INFLUENCE OF GEOMETRY OF THE IMPENETRABLE ELECTRODES ON PROCESS OF FORMATION OF THE CURRENT CRISIS IN THE PLASMA ACCELERATORS

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This paper reports the results of the numerical studies of the axisymmetric flows in the plasma accelerators with the impenetrable equipotential electrodes of the various geometries. The calculations were performed using the two-dimensional two-fluid magnetohydrodynamic model taking into account the Hall effect and the conductivity tensor of the medium. The numerical experiments have allowed to reveal the influence of the electrode form on effect of occurrence of the current crisis.
PACS: 52.30.Cv, 52.59.Dk, 52.65.-y

1. INTRODUCTION

Now the various modifications of the quasi-steady plasma accelerators (QSPA) are tested (see, for example, [1-6]), which allow partially to solve a problem of interaction of the plasma streams with electrodes. In practice the continuous impenetrable electrodes continue to be used in a lot of cases.

Earlier the simplified numerical models which did not include the dependence of coefficients in the equations and boundary conditions from parameter $\omega_e \tau_e$ were used. The present researches are executed on the basis of the full MHD model according to [7]. The various modifications of the two-fluid MHD model answer the statement of the various boundary conditions and have been used earlier for the comparison of the two-dimensional numerical and analytical models [8], and also for the analysis of the ion current transport regime in QSPA with the penetrated electrodes including the additional longitudinal magnetic field [9]. In this case it is a question of numerical researches in QSPA with continuous electrodes of the various geometries at the presence of the unique azimuthal magnetic field.

2. FORMULATION OF PROBLEM

Following [13] we consider that plasma is quasineutral $n_i = n_e = n$ and ignore the inertia of electrons ($m_e << m_i$). We will restrict ourselves to the dynamics of a hydrogen plasma ($Z = 1$, $m = m_i = m_p$) under condition of $T_i \cong T_e = T$.

The initial transport equations using the above assumptions lead to the following system:

$$\frac{\partial \rho}{\partial t} + div \rho \mathbf{V} = 0 \ ; \qquad \rho \frac{d\mathbf{V}}{dt} + \nabla P = \frac{1}{c}\left[\mathbf{j}, \mathbf{H}\right];$$

$$\rho \frac{d\varepsilon}{dt} + P \, div \, \mathbf{V} = Q - div \, \mathbf{q} + \frac{k}{e(\gamma - 1)}(\mathbf{j}, \nabla) T + \frac{P_e}{e} div \frac{\mathbf{j}}{n};$$

$$\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = -rot \, \mathbf{E} \ ; \quad \mathbf{E} = -\frac{1}{c}\left[\mathbf{V}_e, \ \mathbf{H}\right] - \frac{1}{en}\nabla P_e + \frac{1}{en}\mathbf{R};$$

$$\mathbf{j} = \frac{c}{4\pi} rot \, \mathbf{H} = e \, n \left(\mathbf{V}_i - \mathbf{V}_e\right) \ ; \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{V}, \nabla);$$

$$P = P_i + P_e = 2\left(c_p - c_v\right)\rho T \ ; \quad \varepsilon = 2 c_v T,$$ where $\mathbf{V} = \mathbf{V}_i$; P is the total pressure, $\mathbf{q}$ is the heat flux; $\rho = m n$ is the heavy-particle density, and $\mathbf{j}$ is the electric current. The force of friction $\mathbf{R} = \mathbf{R}_\mathbf{j} + \mathbf{R}_T$ is the sum of the friction force $\mathbf{R}_\mathbf{j}$ due to the presence of the relative velocity $\mathbf{u} = \mathbf{V}_e - \mathbf{V}_i = -\mathbf{j}/e\,n$ and the thermal force $\mathbf{R}_T$ dependent on the temperature gradient. The transport coefficients in magnetic field depend on $\omega_e \tau_e$.

