

**ANALYSIS OF THE POSSIBILITY OF ACCOUNTING FOR THE ANTENNA
REFLECTION COEFFICIENT IN DISPLACEMENT MEASUREMENTS BY
PROBE METHODS**

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The aim of this work is to choose a probe method for displacement measurement that could be modified to account for the horn antenna reflection coefficient, which can hardly be neglected if the target–antenna distance is large enough. Four methods were considered: a single-probe method in which the phase ambiguity problem is resolved by using the fact that the target displacement and velocity are continuous time functions, a two-probe method in which the target velocity is determined by differentiation of the detector currents and then integrated to give the displacement, a two-probe method in which the detector currents are differentiated twice to exclude the unknown magnitude of the target reflection coefficient, and a two-probe method in which the target displacement is determined from the quadrature signals using a phase unwrapping technique; as a result, the last-named method was chosen. That method determines the magnitude and the wrapped phase of the complex reflection coefficient of the target theoretically exactly for reflection coefficient magnitudes no greater than $2^{-1/2}$. Because of this, the chosen method allows one to determine the complex reflection coefficient of the horn antenna at the end of the waveguide section with the probes, whose magnitude is rather small, from the detector currents measured with the horn antenna operating into a matched load. The approach underlying the chosen method made it possible to express the quadrature signals, which contain information on the distance to the target, in terms of the detector

currents, the known complex reflection coefficient of the horn antenna, and the unknown magnitude of the complex reflection coefficient of the target and to derive an equation in the last-named. The results obtained may serve as a basis for the development of probe techniques for displacement measurement with due account for the horn antenna reflection coefficient.

Keywords: *complex reflection coefficient, displacement, electric probe, horn antenna, semiconductor detector, waveguide section.*

The solution of current problems in the dynamics of space hardware, power engineering, and transport systems calls for methods to measure the displacement and velocity of mechanical objects. At present, microwave interferometry is widely used in the determination of these motion parameters [1]. This is due to fact that measurements by microwave interferometry are instantaneous, need no mechanical contact with the target, and can be made in dusty or smoky environments. In the last-named regard, microwave interferometry compares favorably with laser Doppler sensors [2–4] or vision-based systems using digital image processing techniques [5]. Microwave interferometry features a simple hardware implementation, and this is especially true for its probe version, which does not require such special devices as, for example, an analog [6] or a digital [7] quadrature mixer. In existing probe methods, it is assumed that only two electromagnetic waves interfere in the waveguide section where the probes are placed: the wave incident on the target and the wave reflected from the target. This assumption neglects the electromagnetic wave reflected from the horn antenna at the end of the waveguide section. This neglect is justified if the reflection coefficient of the target is far greater than that of the horn antenna. However, the target reflection coefficient, which is defined as the ratio of the amplitude of the wave reflected from the target to the incident wave amplitude, depends on the distance between the target and the antenna aperture, and it may become rather small if this distance is large enough. In the case where the reflection coefficient of the target is comparable with that of the horn antenna, neglecting the latter may result in a sizeable error in displacement determination. The aim of this paper is to choose a probe method for displacement measurement that could be modified to account for the horn antenna reflection coefficient.

In [8] and [9], a single-probe displacement method was proposed. The method is based on the following expression for the distance x between the target and the probe

$$x(t) = \pm \frac{\lambda_0}{4\pi} \arccos \frac{1}{2R} [J(t) - 1 - R^2] + \frac{\lambda_0}{4} (2n + 1), \quad n = 0, \pm 1, \pm 2, \dots \quad (1)$$

where t is the time, λ_0 is the free-space operating wavelength, R is the magnitude of the target reflection coefficient, and J is the current of semiconductor detector connected to the probe normalized to its value in the case where there is no reflected wave (this value must be determined before displacement measurements; to do this, a matched load may be connected to the end of the waveguide section).

When determining the relative displacement, any value of x given by Eq. (1) at $t = 0$ may be taken as the initial value $x(0)$. To unambiguously determine $x(t)$ at $t > 0$, i.e., to choose the sign of the first term and the number n in the second term, use is made of the fact that the target coordinate $x(t)$ and velocity

$\dot{x}(t)$ (here and in the following, a dot denotes time differentiation) are continuous time functions.

The method determines the magnitude of the relative displacement, but cannot determine its direction. Besides, the magnitude R of the target reflection coefficient, which appears in Eq. (1), is assumed to be constant and is determined prior to displacement measurements as follows: the target is moved relative to the antenna, the detector current is measured in doing so, and the reflection coefficient magnitude is found as

$$R = \frac{\sqrt{J_{\max}/J_{\min}} - 1}{\sqrt{J_{\max}/J_{\min}} + 1} \quad (2)$$

where J_{\min} and J_{\max} are the minimum and the maximum normalized detector current, respectively.

