ONCE MORE ABOUT CAUCHY PROBLEM FOR EVOLUTION EQUATION

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1. Introduction. The Cauchy problem for linear evolution equations and systems of equation of the arbitrary order have been a subject of a practically infinite number of papers. They contain important different information as about the uniqness and vell-posedness of Cauchy problem for equations of different structure and type as about gualitative properties of the solutions of this problem.

The talk mainly concentrates on the discussion of certain concrete questions about this exstensive now sufficiently traditional, but still attractive region of investigation. We consider:

1. The classical Cauchy problem for parabolic equations form

$$\rho(t,x)\,\partial_t u = a_{ij}(t,x)\,\partial_{x_i}\partial_{x_j}u + b_i(t,x)\,\partial_{x_i}u + c(t,x)\,u, \tag{1}$$

$$(t,x) \in S_T = (0,T] \times \mathbb{R}^n, u|_{t=0} = u_0(x), x \in \mathbb{R}^n. \tag{2}$$

2. The generalized $\left(u(t,x) \in W_{2,t,x,loc}^{0,1}(S_T)\right)$ Cauchy problem (2) for parabolic equations with divergent stractur form

$$\rho(x)\,\partial_t u = \partial_{x_i}\left(a_{ij}(t,x)\,\partial_{x_j}u + a_i(t,x)\,u\right) + b_i(t,x)\,\partial_{x_i}u + c(t,x)\,u \tag{3}$$

3. The weak $(u(t,x) \in L_{1,loc}(S_T))$ Cauchy problem (2) for systems form

$$\partial_t(\rho(t,x)\,u) = \partial_{x_i}\partial_{x_j}(a_{ij}(t,x)\,u) - \partial_{x_i}(b_i(t,x)\,u) + c(t,x)\,u, \tag{4}$$

where $\rho(t,x)$, $a_{ij}(t,x)$, $i,j = 1,...,n$, $b_i(t,x)$, $i = 1,...,n$, $c(t,x)$- are square matrix - functions of the order N.

Everywhere repeated indices mean summation over these indices.

The satisfaction of the initial data in generalized Cauchy problem (3)-(2) and weak Cauchy problem (4)-(2) is understood in the natural topology.

We study how the behaviour of the function ρ for large x influences the uniqueness classes of the problems (1)-(2) and (3)-(2) the behaviour of the matrix ρ for large x influences the triviality classes of weak positive solutions of the systems (4), which are constructed by initial zero data. Let us remind, that some of the class functions is called trivial, if it does not contain functions, which differ from almost equal zero everywhere.

2. Energy estimations of the solutions of parabolic equations with divergent structure and their applications. We investigate the generalized solutions $u(t,x)$ from $W_{2,t,x,loc}^{0,1}(S_T)$ equation (3). The following assumptions will be made:

(H_1) The coefficients of equation (3) are measurable in $\bar{S}_T$ functions

$$\exists\mu > 1 : \mu^{-1}\xi^2 \leq a_{ij}(t,x)\,\xi_i\xi_j \leq \mu\xi^2, \forall\xi \in \mathbb{R}^n, \forall(t,x) \in S_T, \tag{5}$$

$$\xi^2 = \xi_1^2 + ... + \xi_n^2, a_{ij}(t,x) = a_{ji}(t,x), \forall(t,x) \in \bar{S}_T.$$

For some q, $q \in (-\infty, \infty)$, there exists positive constant C_1 and $M \geq 1$ such that

$$\rho(t,x) \geq C_1 \left(1+x^2\right)^{-1+q/2}, \tag{6}$$

$$|a_i(t,x)|, |b_i(t,x)| \leq M \left(1+x^2\right)^{(q-1)/2}, c(t,x) \leq M \left(1+x^2\right)^{q-1}. \tag{7}$$

The obtaining of the energy estimations is fulfilled in two steps. On first original equation is transformed so that the new equation in the sufficiently narrow layer, appears to be dissipative. Then for the solutions of the transformed equation it is established the apriory estimation of the energy form.

