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Optimum Constant-Stress Partially Accelerated Life Test Plans Using Type-I Censored Data from the Inverse Weibull Distribution

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Оптимизация планирования частично ускоренных ресурсных испытаний при постоянных нагрузках и цензурировании данных по типу I для обратного распределения Вейбулла

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Предложенная модель частично ускоренных ресурсных испытаний применена к данным, цензурированным по типу I для обратного распределения Вейбулла. Оценка максимальной вероятности параметров распределения и коэффициента ускорения выполнена в точечном и интервальном видах. Осуществлено планирование ускоренных ресурсных испытаний с оптимизацией доли испытаний, которые необходимо проводить при нормальных эксплуатационных условиях. Численное моделирование тестовой задачи подтвердило точность аналитических расчетов.

Ключевые слова: надежность, обратное распределение Вейбулла, постоянное напряжение, матрица информации Фишера, обобщенная асимптотическая вариация, оптимальный план испытаний.

Notation

PALTs – partially accelerated life tests

IW – inverse Weibull

n – number of test specimens in PALTs

T – lifetime of a specimen under normal conditions

X – lifetime of a specimen under accelerated conditions

η – censoring time of PALTs using type-I censoring

β – acceleration factor ($\beta > 1$)

$\delta_{ui} \equiv I(T_i \le \eta)$

$\delta_{aj} \equiv I(X_j \le \eta)$

π – proportion of allocated sample specimens to run under accelerated conditions

π^* – optimum proportion of allocated sample specimens to run under accelerated conditions

n_u, n_a – number of failed specimens under normal/accelerated conditions, respectively

Introduction. The observed lifetimes of high reliability specimens tend to be very long. So, the time necessary to test a sample of such devices at normal conditions tends to be excessive. Such situations call for testing the sample under stresses which are higher than the usual conditions. The test data are then induced to estimate the life distribution at design stress. So, accelerated life tests (ALT) or partially accelerated life tests (PALT) are usually used to save time and cost. The objective of PALTs is to gather more failure data in a limited time without necessarily using high stress for all test units. The constant-stress PALTs run each specimen at either use or accelerated condition only. That each unit is run at a constant-stress level until the test is finished.

This paper will focus on constant-stress PALTs using type I censored data from the inverse Weibull (IW) distribution. Optimum constant-stress PALTs plans are developed based on the D-optimality criterion.

As indicated by Nelson [1], the constant-stress testing has several advantages. (i) It is easier to maintain a constant-stress level in most tests. (ii) Accelerated test models for constant-stress are better developed for some materials and products. (iii) Data analysis for reliability estimation is well developed.

For an overview of constant-stress PALTs, there are some studies on the estimation and optimally designing of constant-stress PALTs [2–8].

This paper can be organized as follows. Section 1 introduces the IW distribution as a failure time model and discusses the used test method. Section 2 presents the maximum likelihood estimates (MLEs) of the model parameters. Section 3 considers the confidence interval estimation of the model parameters. Section 4 introduces the optimal constant-stress PALTs plans. Section 5 includes simulation studies to illustrate the theoretical results.

1. **The Model and Test Method**.

1.1. ***The Inverse Weibull Distribution***. The IW distribution can be used to model a variety of failure characteristics such as infant mortality, useful life and wear-out periods. The IW model has been introduced as a suitable model for describing the degradation phenomena of mechanical specimens, such as the dynamic components of diesel engines; see, for example, Murthy et al. [9]. Erto and Rapone [10] showed that the IW model provides a good fit to survival data such as the times to breakdown of an insulating fluid subject to the action of constant tension; see also Nelson [11]. Calabria and Pulcini [12] provided an interpretation of the IW distribution in the context of load-strength relationship for a component.

This issue is tackled using the approach earlier developed in [13, 14]. If the random variable Y has Weibull distribution with the probability density function (PDF)

$$f_Y(y;\alpha,\theta\le\eta)=\alpha\theta y^{\alpha-1}e^{-\theta y^{\alpha}}, \qquad y>0,$$

then the random variable $T=1/Y$ has the IW distribution with the PDF

$$f(t)=\alpha\theta(t)^{-(\alpha+1)}e^{-\theta(t)^{-\alpha}}\; t>0, \qquad \alpha>0, \qquad \theta>0. \tag{1}$$

The quantities α and θ are the shape and scale parameters, respectively.

