# STABILIZATION OF THE EXCITED STATES

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The mechanism allowing to stabilize excited states of quantum systems is offered. The mechanism stabilization is similar to quantum Zeno's effect. The difference consists that under system is not made supervision. Instead of it the system is periodically transferred in other quantum state. The speed, necessary for stabilization, of stabilizing transferring is determined.

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## 1. INTRODUCTION

The mechanism of stabilization of the excited states offered in this work, is similar to quantum Zeno's effect [1-2]. Therefore, to clarify the difference between offered mechanism and Zeno's mechanism, we briefly shall describe the contents of Zeno's effect. Let we have a two-level quantum system. The zero level corresponds to stationary, not excited state. The first level corresponds to the excited state. Let now this system is under action of resonant perturbation. We want to consider how the crossing occurs from the zero level to the first level and back. As known, such process is described by the following simple system of the differential equations:

$$i\cdot\hbar\cdot\dot{A}_0 = V_{01}A_1; \qquad i\cdot\hbar\cdot\dot{A}_1 = V_{10}A_0, \qquad (1)$$

where $A_i$ – complex amplitudes of wave functions. The square of the module of these amplitudes defines the probability of a presence of the quantum system at the correspond power level. The matrix elements of the interaction $V_{01}$ and $V_{10}$, generally, depend both on structure of considered system, and from the characteristics of perturbation.

We shall be consider them equal, constant and real. Let at the initial moment of time the quantum system is in the excited state. Then the solutions of the equations (1) will be functions:

$$A_1 = \cos(\Omega\cdot t), \quad A_0 = \sin(\Omega\cdot t), \qquad (2)$$

where $\Omega = V/\hbar$ – Rabi frequency.

The physical contents of the solutions (2) is such: if at the initial moment of time the system was in the excited state, then after the expiration of time $T = 2\pi/\Omega$ she, with probability which is equal unit, will pass in the basic, not excited state. Further the process will repeat, but on each small interval of time this process has probability character.

Therefore, for further it is convenient all interval of time $T$ to divide on small time intervals $\Delta t = T/n$.

Now we shall enter a new element - measurement of a state of investigated system. Let at the moment of time $\Delta t$ we somehow can estimate a state of our system. Probability of that fact that she during the time $\Delta t$ will pass from the excited state to basic state will be equal:

$$w(\Delta t) = 1-(\Omega\cdot\Delta t)^2. \qquad (3)$$

This formula already contains the important element of quantum transitions. It consists in that that the velocity of transitions at small intervals of time is the less the less are these intervals:

$$\frac{dw}{dt}/w = -2\cdot\Omega^2\cdot t. \qquad (4)$$

This result in the theory of quantum Zeno's effect named as nonexponential law of disintegration (see, for example, [2]). After the expiration of the following interval of time we again include process of measurement. The probability of detection of the originally excited system in the initial state will be defined by formula:

$$w(2\cdot\Delta t) = \left(1-(\Omega\cdot\Delta t)^2\right)^2. \qquad (5)$$

Such formula reflects the fact of independence of quantum transitions in each of time intervals $\Delta t$. Eventually, after the large number of measurements the probability of a presence of system in the excited state will be expressed by the formula:

$$w(n\cdot\Delta t) = \left(1-(\Omega\cdot\Delta t)^2\right)^n. \qquad (6)$$

Let us take the logarithm from the left and right parts of expression (6). In result we shall get that in limit of a large number of measurements during time $T$, the probability of detection of system in its initial excited state go to unit:

$$w(T) = \exp\left(-\Omega^2T^2/n\right)\xrightarrow[n\to\infty]{}1. \qquad (7)$$

Thus, the process of supervision of the excited system does not give for this system to pass from the initial excited state in any other state. This fact makes the contents of quantum Zeno's effect.

Now it is easy to explain the basic contents of work. It consists in the following. We refuse from procedure of measurement. However we assume, that near to the excited energy level of system there is an additional level (system becomes three-level). At that, on quantum system except perturbation which can transfer system from excited state in stationary, not excited state the additional external low-frequency perturbation acts. And, the frequency of this perturbation is resonant in relation to transitions between the excited state and new additional energy level. Besides we shall require that the Rabi frequency of transitions between the excited state and new state should be much greater, than return time of transition of the excited system in the not excited stationary state. Thus we shall show that the period of these additional transitions, in which the excited system is involved, will play a role of an interval of time between supervision in quantum Zeno's effect. The large this frequency the smaller probability of transition the



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system from the excited state to the basic state (not excited stationary state).