As the normalizing quantities we use the following dimensional constants: the channel length L, the characteristic values of the concentration n_0 $\left(\rho_0 = m\, n_0\right)$, temperature T_0, and azimuthal magnetic field component at the channel inlet $H_0 = H_\varphi^0 = 2 J_d / c R_0$. Here R_0 is the radius of the outer electrode; J_d is the discharge current in the system. By means of these values the units of the characteristic velocity $V_0 = H_0 / \sqrt{4\pi\rho_0}$ and the current density $j_0 = c H_0 / 4\pi L$ are formed for example. Four dimensionless parameters participate in model: $\xi = \dfrac{c}{eL}\sqrt{\dfrac{m}{4\pi n_0}}$ is the parameter characterizing the role of the Hall effect; $\beta = 8\pi P_0 / H_0^2$ is the ratio of the gas ($P_0 = k\, n_0 T_0$) and magnetic pressures; $\omega_e \tau_e = \xi H / \nu \rho$; and $\nu = 1/\mathrm{Re}_m = c^2 / 4\pi L \, V_0 \sigma$ is the magnetic viscosity which is inverse proportional to the magnetic Reynolds number corresponding to Spitzer conductivity $\mathrm{Re}_m = \sigma_0 T^{3/2}$.

The statement of boundary conditions assumes that at the channel inlet $z = 0$ there is the subsonic plasma flowing with the known distributions of density and temperature.

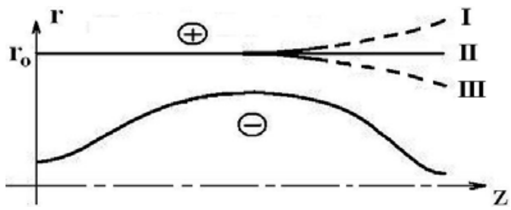


Fig. 1. Geometry of electrodes

Further we believe as $\rho = 1$ and $T = 1$ at the inlet. Without taking into account the equation of an electric circuit we consider that the current has a constant and $|r H_\varphi| = r_0 = const$ at $z = 0$ where $r_0 = R_0 / L$. The boundary conditions assumes that the electrodes are equipotential ($E_\tau = 0$) and impenetrable ($V_n = 0$). The polarity of electrodes specified Fig. 1 answers the standard experimental researches.

The form of the internal cathode is certain according to the analytical model [8] for the external electrode $r_a(z) = r_0 = const$ (line II on Fig. 1). The forms of anode I and III are set by pieces of a parabola at $z > z_0 = 0.5$.

3. CALCULATIONS OF PLASMA FLOWS

We choose as the characteristic units of a problem for calculation of the base variant, for example, the following values: $n_0 = 2 \cdot 10^{16}\,\mathrm{cm}^{-3}$; $T_0 = 3\,eV$; $J_d = 500\ kA$; $L = 0.6\,m$; $R_0 = 0.25\ m$. In this case the values of the dimensionless parameters of the problem are $\beta = 0.15$; $\xi = 0.0027$; $\sigma_0 = 334.8$. If $T = 1$ and $\rho = 1$ we have $\omega_e \tau_e = 0.9$ and $\nu = 0.003$.

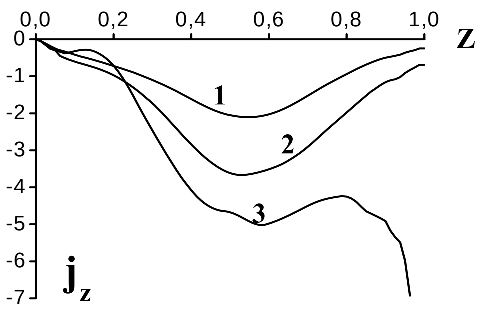
The theoretical analysis [1] of the plasma dynamics across a magnetic field $H = H_\varphi$ in a vicinities of the equipotential impenetrable electrode ($\mathbf{E} \perp \mathbf{V}$, $V_n = 0$, $E_\tau = 0$) has been spent on the basis of the generalized Ohm's law. As $\mathbf{V}_e \neq \mathbf{V}_i$ the account of the Hall effect and parameter $\omega_e \tau_e$ leads to the occurrence of the longitudinal Hall's component of a current $|j_{||}| \cong \omega_e \tau_e |j_\perp|$ and to the pushing of plasma from the anode. In turn the concentration decrease in a vicinity of an electrode increases the parameter $\omega_e \tau_e$ and increases a current along the anode and the pushing of plasma from an electrode even more. As a result under certain conditions there can be a full reorganization of the flow structure and the large-amplitude oscillations occur. In experiments this phenomenon exerts the greatest effect on the volt-ampere characteristics when the discharge current in the system cannot exceed some critical value J_{cr}.

