THEORY OF SURFACE CYCLOTRON X-MODES AT THE SECOND HARMONIC OF ELECTRON CYCLOTRON FREQUENCY EXCITED BY ALTERNATING ELECTRIC FIELD

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Instability of surface cyclotron X-modes at the second harmonic of electron cyclotron frequency (SCXM) in dielectric planar waveguide filled by plasma is studied in kinetic approximation. An external magnetic field is assumed parallel to the plasma surface. Doing that, two components of the SCXM wave vector, which are perpendicular to external magnetic filed have been taken into the account in Vlasov-Boltzmann kinetic equation. Unlike the previous consideration an amplitude value of the alternating electric field is assumed to be either less or approximately equal to unit. Simple expressions for growth rates of the SCXM parametrical instability are calculated. The obtained results can be used for controlling of gas discharges sustained by surface waves under the regime of electron cyclotron resonance.

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1. INTRODUCTION

The work develops theory of the surface electron cyclotron waves' parametrical instability. Parametrical effect of the external alternating electric field on plasmas is studied during a long time [1]. Thus parametrical excitation of the bulk cyclotron waves is examined analytically rather well see e.g. [2,3] as compared with the case of surface waves' (SW) excitation. The SW parametrical instabilities have some peculiarities as compared with the case of bulk waves parametrical instabilities [4]. These instabilities are characterized by different dependences of their growth rates upon the alternating electric fields' amplitude and plasmas' parameters.

Parametric excitation of the SW at the second harmonic of electron cyclotron frequency in planar dielectric waveguides with uniform plasma filling is examined here. The study is carried out in the framework of kinetic description [5]. The SCXM dispersion properties were considered in monograph [6] using simple model of semi-bounded plasma. It was shown, that their skin-depth into plasma is approximately equal to their wavelength in the approach, if plasma density is supposed to be relatively large (Langmuir frequency is larger than electron cyclotron frequency: $\Omega_e > |\omega_e|$) and plasma spatial dispersion is weak, i.e. Larmor radii of the plasma charged particles are essentially less than the SCXM wavelengths $|\vec{k}|\rho_\alpha << 1$, here subscript α describes plasma species: electrons ($\alpha = e$) and positively charged ions ($\alpha = i$). But these results have been obtained by the aid of simplified assumption neglecting the wave vector component, which is oriented perpendicularly to the plasma surface. It was explained by application the approach of weak spatial dispersion of the plasma while solving the kinetic equation.

Unlike the indicated simplified assumption of the [6], here we take into the account both components of the wave vector: $\vec{k} = \vec{k}_1 + \vec{k}_2$ (they are oriented along $\vec{X}$ and $\vec{Y}$ coordinates, respectively), the wave vector is perpendicular to an external magnetic field $\vec{B}_0$, which is directed along the $\vec{Z}$ axis. It allows us to make some correction of numerical coefficient in analytical expression for the SCXM eigen frequency and to improve theory of parametrical influence of an external alternating electric field on these modes.

The paper is organized as follows. In Section 2 we study dispersion properties of the SCXM propagating across an external steady magnetic field in plasma filled planar dielectric waveguides at the second electron cyclotron harmonic. In Section 3 we examine the parametrical instability of the electron SCXM excited by external alternating electrical field. Simple analytical expressions for their growth rates' values are derived. The paper is concluded by a short Summary.

2. DISPERSION OF THE SCXM

Let's consider uniform plasma that occupies the half-space $0 \le x$ and that is bounded on the plane $x = 0$ by a dielectric medium with dielectric coefficient: ε_d. The model of uniform plasma is applicable if a scale of non-uniformity is much larger than wave's penetration depth into the plasma region. On the other hand, as far as we consider here electromagnetic perturbations just at the harmonics of electron cyclotron frequency then they are affected mainly just by an external magnetic field value but not by a plasma density non-uniformity. External steady magnetic field $\vec{B}_0$ is parallel to the plasma-dielectric interface. The studied surface modes with components: E_x, E_y, H_z propagate along the $\vec{Y}$ direction across $\vec{B}_0$. Plasma particles behavior is governed by kinetic Vlasov-Boltzmann equation and the SCXM fields are described by Maxwell equations. Dependence of the SCXM fields upon time and coordinates is chosen in the following form:

$$E_{x,y}, H_z \propto f(x)\exp(ik_2 y - i\omega t).$$

We supposed that there is no dependence upon Z coordinate. One can't derive dispersion equation for the SCXM in general form (for implicit value of the electron cyclotron harmonics' number S). So let's consider the case $S = 2$ because of its practical significance.