The distribution function of t is given by

$$F(t)=e^{-\theta t^{-\alpha}}, \qquad t>0. \tag{2}$$

The reliability function (RF) is then

$$R(t) = 1 - e^{-\theta t^{-\alpha}}, \qquad t > 0. \tag{3}$$

The related failure rate function is expressed as

$$h(t) = \frac{f(t)}{R(t)} = \frac{\alpha\theta t^{-(\alpha+1)} e^{-\theta t^{-\alpha}}}{1 - e^{-\theta t^{-\alpha}}} = \frac{\alpha\theta t^{-(\alpha+1)}}{e^{\theta t^{-\alpha}} - 1}. \tag{4}$$

1.2. ***Constant-Stress PALTs***. The test procedure can be described as follows:

(1) The total sample size of test specimens (n) is divided into two parts based on a pre-specified proportion π. The first part includes $n\pi$ items selected randomly to run under accelerated conditions, while the remaining items are allocated to run under normal conditions.

(2) Each test specimen is run until the censoring time is reached or the specimen fails and the test condition is not changed.

It is assumed that:

(1) The lifetimes T_i, $i = 1, \ldots, n(1-\pi)$ of items allocated to run under normal conditions, are i.i.d. r.v.'s.

(2) The lifetimes X_j, $j = 1, \ldots, n\pi$ of items allocated to run under accelerated conditions, are i.i.d r.v.'s.

The PDF under accelerated conditions is given by

$$f(x) = \alpha\theta\beta(x\beta)^{-(\alpha+1)} e^{-\theta(x\beta)^{-\alpha}}; \qquad x > 0, \qquad \alpha, \theta > 0, \qquad \beta > 1, \tag{5}$$

where $X = \beta^{-1} T$.

2. **MLEs of the Parameters**. In this section, the MLEs of the CSPALTs model parameters using type-I censored data from the IW distribution are obtained.

Now, let us define the indicator functions:

$$\delta_{ui} = \begin{cases} 1 & t_i \le \eta, \\ 0 & \text{otherwise}, \end{cases} \qquad i = 1, 2, \ldots, n(1-\pi)$$

and

$$\delta_{aj} = \begin{cases} 1 & x_j \le \eta, \\ 0 & \text{otherwise}, \end{cases} \qquad j = 1, 2, \ldots, n\pi.$$

Then the total likelihood function for (t_i, δ_{ui}) and (x_j, δ_{aj}) under CSPALTs is given by

$$L(t, x, \alpha, \theta, \beta) = \prod_{i=1}^{n(1-\pi)} L_{ui}(t_i, \delta_{ui}) \prod_{j=1}^{n\pi} L_{aj}(x_j, \delta_{aj}) =$$

$$= \prod_{i=1}^{n(1-\pi)} [\alpha\theta t_i^{-(\alpha+1)} e^{-\theta t_i^{-\alpha}}]^{\delta_{ui}} [1 - e^{-\theta\eta^{-\alpha}}]^{\bar{\delta}_{ui}} \times$$

$$\times \prod_{j=1}^{n\pi} [\alpha\theta\beta(x_j\beta)^{-(\alpha+1)} e^{-\theta(x_j\beta)^{-\alpha}}]^{\delta_{aj}} [1 - e^{-\theta(\beta\eta)^{-\alpha}}]^{\bar{\delta}_{aj}}, \tag{6}$$

where

$$\bar{\delta}_{ui} = 1 - \delta_{ui} \qquad \text{and} \qquad \bar{\delta}_{aj} = 1 - \delta_{aj}.$$