However, the magnitude of the target reflection coefficient depends on the distance between the target and the antenna aperture, and the reflection coefficient magnitude vs. distance relationship may exhibit sizeable oscillations due to multiple reflections between the target and the antenna. Besides, because the minimum and the maximum detector current, from which the reflection coefficient magnitude is found, can only be determined by moving the target, the method does not allow one to measure the reflection coefficient of the horn antenna fixed at the end of the waveguide section.

To determine both the displacement magnitude and the displacement direction, at least two probes are needed. In what follows, the consideration will be restricted to the case of two probes because the use of as few as two probes allows one to simplify the design of the waveguide section, simplify its manufacture due to the fact that only one interprobe distance must be held to permitted strict tolerances, and alleviate the parasitic effect of multiple reflections between the probes.

The method proposed in [10] uses two probes placed $\lambda_g/8$ apart where λ_g is the guided operating wavelength (here and in the following, the probe situated farther from the antenna and the probe situated closer to the antenna will be referred to as probe 1 and probe 2, respectively). In that method, the target velocity \dot{x} is determined first, and then the target displacement is found by integrating the velocity. The target velocity is expressed in terms of the detector currents and their time derivatives as follows

$$\dot{x} = \begin{cases} \frac{\lambda_0(1+R)^2 J_1}{4\pi[1+R^2 - J_2(1+R)^2]}, & |1+R^2 - J_2(1+R)^2| \geq |1+R^2 - J_1(1+R)^2| \\ -\frac{\lambda_0(1+R)^2 J_2}{4\pi[1+R^2 - J_1(1+R)^2]}, & |1+R^2 - J_2(1+R)^2| < |1+R^2 - J_1(1+R)^2| \end{cases}$$

where J_1 is the normalized current of the detector connected to probe 1, and J_2 is the normalized current of the detector connected to probe 2.

Given the velocity \dot{x} , the target displacement relative to the target position at $t = 0$ is determined by integrating the velocity over the time

$$\Delta x = \int_0^t \dot{x} dt .$$

The magnitude of the target reflection coefficient is also assumed to be constant and has to be determined prior to displacement measurements from Eq. (2).

In the method proposed in [11], the magnitude of the target reflection coefficient is eliminated by double differentiation of the detector currents. In the case of two probes 1 and 2 placed $l\lambda_g/8$ apart, the target velocity is expressed in terms of the first and the second detector current derivatives as follows:

$$\dot{x} = \begin{cases} 0, & j_1^2 + j_2^2 = 0, \\ \frac{\lambda_0}{4\pi} \frac{\ddot{j}_2 j_1 - \ddot{j}_1 j_2}{j_1^2 + j_2^2}, & j_1^2 + j_2^2 \neq 0. \end{cases}$$

However, in that method the target reflection coefficient magnitude is assumed to be constant too. Besides, the detector currents may have a sizeable noise component. Because of this, double differentiation may introduce a large error.

All the methods considered above, both the single-probe one and the two-probe ones, cannot determine the reflection coefficient of the horn antenna and depend on the following relationship between the normalized detector current J and the magnitude R and phase ψ of the complex reflection coefficient of the target at the location of the probe connected to the semiconductor detector

$$J = 1 + R^2 + 2R \cos \psi .$$

This relationship is derived in the assumption that the only reflected wave in the waveguide section where the probes are placed is the wave reflected from the target, i.e., in the assumption that the wave reflected from the horn antenna is ignored. Because of this, the above-considered methods cannot be used in the case where the reflection coefficient of the target is comparable with that of the horn antenna.

The method proposed in [12] and [13] determines the displacement using two probes placed $l\lambda_g/8$ apart without differentiation of the detector currents. In that method, the quadrature signals $\sin \psi$ and $\cos \psi$, which contain information on the distance to the target, are expressed in terms of the normalized detector currents and the magnitude R of the target reflection coefficient as follows:

$$\cos \psi = \frac{J_1 - 1 - R^2}{2R}, \quad (3)$$

$$\sin \psi = \frac{J_2 - 1 - R^2}{2R}. \quad (4)$$

Given $\sin \psi$ and $\cos \psi$, the displacement Δx of the target at time t_n , $n=0,1,2,\dots$, relative to its initial position $x(t_0)$ is determined using the following phase unwrapping algorithm [14]

$$\varphi(t_n) = \begin{cases} \arctan \frac{\sin \psi(t_n)}{\cos \psi(t_n)}, & \sin \psi(t_n) \geq 0, \cos \psi(t_n) \geq 0, \\ \arctan \frac{\sin \psi(t_n)}{\cos \psi(t_n)} + \pi, & \cos \psi(t_n) < 0, \\ \arctan \frac{\sin \psi(t_n)}{\cos \psi(t_n)} + 2\pi, & \sin \psi(t_n) < 0, \cos \psi(t_n) \geq 0, \end{cases} \quad (5)$$