Using method of Fourier transformation one can find out expression for Fourier coefficient of tangential

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component of SCXM electric field. By the aid of theory of residuals one can obtain expression for plasma impedance and then by the aid of linear boundary conditions for tangential component of the waves' fields derive dispersion equation for SCXM propagating at the second harmonic of electron cyclotron frequency:

$$\varepsilon_1+\varepsilon_2\eta+1=\frac{k_2 q_0(\varepsilon_{12}(q_0)+i\eta)}{(q_0^2+k_2^2)[1-\Omega_e^2 k_2^2\rho_e^2(4\varepsilon_1\omega^2 h)^{-1}]}, \quad (1)$$

here $\varepsilon_{1,2}$ are components of plasma permeability in approach of cold magneto-active plasma, ε_{12} is non-diagonal component of plasma permeability tensor that is obtained in a kinetic approach [5], q_0 is root of the equation $\varepsilon_{11}(k_1=q_0)=0$, which is located in upper semi-plane of the complex plane of wave vector component $\vec{k}_1$, $h=1-2|\omega_e|/\omega$ is the shift of SCXM frequency from the second electron cyclotron harmonic. Appearance of multiplier in square brackets in the denominator of equation (1) makes it differ from that one, which was obtained in monograph [6]. Nevertheless its solution is not sufficiently differed from that one, which is indicated in [6]. It is explained by the following circumstance: dispersion equation represented in [6] has been derived in supposition that: $\varepsilon_{11}(k_1=i|k_2|)=\varepsilon_1$, which is correct for the presented consideration. Because of that relative error for the shift of SCXM frequency, which was obtained in [6] under the assumption about smallness of $|q_0|$, has been found small. In the present consideration, value of this root is $q_0\approx i|k_2|/3$ for the approach of dense plasma ($\Omega_e^2>>\omega_e^2$). The main distinguishing feature of the presented dispersion equation (1) is absence the wave, which is propagating along $+\vec{Y}$ direction. So electron SCXM propagate along direction of Larmor precession for electrons located just nearby the plasma surface, i.e. their wave number k_2<0 and frequency value decreases with increasing $|k_2\rho_e|$. Dispersion relation for the SCXM under considered conditions is as follows:

$$\omega\approx 2|\omega_e|/(1+k_2^2\rho_e^2/6). \quad (2)$$

Using asymptotical expressions of the non-integrant kernel of the plasma conductivity tensor values [7] one can calculate the SCXM damping decrement caused by interaction between plasma particles and plasma boundary. Its value is approximately equal to $k_2 V_T h^{1/2} c^{-1}$, as it was indicated in [6]. Collisions between plasma particles carry on their contribution into the process of the SCXM damping. The value of the collisional damping rate is proportional to the effective value of the collision frequency of electrons: $\kappa_{col}\approx\nu k_2/(2\omega h)$. Estimating the integral damping rate of these modes one can conclude that they are weakly damped, the waves with long wavelengths damp more strongly than the modes with short wave lengths.