In model the decrease n_0 leads to the qualitative reorganization of flow and to formation of the obviously expressed layer in the vicinity of anode adequating to the more high-velocity stream of the rarefied plasma.

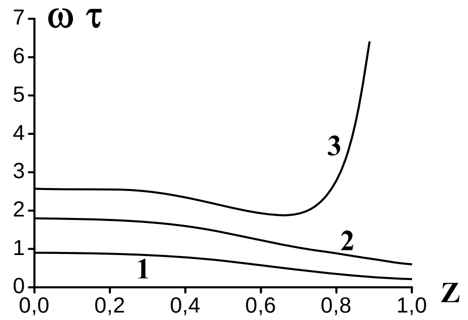
The distributions of a) longitudinal components of a current j_z and b) parameter $\omega_e \tau_e$ along the anode for three values n_0, other parameters specified above and $r_a(z) = r_0 = const$ are presented in Fig. 2. The distribution of a radial current does not vary practically. At the same time the decrease of parameter n_0 leads to the essential growth of value $|j_z|$. The essential growth of a variable $\omega_e \tau_e$ is simultaneously observed. The given distributions answer the stationary flows calculated by means of the relaxation method.

Even greater decrease of the characteristic concentration of particles up to value $n_0 = 0.69 \cdot 10^{16}\,\mathrm{cm}^{-3}$ conducts to the qualitative reorganization of processes. The flow is not become stationary. The fast increase of values $|j_z|$ and $\omega_e \tau_e$ in the vicinity of the outlet part of the anode is observed in the full conformity with the theory. Accordingly for the given magnitude of a discharge current $J_d = 500\,kA$ the value $n_0 = 0.7 \cdot 10^{16}\,\mathrm{cm}^{-3}$ can be considered as the critical value of the characteristic concentration n_{cr}. If $n_0 > n_{cr}$ we have the laminar stationary flows. In case of $n_0 < n_{cr}$ the quickly increasing instability is observed.

As a result of a series of calculations for the various values of the discharge current J_d the critical values of the characteristic plasma concentration n_{cr} at the inlet in the accelerator channel presented in Fig. 3 have been certain accordingly for three different profiles of anode I, II and III represented on Fig. 1. It has appeared that in a plane of variables (J_d, n_{cr}) the boundary between the laminar and unstable modes is the linear function. We can see that values n_{cr} decrease at transition from a profile of electrode I to geometry II and accordingly from the form of an electrode II to III. In other words the narrowing of the accelerating channel expands the area of the parameters adequating to the laminar flows and interferes with development of the current crisis.



a



b

Fig. 2. Distributions of a) longitudinal current component; b) parameter $\omega_e \tau_e$ along anode: 1- $n_0 = 2\times10^{16}\ cm^{-3}$, 2- $n_0 = 10^{16}\ cm^{-3}$, 3- $n_0 = 0.7\times10^{16}\ cm^{-3}$