$$\Delta\varphi(t_n) = \varphi(t_n) - \varphi(t_{n-1}), \quad (6)$$

$$\theta(t_n) = \begin{cases} 0, & n = 0, \\ \theta(t_{n-1}) + \Delta\varphi(t_n), & |\Delta\varphi(t_n)| \leq \pi, \quad n = 1, 2, \dots, \\ \theta(t_{n-1}) + \Delta\varphi(t_n) - 2\pi \operatorname{sgn}[\Delta\varphi(t_n)], & |\Delta\varphi(t_n)| > \pi, \quad n = 1, 2, \dots, \end{cases} \quad (7)$$

$$\Delta x(t_n) = \frac{\lambda_0}{4\pi} \theta(t_n), \quad n = 0, 1, 2, \dots, \quad (8)$$

where φ and θ are the wrapped and the unwrapped phase, respectively.

Eqs. (3) and (4) contain the unknown magnitude R of the target reflection coefficient. Combining the squares of these equations and using the basic trigonometric identity gives the following biquadratic equation in R

$$R^4 - (J_1 + J_2)R^2 + \frac{(J_1 - 1)^2 + (J_2 - 1)^2}{2} = 0. \quad (9)$$

This equation has two positive roots:

$$R_1 = \left[\frac{J_1 + J_2}{2} + \sqrt{\frac{(J_1 + J_2)^2}{4} - \frac{(J_1 - 1)^2 + (J_2 - 1)^2}{2}} \right]^{1/2},$$

$$R_2 = \left[\frac{J_1 + J_2}{2} - \sqrt{\frac{(J_1 + J_2)^2}{4} - \frac{(J_1 - 1)^2 + (J_2 - 1)^2}{2}} \right]^{1/2}. \quad (10)$$

One of these roots is extraneous. As shown in [12], the extraneous root R_{ext} is given by the expression

$$R_{ext} = \left[R^2 + 2\sqrt{2}R \sin\left(\psi + \frac{\pi}{4}\right) + 2 \right]^{1/2}.$$

The analysis made in [12] using this expression for the extraneous root showed that the magnitude R of the target reflection coefficient is given by the smaller root R_2 at any phase of the target reflection coefficient in the case where R satisfies the condition

$$R \leq \frac{1}{\sqrt{2}}. \quad (11)$$

In the case $R > 1/\sqrt{2}$, the magnitude of the target reflection coefficient is given by the root R_2 at the following values of the wrapped phase φ of the target reflection coefficient

$$0 \leq \varphi \leq \varphi_{left} = \frac{3\pi}{4} + \arcsin \frac{1}{\sqrt{2R}},$$

$$\frac{7\pi}{4} - \arcsin \frac{1}{\sqrt{2R}} = \varphi_{right} \leq \varphi \leq 2\pi.$$

At $\varphi_{left} < \varphi < \varphi_{right}$, the magnitude of the target reflection coefficient is given by the greater root R_1 .

As can be seen from the aforesaid, the root R_2 gives the actual magnitude of the target reflection coefficient in a far wider range of R and φ than the root R_1 does. Because of this, in the method proposed in [12] and [13] the root R_2 is taken as the magnitude of the target reflection coefficient. The maximum value of the error introduced in the case where the root R_2 is extraneous is about 4.4 % of the free-space operating wavelength.

The magnitude of the complex reflection coefficient of a horn antenna usually does not exceed 0.1. Because of this, in the case where the only reflected wave in the waveguide section is the wave reflected from the horn antenna, the condition of (11) is certainly satisfied (technically, this case may be implemented, for example, with the horn antenna operating into a matched load). So the method proposed in [12] and [13] allows one to determine the magnitude and wrapped phase of the horn antenna reflection coefficient from Eqs. (3), (4), and (10).

It should be noted at this point that the traditional method for the determination of the complex reflection coefficient involves the use of vector reflectometers, which are rather sophisticated devices, such as:

- matched 12-port measuring transducer [15], which requires special calibration methods and means [16];
- six-port vector analyzer, whose output signals are processed using the principle of holography with three reference signals and Tikhonov's regularization [17];
- two-channel two-detector slotted-waveguide transducer [18], in which the determination of the reflection coefficient phase calls for measurements at two frequencies close to each other.

So the determination of the antenna reflection coefficient by the two-probe method proposed in [12] and [13] is far simpler in terms of both hardware implementation and measuring data processing. Notice that since the classic text by Tischer [19] it has been universally believed that at least three probes are needed to determine or eliminate the unknown reflection coefficient [20, 21].