3. PARAMETRIC EXCITATION OF THE SCXM

Let's suppose in this section that considered waveguide structure is affected by alternating electrical field: $\vec{E}_0\cos(\omega_0 t)\perp\vec{Z}$. Solution of the kinetic equation allows one to drive two components of electrical conductivity tensor of magneto-active plasma calculated in kinetic approach, which are applied:

$$\sigma_{11}(p)=\sum_\alpha\sum_{s,m,n=-\infty}^{+\infty}\frac{i\Omega_\alpha^2 s^2 e^{-y_\alpha}I_s(y_\alpha)}{4\pi(\omega_{n+m}-s\omega_\alpha)y_\alpha}\times J_m(g)J_{m-n}(g), \quad (3)$$

$$\sigma_{12}(p)=-\sum_\alpha\frac{\Omega_\alpha^2}{4\pi}\sum_{s,m,n=-\infty}^{+\infty}\frac{\Omega_\alpha^2 s e^{-y_\alpha}}{4\pi(\omega_{n+m}-s\omega_\alpha)}\times$$
$$\times[I_s'(y_\alpha)-I_s(y_\alpha)]J_m(g)J_{m-n}(g), \quad (4)$$

here $g=\dfrac{k_\perp e_\alpha E_0}{m_\alpha(\omega_0^2-\omega_\alpha^2)}=g_0 k_{\perp\rho_e}$, $y_\alpha=\dfrac{k_\perp^2\rho_\alpha^2}{2}<<1$, $J_m(x)$ is Bessel function of the first type, summing up over indexes s,m,n can be carried out independently from $-\infty$ to $+\infty$, $\omega_{p+m}=\omega+(p+m)\omega_0$, $I_n(z)$ and $I_m'(y)$ are modified Bessel function and its derivative over argument, accordingly.

In order to derive set of equations, which describes excitation of electron SCXM one can apply the following boundary conditions: continuity of the tangential component of the SCXM electric field on the plasma-dielectric boundary and discontinuity of the magnetic field on the plane: $x=0$, which is caused by nonlinear surface electric current flowing along the plasma surface. To formulate this non-linear boundary condition one can solve set of Maxwell equation using Fourier method. Doing that, reverse Fourier transformation can be carried out according to theory of residues. Thus discontinuity of magnetic component of the SCMX field can be written in the following form:

$$H_z(x=0,n)\approx\frac{i\omega_n}{c|k_2|}E_y(x=0,n)-\frac{4\pi}{c}\int_{-0}^{+0}j_y^{(n+l)}dx\Big|_{l\neq 0}. \quad (5)$$

Application of the indicated boundary conditions allows one to derive infinite set of equations for satellite harmonics of tangential component of SCXM electric field, which include in themselves influence of external alternating electric field on plasma. Let's supposed that the following resonant condition is realized:

$$\omega=2|\omega_e|+\Delta_T+\delta\omega, \quad (6)$$

here Δ_T is the frequency shift determined by the plasma particles thermal motion (see eq.(2)) and $\delta\omega$ is correction to the electron SCXM eigen frequency caused by affect of alternating electric field. Under the resonant condition (6), taking into the account only the nearest satellite harmonics of the tangential component of the electron SCXM field one can reduce the mentioned above infinite set of equations to the following one, which is formulated for determinant constructed of coefficients herewith harmonics of the tangential component of the SCXM electric field:

$$\begin{vmatrix}F^{(-1)} & F^{(-1,+1)} & F^{(-1,+2)}\\ F^{(0,-1)} & F^{(0)} & F^{(0,+1)}\\ F^{(+1,-2)} & F^{(+1,-1)} & F^{(+1)}\end{vmatrix}=0, \quad (7)$$

here

$$F^{(n,l)} = \sum_{m,l=-\infty,l\neq 0}^{+\infty} \frac{\Omega_e^2 (0.47 g_0 k_2 \rho_e)^{|m|+|m-l|}}{2m!(m-l)!\omega_n} \times$$
$$\times [\frac{3|\omega_e| + 9\omega_{n+m}}{\omega_{n+m}^2 - \omega_e^2} + \frac{2k_2^2 \rho_e^2 / 3}{\omega_{n+m} - 2|\omega_e|}] ,$$
$$F^{(n)} = (\varepsilon_1 + \varepsilon_2 - 1) i |k_2| / q_0 + (\varepsilon_1 - \varepsilon_2 + 1) .$$

Solution of the equation (7) for the limiting case of dense plasma ($\Omega_e^2 >> \omega_e^2$) has the following form:

$$\delta\omega \approx -2\Delta_T / 3 + i0.3 |g_0 k_2 \rho_e \Delta_T| . \quad (8)$$

Expression (8) coincides with the result obtained in the framework of the previous simplified model with accuracy to within numerical multipliers. Thus general features of the SCXM growth rate remain the same and its value is larger than the SCXM damping rate for the range of relatively short wavelengths: $|k_2 \rho_e| > 0.25$ [8].

