force acts on particles, as a result of which rout-mean-square value of velocity increases as $\tau^{1/2}$.

The comparison of the Eqs. (15) and (16) with the Eq. (14) shows, that at a^4kR>Λ the diffusion in velocity will be caused mainly by the radiation interaction of particles.

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ДИФФУЗИЯ ПО СКОРОСТЯМ ЗАРЯЖЕННЫХ ЧАСТИЦ, РАССЕИВАЮЩИХ ЭЛЕКТРОМАГНИТНУЮ ВОЛНУ

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Рассмотрено рассеяние внешней монохроматической электромагнитной волны идентичными заряженными частицами с учетом влияния рассеянного излучения на движение частиц. Исследована диффузия по скоростям частиц, вызванная столкновениями частиц и полями рассеянного излучения. Установлена зависимость среднеквадратичного разброса частиц по скоростям от времени, параметров волны и системы заряженных частиц.

ДИФУЗІЯ ПО ШВИДКОСТЯМ ЗАРЯДЖЕНИХ ЧАСТИНОК, ЩО РОЗСІЮЮТЬ ЕЛЕКТРОМАГНІТНУ ХВИЛЮ

В.В. Огнівенко

Розглянуто розсіяння зовнішньої монохроматичної електромагнітної хвилі ідентичними зарядженими частинками з урахуванням впливу розсіяного випромінювання на рух частинок. Досліджено дифузію по швидкостям частинок, що викликана зіткненнями частинок і полями розсіяного випромінювання. Встановлено залежність середньоквадратичного розкиду частинок по швидкостях від часу, параметрів хвилі і системи заряджених частинок.