It is easier to maximize the natural logarithm of the likelihood function instead of the likelihood function itself. The natural logarithm of the likelihood function in (6) can be written as

$$\ln L = \sum_{i=1}^{n(1-\pi)} \delta_{ui}[\ln\alpha + \ln\theta - (\alpha+1)\ln t_i - \theta t_i^{-\alpha}] + \sum_{i=1}^{n(1-\pi)} \bar{\delta}_{ui} \ln[1 - e^{-\theta\eta^{-\alpha}}] +$$

$$+ \sum_{j=1}^{n\pi} \delta_{aj}[\ln\alpha\theta\beta - (\alpha+1)\ln(x_j\beta) - \theta(x_j\beta)^{-\alpha}] + \sum_{j=1}^{n\pi} \bar{\delta}_{aj} \ln[1 - e^{-\theta(\beta\eta)^{-\alpha}}]. \tag{7}$$

The first partial derivatives of the natural logarithm of the likelihood function defined in Eq. (7) with respect to (w.r.t.) α, β, and θ are given by

$$\frac{\partial \ln L}{\partial\alpha} = \frac{n_u + n_a}{\alpha} - \sum_{i=1}^{n(1-\pi)} \delta_{ui} \ln t_i + \theta \sum_{i=1}^{n(1-\pi)} \delta_{ui} t_i^{-\alpha} \ln t_i - \frac{\theta\eta^{-\alpha} \ln\eta}{e^{\theta}\eta^{-\alpha} - 1}(n(1-\pi) - n_u) +$$

$$+ \sum_{j=1}^{n\pi} \delta_{aj} \ln(x_j\beta) + \theta \sum_{j=1}^{n\pi} \delta_{aj}(x_j\beta)^{-\alpha} \ln(x_j\beta) - \frac{\theta(\beta\eta)^{-\alpha}\ln(\beta\eta)}{e^{\theta(\beta\eta)^{-\alpha}} - 1}(n\pi - n_a), \tag{8}$$

where $n_u = \sum_{i=1}^{n(1-\pi)} \delta_{ui}$ and $n_a = \sum_{j=1}^{n\pi} \delta_{aj}$,

$$\frac{\partial \ln L}{\partial\beta} = -\frac{\alpha n_a}{\beta} + \theta\alpha \sum_{j=1}^{n\pi} \delta_{aj} x_j (x_j\beta)^{-\alpha-1} - \frac{\theta\alpha\eta^{-\alpha}(\beta)^{-\alpha-1}}{e^{\theta(\beta\eta)^{-\alpha}} - 1}(n\pi - n_a), \tag{9}$$

$$\frac{\partial \ln L}{\partial\theta} = \frac{n_u + n_a}{\theta} - \sum_{i=1}^{n(1-\pi)} \delta_{ui} t_i^{-\alpha} + \frac{\eta^{-\alpha}(n(1-\pi) - n_u)}{e^{\theta\eta^{-\alpha}} - 1} - \sum_{i=1}^{n\pi} \delta_{aj}(x_j\beta)^{-\alpha} +$$

$$+ \frac{(\beta\eta)^{-\alpha}(n\pi - n_a)}{e^{\theta(\beta\eta)^{-\alpha}} - 1}. \tag{10}$$

Now, we have a system of three non-linear equations which can be solved numerically to obtain the MLEs of the model parameters.

Regarding the corresponding variance-covariance matrix of the MLEs of α, β, and θ, it can be obtained by inverting the Fisher information matrix F. The matrix F is composed of the negative second partial derivatives of the natural logarithm of the likelihood function evaluated at the MLEs. The second-order partial derivatives can be obtained as follows:

$$\frac{\partial^2 \ln L}{\partial\alpha^2} = -\frac{n_u + n_a}{\alpha^2} - \theta \sum_{i=1}^{n(1-\pi)} \delta_{ui} t_i^{-\alpha}(\ln t_i)^2 + (n(1-\pi) - n_u) \times$$

$$\times\frac{\theta\eta^{-\alpha}(\ln\eta)^2(e^{\theta\eta^{-\alpha}}-1)-\theta^2\eta^{-2\alpha}(\ln\eta)^2e^{\theta\eta^{-\alpha}}}{(e^{\theta\eta^{-\alpha}}-1)^2}-$$

$$-\theta\sum_{j=1}^{n\pi}\delta_{aj}(x_j\beta)^{-\alpha}(\ln x_j\beta)^2+(n\pi-n_a)\times$$