## 2. STATEMENT OF A PROBLEM AND BASIC EQUATIONS

Let's consider quantum system, which is described by Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{H}_1(t). \qquad (8)$$

Second addend in the right part describes perturbation. The wave function of system (8) obeys to the Schrödinger equation, which solution we shall search as a row of own functions of the not perturbed equation:

$$\psi(t) = \sum_n A_n(t) \cdot \varphi_n \cdot \exp(i\omega_n t), \qquad (9)$$

where $\omega_n = E_n / \hbar$; $\quad i\hbar \dfrac{\partial \varphi_n}{\partial t} = \hat{H}_0 \varphi_n = E_n \cdot \varphi_n$.

Let's substitute (9) in the Schrödinger equation and by usual way we shall receive system of the connected equations for a finding of complex amplitudes $A_n$:

$$i\hbar \cdot \dot{A}_n = \sum_m U_{nm}(t) \cdot A_m, \qquad (10)$$

where $U_{nm} = \int \varphi_m^* \cdot \hat{H}_1(t) \cdot \varphi_n \cdot \exp[i \cdot t \cdot (E_n - E_m)/\hbar] \cdot dq$.

Let's consider more simple case – the case of harmonic perturbation:

$$\hat{H}_1(t) = \hat{U}_0 \cdot \exp(i\omega_0 t) + \hat{U}_1 \cdot \exp(i\omega_1 t).$$

Then the matrix elements of interaction will get the following expression:

$$U_{nm} = V_{nm} \exp\{i \cdot t \cdot [(E_n - E_m)/\hbar + \Omega]\},$$

$$V^{(k)}{}_{nm} = \int \varphi_n^* \cdot \hat{U}_k \cdot \varphi_m dq. \qquad (11)$$

Let's consider of the dynamic three-level system: ($|0\rangle, |1\rangle, |2\rangle$). We shall consider that frequency of external perturbation and the own meanings of energy of these levels satisfy to such relations:

$$m = 1,\ n = 0, \quad \hbar\omega_0 = E_1 - E_0; \quad m = 2,\ n = 0$$

$$\hbar(\omega_0 + \delta) = E_2 - E_0 \quad |\delta| \ll \omega_0,\ \hbar\omega_1 = E_2 - E_1 \ |\delta| \sim \omega_1. \qquad (12)$$

These relations indicate that fact, that the frequency $\omega_0$ of external perturbation is resonant for transitions between zero and first levels, and the frequency $\omega_1$ is resonant for transitions between the first and second levels. Using these relations in system (10), it is possible to be limited by three equations:

$$i \cdot \hbar \cdot \dot{A}_0 = V_{01} A_1 + V_{02} A_2 \cdot \exp(i \cdot \delta \cdot t);$$

$$i \cdot \hbar \cdot \dot{A}_1 = V_{10} A_0 + V_{12} A_2;$$

$$i \cdot \hbar \cdot \dot{A}_2 = V_{21} A_1 + V_{20} A_0 \cdot \exp(-i \cdot \delta \cdot t). \qquad (13)$$

The system of the equations (13) is that system, which we shall analyze. The scheme of energy levels for system (13) is represented in Fig.1.

## 3. DYNAMICS IN ABSENCE OF DETUNING

Let's consider, first of all, case, when detuning is enough large and the terms in system of the equations (13), which contain detuning, can be neglected. Besides we shall consider that the matrix elements of direct and return transitions are equal ($V_{12} = V_{21}, V_{10} = V_{01}$).Besides we shall consider that the matrix elements of transitions between the first and second levels are much more, than matrix elements of transitions between zero and first levels ($V_{12}/V_{10} \equiv \mu >> 1$). For further it is convenient to introduce dimensionless time $\tau = V_{10} \cdot t / \hbar$.

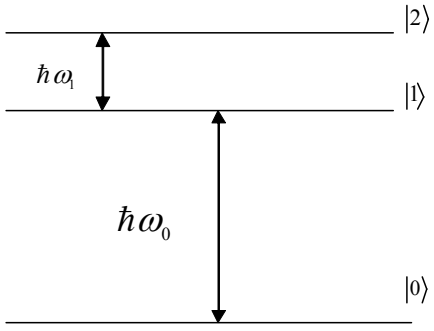


*Fig.1. Schema of energetic levels*

Take into account these reasons the system of the equations (13) becomes elementary simple:

$$i\dot{A}_0 = A_1, \quad i\dot{A}_1 = A_0 + \mu A_2, \quad i\dot{A}_2 = \mu A_1. \qquad (14)$$