Analyzing expression (8) one can see, that parametrical influence of the external alternating electric field on the SCXM dispersion leads to decreasing of their frequency. The SCXM growth rates strongly decrease with increasing their wavelength and decreasing amplitude of the alternating electric field. It's interesting to underline that: – the electron SCXM excitation by an alternating electric field can be realized only for the waves with $k_2 < 0$ (it is true both for the present model and for the previous model); – unlike the result obtained in [8], amplitude of an external alternating electric field can be relatively larger $g_0 \leq 1$ in the present approach.

SUMMARY

Motivated by problem of the plasma edge physics dispersion properties of the SCXM and their parametrical instability caused by affect of an external alternating electric field have been studied. We have derived equations that consistently describe the SCXM dispersion and their parametrical excitation. Simple analytical expressions for their eigen frequencies, their damping rates due to collisions between plasma particles and growth rates due to parametric instability are found out. The predictions of the presented theory can be interesting for the plasma edge physics and experiments devoted to gas discharges sustained by surface waves [9].

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ТЕОРИЯ ПОВЕРХНОСТНЫХ ЦИКЛОТРОННЫХ X-МОД НА ВТОРОЙ ГАРМОНИКЕ ЭЛЕКТРОННОЙ ЦИКЛОТРОННОЙ ЧАСТОТЫ, ВОЗБУЖДАЕМЫХ ПЕРЕМЕННЫМ ЭЛЕКТРИЧЕСКИМ ПОЛЕМ

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Неустойчивость поверхностных циклотронных X-мод на второй гармонике электронных циклотронных частот (ПЦХМ) в диэлектрических планарных волноводах с плазменным наполнением исследована в кинетическом приближении. Внешнее магнитное поле считалось параллельным поверхности плазмы. При этом две компоненты волнового вектора ПЦХМ, перпендикулярные внешнему магнитному полю, были учтены в кинетическом уравнении Власова-Больцмана. В отличие от предыдущего подхода к решению этой проблемы амплитуда внешнего переменного электрического поля предполагалась меньшей или приблизительно равной единице. Простые выражения для инкрементов параметрической неустойчивости ПЦХМ выведены. Полученные результаты могут быть использованы при управлении газовыми разрядами, которые поддерживаются поверхностными волнами в режиме электронного циклотронного резонанса.

ТЕОРІЯ ПОВЕРХНЕВИХ ЦИКЛОТРОННИХ X-МОД НА ДРУГІЙ ГАРМОНІЦІ ЕЛЕКТРОННОЇ ЦИКЛОТРОННОЇ ЧАСТОТИ, ЩО ЗБУДЖУЮТЬСЯ ЗМІННИМ ЕЛЕКТРИЧНИМ ПОЛЕМ

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Нестійкість поверхневих циклотронних X-мод на другій гармоніці електронних циклотронних частот (ПЦХМ) у діелектричних планарних хвилеводах із плазмовим наповненням досліджено у кінетичному наближенні. Зовнішнє магнітне поле вважалося паралельним межі плазми. При цьому дві компоненти хвильового вектора ПЦХМ, що є перпендикулярними до зовнішнього магнітного поля, були враховані у кінетичному рівнянні Власова-Больцмана. На відміну від попереднього підходу до розв'язання цієї проблеми амплітуда зовнішнього змінного електричного поля вважалася меншою або приблизно рівною одиниці. Прості вирази для інкрементів параметричної нестійкості ПЦХМ виведено. Здобуті результати можна використати при керуванні газовими розрядами, що підтримуються поверхневими хвилями у режимі електронного циклотронного резонансу.