$$\times\frac{(\theta(\beta\eta)^{-\alpha}(\ln\beta\eta)^2(e^{\theta(\beta\eta)^{-\alpha}}-1)-\theta^2(\beta\eta)^{-2\alpha}(\ln(\beta\eta))^2)e^{\theta(\beta\eta)^{-\alpha}}}{(e^{\theta(\beta\eta)^{-\alpha}}-1)^2}, \qquad (11)$$

$$\frac{\partial^2\ln L}{\partial\theta^2}=-\frac{n_u+n_a}{\theta^2}-\frac{\eta^{-2\alpha}e^{\theta\eta^{-\alpha}}(n(1-\pi)-n_u)}{(e^{\theta\eta^{-\alpha}}-1)^2}-\frac{(\beta\eta)^{-2\alpha}e^{\theta(\beta\eta)^{-\alpha}}(n\pi-n_a)}{(e^{\theta(\beta\eta)^{-\alpha}}-1)^2}, \qquad (12)$$

$$\frac{\partial^2\ln L}{\partial\beta^2}=\frac{\alpha n_a}{\beta^2}+\theta\alpha\sum_{j=1}^{n\pi}\delta_{aj}(-\alpha-1)x_j^{-\alpha}(\beta)^{-\alpha-2}-$$

$$-\frac{\theta\alpha\eta^{-\alpha}(-\alpha-1)\beta^{-\alpha-2}(e^{\theta(\beta\eta)^{-\alpha}}-1)+\theta^2\alpha^2\eta^{-2\alpha}(\beta)^{2(-\alpha-1)}e^{\theta(\beta\eta)^{-\alpha}}}{(e^{\theta(\beta\eta)^{-\alpha}}-1)^2}(n\pi-n_a), \qquad (13)$$

$$\frac{\partial^2\ln L}{\partial\alpha\partial\theta}=\sum_{i=1}^{n(1-\pi)}\delta_{ui}t_i^{-\alpha}\ln t_i-(n(1-\pi)-n_u)\frac{\eta^{-\alpha}\ln\eta(e^{\theta\eta^{-\alpha}}-1)-\theta\eta^{-2\alpha}\ln\eta e^{\theta\eta-\alpha}}{(e^{\theta\eta^{-\alpha}}-1)^2}+$$

$$+\sum_{j=1}^{n\pi}\delta_{aj}(x_j\beta)^{-\alpha}\ln(x_j\beta)-(n\pi-n_a)\times$$

$$\times\frac{(\beta\eta)^{-\alpha}\ln(\beta\eta)(e^{\theta(\beta\eta)^{-\alpha}}-1)-\theta(\beta\eta)^{-2\alpha}\ln(\beta\eta)e^{\theta(\beta\eta)^{-\alpha}}}{(e^{\theta(\beta\eta)^{-\alpha}}-1)^2}, \qquad (14)$$

$$\frac{\partial^2\ln L}{\partial\beta\partial\theta}=\alpha\sum_{j=1}^{n\pi}\delta_{aj}x_j(x_j\beta)^{-\alpha-1}-$$

$$-(n\pi-n_a)\frac{\alpha\eta^{-\alpha}(\beta)^{-\alpha-1}(e^{\theta(\beta\eta)^{-\alpha}}-1)-\theta\alpha\eta^{-2\alpha}(\beta)^{-2\alpha-1}e^{\theta(\beta\eta)^{-\alpha}}}{(e^{\theta(\beta\eta)^{-\alpha}}-1)^2}, \qquad (15)$$

$$\frac{\partial^2\ln L}{\partial\alpha\partial\beta}=-\frac{1}{\beta}\sum_{j=1}^{n\pi}\delta_{aj}+\theta\sum_{j=1}^{n\pi}\delta_{aj}\left(-\alpha x_j^{-\alpha}\beta^{-\alpha-1}\ln(x_j\beta)+\frac{(x_j\beta)^{-\alpha}}{\beta}\right)-(n\pi-n_a)\times$$

$$\times\frac{((-\alpha\theta\eta^{-\alpha}\beta^{-\alpha-1}\ln(\beta\eta)+\theta\beta^{-\alpha-1}\eta^{-\alpha})(e^{\theta(\beta\eta)^{-\alpha}}-1)+\alpha\theta^2\eta^{-2\alpha}\beta^{-2\alpha-1}\ln(\beta\eta)e^{\theta(\beta\eta)^{-\alpha}}}{(e^{\theta(\beta\eta)^{-\alpha}}-1)^2}. \qquad (16)$$