Let at the initial moment of time ($t = 0$) the considered quantum system is on first, excited level. Then, as it is easy to see, the solution of system (14) will be functions:

$$A_0 = \frac{1}{i \cdot \mu} \sin(\mu \cdot t), \ A_1 = \cos(\mu \cdot t), \ A_2 = -i \sin(\mu \cdot t). \qquad (15)$$

From the solution (15) follows, that than there will be the more parameter $\mu$, the there will be less probability, that the system from the excited state will pass in not excited, stationary state. It is necessary to say a little words about parameter $\mu$. Physically this parameter defines the relation of number of quantum of low-frequency perturbation which is responsible for transitions between the first and second levels to number of quantum of high-frequency perturbation which defines transitions between the first and zero levels. Than there will be more this ratio, the there will be less a probability that the excited system will pass in the not excited state. The account of influence of the terms containing detuning can be made by numerical methods. Such analysis was carried out. He has shown, that presences even enough large detuning only little changes the result. And, the more size of parameter $\mu$, the smaller appears this influence. In a Fig.2 is given characteristic dependence of module of amplitude $A_0$ from time, and in Fig.3 – time dependence of module of amplitude $A_1$ when there is detuning, which was equal 0.1. Parameter $\mu$ at this calculation was equal 100.

At the initial moment of time the system practically was at the first excited level. The dependence of the module of amplitude $A_2$ from time does not differ practically from dependence submitted in a Fig.3. One can see from these figures that initial probability to be systems in the not excited state (at a zero level) during the time practically does not vary. She is very small and in all cases inversely to a square of parameter $\mu$. Shall note, that result is a little varies if parameter detuning to increase up to 0.5.

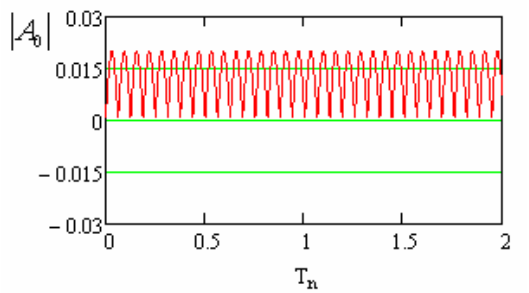


*Fig.2.*

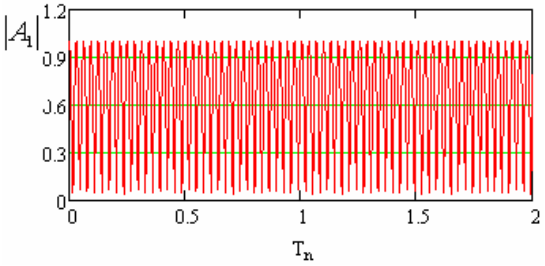


*Fig.3.*

## 4. TRANSITIONS AT SPONTANEOUS RADIATION

In the previous section we have considered quantum system, the transitions between which levels were caused by induced processes. The probabilities of induced processes are considerably large than spontaneous, however spontaneous transitions can be occur on all underlying energy levels. Such transitions, generally, can be much. There is a question, as the process of stabilization of the excited state will proceed in this case. Whether he can be realized in this case? For the answer to the put question we shall consider a concrete case of stabilization excited state in synchrotron. At this we shall use results stated in [3]. Thus, we shall consider synchrotron radiation. In difference to consideration in [3], we shall study not two-level system, but three-level system. And, we shall assume, that on considered system acts the external periodic perturbation causing induce resonant transitions between two top levels (between the level one and the level two in Fig.1). The transitions between level 1 and zero level occur spontaneously. System of the equations describing dynamics of amplitudes of wave functions for such system can be represented as:

$$i\hbar\dot{A}_0 = V^+ A_1 \exp\left(-i\cdot\Delta E\cdot t/\hbar\right),$$
$$i\hbar\dot{A}_1 = V^- A_0 \exp\left(i\cdot\Delta E\cdot t/\hbar\right) + V_{12}\cdot A_2,$$
$$i\hbar\dot{A}_2 = V_{21}\cdot A_1. \qquad (16)$$

Here $V^+ = \int \psi_0^+ U^+ \psi_1 d^3x$, $V^- = \int \psi_1^+ U^- \psi_0 d^3x$, $V_{12} = \int \psi_1^+ U \psi_2 d^3x$, operators $U^\pm$ are defined in [3].