Consider two probes with semiconductor detectors placed $\lambda_g/8$ apart in a waveguide section between a microwave oscillator and a target. The waveguide section has a horn antenna at its end facing the target. Three electromagnetic waves will interfere with one another in the waveguide section: the incident wave generated by the microwave oscillator, the wave reflected from the target, and the wave reflected from the antenna. The resulting electric field amplitude E will be

$$E = E_{in} \left(e^{j\gamma z} + R_a e^{j\psi_a} e^{-j\gamma z} + R e^{j\psi} e^{-j\gamma z} \right) \quad (12)$$

where E_{in} is the incident wave amplitude, j is the imaginary unit, $\gamma = 2\pi/\lambda_g$ is the propagation constant, z is the coordinate along the waveguide section reckoned from probe 1 towards the horn antenna, and R_a and ψ_a are the magnitude and the phase of the horn antenna reflection coefficient, which can be measured as described above.

For a square-law semiconductor detector, the detector current I is proportional to the squared magnitude of the electric field amplitude

$$I = k|E|^2 = kE\bar{E} \quad (13)$$

where k is a proportionality factor, and the bar denotes complex conjugation.

The complex conjugate amplitude \bar{E} is

$$\bar{E} = \bar{E}_{in} \left(e^{-j\gamma z} + R_a e^{-j\psi_a} e^{j\gamma z} + R e^{-j\psi} e^{j\gamma z} \right). \quad (14)$$

The following expression for the detector current I results from Eqs. (12) to (14):

$$I = k|E_{in}|^2 \left[1 + R_a^2 + R^2 + 2R_a R \cos(\psi - \psi_a) + 2R_a \cos(\psi_a - 2\gamma z) + 2R \cos(\psi_a - 2\gamma z) \right]. \quad (15)$$

It follows from Eq. (15) that the current J_1 of the detector connected to probe 1 ($z = 0$) and the current J_2 of the detector connected to probe 2 ($z = \lambda_g/8$) normalized to their values in the absence of reflected waves (these values have to be determined prior to displacement measurements, for example, using a matched load at the end of the waveguide section) will be

$$J_1 = 1 + R_a^2 + R^2 + 2R_a R \cos(\psi - \psi_a) + 2R_a \cos \psi_a + 2R \cos \psi, \quad (16)$$

$$J_2 = 1 + R_a^2 + R^2 + 2R_a R \cos(\psi - \psi_a) + 2R_a \sin \psi_a + 2R \sin \psi. \quad (17)$$

Eqs. (12) and (13) may be rearranged as follows:

$$2R(1 + R_a \cos \psi_a) \cos \psi + 2RR_a \sin \psi_a \sin \psi = a, \quad (18)$$

$$2R \cos \psi - 2R \sin \psi = b \quad (19)$$

where

$$a = J_1 - 1 - 2R_a \cos \psi_a - R_a^2, \quad b = J_1 - J_2 - 2R_a (\cos \psi_a - \sin \psi_a).$$

Solving Eqs. (18) and (19) for the quadrature signals $\sin \psi$ and $\cos \psi$ yields

$$\cos \psi = \frac{A_1 - R^2}{2RB}, \quad (20)$$

$$\sin \psi = \frac{A_2 - R^2}{2RB} \quad (21)$$

where

$$A_1 = a + bR_a \sin \psi_a, \quad A_2 = a - b(1 + R_a \cos \psi_a),$$

$$B = 1 + \sqrt{2}R_a \sin\left(\psi_a + \frac{\pi}{4}\right).$$

Combining the squares of Eqs. (20) and (21) gives the following biquadratic equation in the unknown magnitude R of the target reflection coefficient

$$R^4 - R^2(2B^2 + A_1 + A_2) + \frac{A_1^2 + A_2^2}{2} = 0. \quad (22)$$

Given R , the quadrature signals can be found from Eqs. (20) and (21), and then the target displacement can be found using the phase unwrapping algorithm of (5) to (8).

Since squaring was used in the derivation of Eq. (22), this equation may have extraneous roots. Because of this, to answer the question which root of this equation gives the actual magnitude of the target reflection coefficient, an analysis of its roots is needed. Such an analysis may be made using the approach employed in the analysis of the roots of Eq. (9) and described in [12].

So the two-probe displacement measurement method proposed in [12] and [13] allows one to determine the horn antenna reflection coefficient. The approach underlying this method has made it possible to express the quadrature signals, which contain information on the distance to the target, in terms of the detector currents, the known reflection coefficient of the horn antenna, and the unknown magnitude of the target reflection coefficient and to derive an equation in the last-named. The results obtained may form a basis for the development of probe techniques for displacement measurement with due account for the horn antenna reflection coefficient.

This work was funded by the Ukrainian Budget Program “Support of the Development of Priority Lines of Research” (KPKVK 6541230).

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Received 12.02.2019,
in final form 28.03.2019