Accordingly, the matrix F can be expressed as

$$F=\begin{bmatrix} -\dfrac{\partial^2 \ln L}{\partial^2\beta^2} & -\dfrac{\partial^2 \ln L}{\partial\beta\partial\alpha} & -\dfrac{\partial^2 \ln L}{\partial\beta\partial\theta} \\ -\dfrac{\partial^2 \ln L}{\partial\alpha\partial\beta} & -\dfrac{\partial^2 \ln L}{\partial^2\alpha^2} & -\dfrac{\partial^2 \ln L}{\partial\alpha\partial\theta} \\ -\dfrac{\partial^2 \ln L}{\partial\theta\partial\beta} & -\dfrac{\partial^2 \ln L}{\partial\theta\partial\alpha} & -\dfrac{\partial^2 \ln L}{\partial^2\theta^2} \end{bmatrix}. \tag{17}$$

3. **Confidence Bounds of the Parameters**. The most common method to set confidence bounds for the parameters is to use the large-sample normal distribution of the MLEs. For a large sample size, the maximum likelihood estimators are consistent and asymptotically normally distributed. Therefore, the two-sided approximate confidence limits for the model parameters are obtained.

4. **Optimum Constant-Stress Test Plans**. In this section, the problem of determining the optimum proportion of allocated sample specimens to run under accelerated condition π^* is considered. Minimization of the generalized asymptotic variance (GAV) of the MLEs of the model parameters is used as an optimality criterion. The GAV of the MLEs of the model parameters is defined as the reciprocal of the determinant of the Fisher information matrix (F), see Bai et al. [3]. That is,

$$GAV(\hat{\beta}, \hat{\alpha}, \hat{\theta})=\frac{1}{|F|}. \tag{18}$$

Equivalently, we can obtain the optimum value π^* that minimizes the GAV as the value that maximizes the determinant of matrix F.

Thus, the associated expected numbers of failed specimens under use and accelerated conditions are, respectively, obtained by

$$n_u^* = n(1-\pi^*)P_u,$$

$$n_a^* = n\pi^* P_a,$$

where P_u is the probability that a tested specimen under normal conditions fails by the censoring time η, while P_a is the probability that a tested specimen under accelerated conditions fails by η.

5. **Simulation Studies**. In this section, simulation studies are prepared to validate the theoretical results obtained in this paper. Three data sets are simulated with samples of sizes 20, 25, 30, 45, 50, 75, and 100 generated from the IW distribution with 20,000 replications. The average of the results is taken to represent an estimate of each parameter, and the mean square error (MSE) is calculated. After obtaining the estimates, the lower and upper confidence limits for each parameter are also obtained. Moreover, optimum constant-stress PALTs plans that determine the optimal proportion of test units π^* allocated to the high stress are developed. Such optimum test plans minimize the GAV of the MLEs of the model parameters. Accordingly, the values of optimal expected numbers of failed specimens under normal and accelerated conditions, n_u^* and n_a^*, respectively, and the optimal GAV are obtained for each sample size.

T a b l e 1

Average Values of the Estimates, Variances, MSE, and Confidence Limits for (α, β, θ) Set at (1, 1.5, 3) Given $\pi = 0.3$ and $\eta = 15$