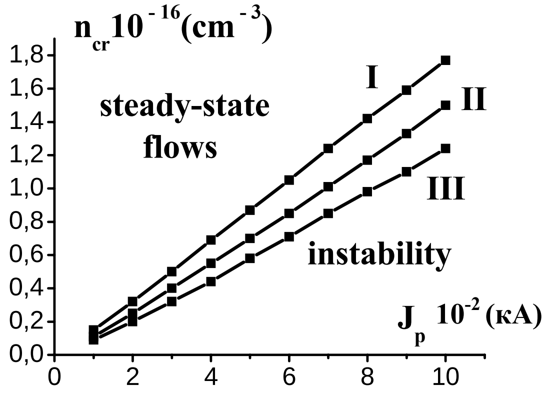


Fig. 3. The critical values of the characteristic plasma concentration at the inlet in the accelerator channel for different profiles of the anode (Fig. 1)

Earlier the experiments [1] have led to a following approximate relation $J_{cr}^2 / J_m \approx K$ in which the critical value of a discharge current J_{cr} participates, the constant K depends basically on the geometry of the channel, and $J_{\dot{m}} = e\,\dot{m} / m_i$ is the mass flux $\dot{m}$ (g/s) expressed in the current units. This flux is easy for defining in calculations.

To each point of the graph represented in Fig. 3 there corresponds the magnitude $\dot{m}$, J_m and $\ln J_m$. It has appeared that the corresponding graph in the plane of variables ($\ln J_d, \ln J_m$) defines also the linear dependences. In this plane of variables the straight lines adequating to three various forms of the anode are in essence parallel each other. On an inclination of these straight lines it is easy to calculate the coefficient α in relations $\ln J_m = \alpha \cdot \ln J_d - \ln K$ or $J_d^{\alpha} / J_m \approx K$ which define the critical values of the mass flux at the certain discharge current or on the contrary the critical values of the discharge current at the known mass flux. Calculations lead to value $\alpha \cong 1.38$ for profile I on Fig. 1, $\alpha \cong 1.42$ for form of electrode II and $\alpha \cong 1.47$ for geometry III. The results of the numerical experiments allow to speak about the comprehensible qualitative conformity with available approximate experimental data.

4. CONCLUSIONS

Researches have confirmed the theory of phenomenon of the current crisis in the vicinity of anode in the plasma accelerator channel with the continuous electrodes. The formation of a layer near to the anode and the occurrence of the processes which proceed to the current crisis are revealed. The comparison to the experimental data defining the presence of the critical modes has been executed in the terms of the values J_d and J_m. The transition from the traditional geometry of the external electrode $r_a(z) = r_0 = const$ to the extending anode (profile I on Fig. 1) promotes the occurrence of the phenomenon of the current crisis.

The work has been executed at the financial support of Russian Foundation of Basic Research (grant 09-01-12056) and Russian Academy of Science (the program No. 14 (1) of Presidium RAS).

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Article received 13.09.10

ВЛИЯНИЕ ГЕОМЕТРИИ НЕПРОНИЦАЕМЫХ ЭЛЕКТРОДОВ НА ПРОЦЕСС ФОРМИРОВАНИЯ КРИЗИСА ТОКА В ПЛАЗМЕННЫХ УСКОРИТЕЛЯХ

А.Н. Козлов

Представлены результаты численных исследований осесимметричных потоков в плазменном ускорителе с непроницаемыми эквипотенциальными электродами различной геометрии. Расчеты выполнены в рамках двумерной двухжидкостной МГД-модели с учетом эффекта Холла и тензора проводимости среды. Численные эксперименты позволили выявить влияние формы электродов на эффект возникновения кризиса тока.

ВПЛИВ ГЕОМЕТРІЇ НЕПРОНИКНИХ ЕЛЕКТРОДІВ НА ПРОЦЕС ФОРМУВАННЯ КРИЗИ ТОКУ В ПЛАЗМОВИХ ПРИСКОРЮВАЧАХ

А.М. Козлов

Представлено результати чисельних досліджень вісесиметричних потоків у плазмовому прискорювачі з непроникними еквіпотенціальними електродами різної геометрії. Розрахунки виконано в рамках двовимірної дворідинної МГД-моделі з урахуванням ефекту Холу і тензора провідності середовища. Чисельні експерименти дозволили виявити вплив форми електродів на ефект виникнення кризи струму.