Let's note only, that the operator $U^+$ is proportional to the operator of birth, and the operator $U^-$ is proportional to the operator of destruction. The operator $U$ defines potential of external periodic perturbation.

We shall consider that the transitions between levels one and two are caused by induce processes and occur during the times which are considerably smaller times of spontaneous transitions. In this case last two equations can be considered independently from first equation. Besides, taking into account, that transitions between two top levels are induce, the matrix elements of transitions upwards and downwards are equal to each other ($V_{12} = V_{21}$). In this case solution for amplitudes of wave functions will look like:

$$A_1 = \cos\left(\Omega\cdot t\right),\quad A_2 = -i\sin\left(\Omega\cdot t\right), \qquad (17)$$

where $\Omega = V_{12}/\hbar$ – Rabi frequency.

For the analysis of efficiency of spontaneous transitions we should use the first equation of system (16). At this in the usual theory of perturbation instead function $A_1$ substitute a constant which is equal to unit. It corresponds to that fact, that at the initial moment of time the system is in the excited condition at the first level. On logic of things, we should substitute in this equation the solution (17). If we shall act thus, the result of stabilization will be absent, only the spectra of spontaneous radiation will slightly change. Actually it is necessary to take into account that the investigated system at a level one exists only during the limited intervals of time, which size is order $\Delta\tau \sim \hbar/V_{12}$. During each of these intervals the system can spontaneously pass to a zero level. The probability of such transition will be proportional to a square of size of this interval of time. Taking into account, that each of such transitions is casual process, and also that fact, that all these casual transitions are independent from each other, we shall receive result, which is described in introduction. Thus, it is possible to expect, that when frequency of transitions between the first and second levels (the Rabi frequency) will be much greater, than return time of life of the excited state one, the process synchrotron radiation will be put down. It is interesting to estimate size and characteristics of external perturbation which can result in such suppression. It is known (see, for example, [3]), that the time of life of the excited state can be estimated by the formula:

$$\Delta t = \frac{\hbar\cdot R}{r_0\cdot mc^2\cdot\gamma}, \qquad (18)$$

where $r_0$ – classical radius of electron; $R$ – radius of electron orbit in synchrotron.

If as an example we shall take $R = 100$ cm, $E = mc^2\cdot\gamma = 500$ MeV the time of life will be about $10^{-9}$ sec. Thus, for observation of suppression of synchrotron radiation it is necessary, that the Rabi frequency for transitions between the first and second levels was much more, than $10^9$.

Let for definiteness she is equal $10^{10}$. Then, those conclusions of the theory, which was used by us above (theory of perturbation) it is necessary, that the frequency of perturbation was greater, than $10^{11}$. It is clear also, that the large Rabi frequency and higher the frequency perturbation the more appreciable will be effect of suppression of synchrotron radiation.

## CONCLUSIONS

Thus, it is possible to formulate the following recommendations for stabilization of the excited quantum systems. First of all, it is necessary to know time of life

of this excited state. Further it is necessary to pick up the appropriate energy levels located not too far from an excited state. Further it is necessary to pick up perturbation which frequency will correspond to transitions between the excited state and this additional level. The intensity of this perturbation should be such, that the appropriate Rabi frequency was as greater as possible in comparison with return time of life of the excited system. It is necessary to say, that such stabilization of quantum systems is similar to stabilization of classical systems. This mechanism is similar to those which appear in movements gyroscope and in the Kapitsa turned pendulum. For this reason it is possible to name offered effect of stabilization as effect of a quantum gyroscope (whirligig).

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## СТАБИЛИЗАЦИЯ ВОЗБУЖДЕННЫХ СОСТОЯНИЙ

***В.А. Буц***

Предлагается механизм, позволяющий стабилизировать возбужденные состояния квантовых и классических систем. Механизм стабилизации аналогичен квантовому эффекту Зенона. Отличие заключается в том, что над системой не производится наблюдения. Вместо этого система периодически переводится в другое квантовое состояние. Определена необходимая для стабилизации скорость стабилизирующих переходов.

## СТАБІЛІЗАЦІЯ ЗБУДЖЕНИХ СТАНІВ

***В.О. Буц***

Пропонується механізм, який дозволяє стабілізувати збуджені стани квантових та класичних систем. Механізм стабілізації аналогічний до квантового ефекту Зенона. Відмінність полягає у тому, що над системою не наглядають. Замість цього система періодично переводиться в інший квантовий стан. Знайдена необхідна для стабілізації швидкість переходів, які стабілізують систему.