n	Parameter	Estimate	Variance	MSE	95%	
					LCL	UCL
20	α	1.1207	0.0546	0.0691	0.6602	1.5759
	β	1.6133	0.6892	0.7020	0.2490	3.5034
	θ	3.7126	3.3531	3.8607	2.4948	9.6729
25	α	1.0962	0.0415	0.0507	0.6881	1.4867
	β	1.5926	0.5727	0.5812	0.6345	3.6010
	θ	3.5412	1.9132	2.2060	1.8011	7.2232
30	α	1.0747	0.0312	0.0368	0.6814	1.3740
	β	1.5733	0.4170	0.4224	0.6775	3.2090
	θ	3.3949	1.1735	1.3293	1.5273	5.7738
45	α	1.0493	0.0190	0.0215	0.6042	1.1450
	β	1.5484	0.2768	0.2791	0.3542	2.4165
	θ	3.2581	0.5930	0.6596	1.3901	4.4087
50	α	1.0440	0.0165	0.0185	0.7937	1.2977
	β	1.5465	0.2355	0.2376	0.0143	1.9165
	θ	3.2213	0.4865	0.5355	1.1876	3.9219
75	α	1.0288	0.0105	0.0114	0.9980	1.4003
	β	1.5291	0.1552	0.1561	0.7096	2.2540
	θ	3.1423	0.2830	0.3033	2.1055	4.1909
100	α	1.0223	0.0076	0.0081	0.9951	1.3376
	β	1.5222	0.1114	0.1119	1.0385	2.3470
	θ	3.1058	0.1935	0.2047	2.5286	4.2530

T a b l e 2

Results of Optimal Design of the Life Test for Different-Sized Samples under Type-I Censoring in Constant-Stress PALTs Based on the Estimation Results from Table 1

n	π^*	n_u^*	n_a^*	Optimal GAV
20	0.3767	7	9	0.01536
25	0.3852	8	12	0.00751
30	0.4558	10	14	0.00227
45	0.5448	16	21	0.00045
50	0.5836	17	25	0.00044
75	0.6180	23	40	0.00014
100	0.6738	26	59	0.00006

It can be observed from Tables 1, 3, and 5 that the MSE of the estimates decreases with the sample size, while the confidence intervals become narrower with the sample size. Moreover, as shown by the numerical results presented in Tables 2, 4, and 6, an increase in sample size n rises the fraction of test specimens allocated to run under accelerated

T a b l e 3

Average Values of the Estimates, Variances, MSE, and Confidence Limits for (α, β, θ) Set at (1.2, 1.5, 3) Given $\pi = 0.3$ and $\eta = 15$

n	Parameters	Estimate	Variance	MSE	95%	
					LCL	UCL
20	α	1.3389	0.0743	0.0936	0.4916	1.5604
	β	1.5727	0.4457	0.4510	0.0721	2.6893
	θ	3.6834	2.8860	3.3528	0.7120	7.3714
25	α	1.3143	0.0572	0.0702	0.7896	1.7271
	β	1.5494	0.3612	0.3636	0.4879	2.8438
	θ	3.5350	1.8138	2.0999	1.2046	6.4840
30	α	1.2890	0.0423	0.0503	0.6708	1.4776
	β	1.5442	0.2814	0.2834	0.4252	2.5047
	θ	3.3980	1.1193	1.2777	0.5259	4.6731
45	α	1.2587	0.0259	0.0293	0.7592	1.3897
	β	1.5331	0.1848	0.1859	1.3018	2.9869
	θ	3.2511	0.5679	0.6309	1.5675	4.5217
50	α	1.2510	0.0221	0.0247	1.0432	1.6259
	β	1.5273	0.1622	0.1630	0.7894	2.3684
	θ	3.2068	0.4600	0.5027	2.3277	4.9863
75	α	1.2338	0.0142	0.0153	1.0805	1.5471
	β	1.5174	0.1067	0.1070	1.0227	2.3029
	θ	3.1365	0.2695	0.2881	1.8772	3.9121
100	α	1.2257	0.0104	0.0110	0.8335	1.2328
	β	1.5144	0.0765	0.0767	1.2710	2.3552
	θ	3.1032	0.1933	0.2039	1.9239	3.6474

T a b l e 4

Results of Optimal Design of the Life Test for Different-Sized Samples under Type-I Censoring in Constant-Stress PALTs Based on the Estimation Results in Table 3

n	π^*	n_u^*	n_a^*	Optimal GAV
20	0.4088	6	11	0.00777
25	0.4512	9	13	0.00376
30	0.5156	9	12	0.00052
45	0.5582	16	23	0.00046
50	0.6411	15	28	0.00042
75	0.6535	23	44	0.00003
100	0.6613	30	59	0.00001

conditions. Thus, the experimenter can achieve more savings in time and cost, because more failure-time data can be obtained in a limited time period. Also, it can be observed from the numerical results, via the values of n_u^* and n_a^* listed in Tables 2, 4 and 6 that the PALTs model is quite appropriate, because failed specimens are obtained at both normal

T a b l e 5

Average Values of the Estimates, Variances, MSE, and Confidence Limits for (α, β, θ) Set at (0.8, 1.5, 3) Given $\pi = 0.3$ and $\eta = 15$

n	Parameters	Estimate	Variance	MSE	95%	
					LCL	UCL
20	α	0.9048	0.0399	0.0509	0.4839	1.2671
	β	1.7096	1.2854	1.3292	1.0435	5.4878
	θ	3.7857	4.1084	4.7255	1.0200	8.9656
25	α	0.8829	0.0300	0.0369	0.3825	1.0617
	β	1.6772	1.0074	1.0387	0.0811	4.0164
	θ	3.5721	2.0515	2.3786	0.5361	6.1507
30	α	0.8666	0.0228	0.0273	0.5335	1.1260
	β	1.6371	0.7459	0.7646	0.1885	3.5739
	θ	3.4417	1.2587	1.4547	0.8586	5.2566
45	α	0.8425	0.0140	0.0158	0.6516	1.1156
	β	1.5926	0.4633	0.4719	0.7396	3.4078
	θ	3.2651	0.6019	0.6721	1.3652	4.4065
50	α	0.8376	0.0119	0.0133	0.5672	0.9952
	β	1.5786	0.3896	0.3958	0.4733	2.9201
	θ	3.2331	0.5042	0.5586	1.8713	4.6549
75	α	0.8246	0.0075	0.0082	0.5744	0.9152
	β	1.5471	0.2541	0.2563	0.9321	2.9082
	θ	3.1340	0.2776	0.2955	2.7478	4.8131
100	α	0.8174	0.0054	0.0057	0.6304	0.9181
	β	1.5383	0.1806	0.1821	0.4331	2.0992
	θ	3.0996	0.19367	0.2036	2.2084	3.9335

T a b l e 6

Results of Optimal Design of the Life Test for Different-Sized Samples under Type-I Censoring in Constant-Stress PALTs Based on the Estimation Results from Table 5

n	π^*	n_u^*	n_a^*	Optimal GAV
20	0.3820	8	5	0.00797
25	0.3927	11	7	0.00591
30	0.4024	12	9	0.00230
45	0.4901	19	14	0.00069
50	0.5215	22	15	0.00041
75	0.6312	41	16	0.00017
100	0.6556	44	34	0.00006

and accelerated conditions. In addition, Tables 2, 4, and 6 present the optimal GAV of the maximum likelihood estimators of the model parameters that were obtained numerically with π^* in place of π for different-sized samples. As indicated by the results, the optimal GAV decreases with the sample size n.

Conclusions. In this paper, the optimal design problem of the constant-stress PALTs was solved using type-I censored data from the inverse Weibull distribution as a lifetime distribution of the test specimens. The maximum likelihood approach was used to estimate the model parameters. The performance of the estimators has been examined in terms of their MSE via simulation studies. Accordingly, optimum test plans were developed by minimizing the GAV of the MLEs of the model parameters. These test plans determine the optimal proportion of test specimens π^* allocated to run under high-stress accelerated conditions. The application of the proposed optimum test plans can save time and cost of the required experimental tests and guarantee a high level of quality of the statistical inference.

Резюме

Запропоновану модель частково прискорених ресурсних випробувань застосовано до даних, цензурованих по типу I для зворотного розподілу Вейбулла. Оцінку максимальної ймовірності параметрів розподілу і коефіцієнта прискорення виконано в точковому й інтервальному вигляді. Виконано планування прискорених ресурсних випробувань з оптимізацією частки випробувань, які необхідно проводити при нормальних експлуатаційних умовах. Чисельне моделювання тестової задачі підтвердило точність аналітичних розрахунків.

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