We introduce the function:

$$H(t,x) = \frac{-C_1 \left(1+x^2\right)^{q/2}}{\chi\left[2(s-\eta)-(t-\eta)\right]}, 0 < \eta < t \leq s \leq T, \quad \chi > 0. \tag{8}$$

LEMMA 1. *The condition* H_1 *is hold,* $u(t,x)$ *is the generalised solution of the equations* (3) *in* S_T.
Then function

$$v(t,s) = u(t,x) \exp H(t,x)$$

satisfies to the equation

$$\rho(x)\,\partial_t v = \partial_{x_i}\left(a_{ij}(t,x)\,\partial_{x_j} v + A_i(t,x)\,v\right) + B_i(t,x)\,\partial_{x_i} v + C(t,x)\,v \tag{9}$$

in $S_{(t,s]} = (t,s] \times R^n$. *And also if*

$$\chi \geq \max\left(80n\mu^2 M^2 q^2,\ (3\mu+1)\, 2^{\left|\frac{q}{2}-1\right|} q^2\right), \quad s-\eta \leq \frac{C_1 |q|}{\chi}, \tag{10}$$

then the coefficients $A_i(t,x)$, $B_i(t,x)$, $i = 1,...,n$, $C(t,x)$ *satisfy to the inequality*

$$2\mu \sum_{i=1}^{n} \left(B_i(t,x) - A_i(t,x)\right)^2 + C(t,x) + 2\sum_{i=1}^{n} A_i^2(t,x) \leq 0 \tag{11}$$

The proof of the lemma consists of the calculation of the new coefficients A_i, B_i, $i = 1,...,n$, C and of the proof of the correctness of the basic inequalites (11) (by means of (5)-(7)).

We pass to the obtaining of the energy estimates of the generalized solutions $v(t,x)$ of the equation (9).

Let $\xi(x,R)$ is the function, which has the following properties: $\xi(x,R) \in C_0^\infty(\mathbb{R}^n)$, $0 \leq \xi(x,R) \leq 1$, $|\xi_x(x,R)| \leq 1$, $\xi(x,R) = 1$ at $|x| \leq R$, $\xi(x,R) = 0$ at $|x| \geq R+2$.

Obtaining of the energy estimations starting with notation of the integral identity for solutions of equation (9) in which the test function is defined in the form

$$v(t,x)\,\xi(x,R)\,Q(t,x), Q(t,x) = \exp\{2H(t,x)\}.$$

Consequent transformation and estimations result in the following statment:

THEOREM 1. *Let coefficients of equation* (9) *satisfy to the condition* (5), (6), (11) *in* $S_{(\eta,s]}$ *and fulfill to the inequality* (10). *Then for any generalized solution* $v(t,x)$ *of the equation* (9) *there is valid apriori estimation*

$$1/2 \int_{\mathbb{R}^n} \rho v^2 \xi^2 Q dx\Big|_{t_1}^{t_2} + \frac{5}{24\mu} \int_{t_1}^{t_2} dt \int_{\mathbb{R}^n} \sum_{i=1}^{n} \left(\partial_{x_i} v\right)^2 \xi^2 Q dx$$

$$\leq (3\mu+1) \int_{t_1}^{t_2} dt \int_{\mathbb{R}^n} v^2 \sum_{i=1}^{n} \left(\partial_{x_i} \xi\right)^2 Q dx, \eta \leq t_1 \leq t_2 \leq s. \tag{12}$$

The immediate sequence of the theorem 1 is

THEOREM 2. *Let coefficients of the equation* (3) *satisfy to the condition* H_1 *and parametrs of the function* $H(t,x)$ — *to the condition* (10). *Then in any layer* $\{0 \le \eta < t \le s \le T\} \times R^n$ *for generalized solutions of the equation* (3) *an apriori estimation*

$$1/2 \int\limits_{\mathbb{R}^n} \rho u^2 \xi^2 Q^2 dx|_{t_1}^{t_2} \le (3\mu + 1) \int\limits_{t_1}^{t_2} dt \int\limits_{\mathbb{R}^n} u^2 \sum_{i=1}^{n} (\partial_{x_i}\xi)^2 Q^2 dx, \tag{13}$$

$$\eta \le t_1 < t_2 \le s$$

is valid.

Apriori estimations (13) is an inequality of the type integral maximum principle. From it we obtain this principle by limiting transition under furtner supposition about behavior of the $u(t,x)$ at $|x| \to \infty$.

Let us pass to the consideration of the uniqueness theorem of the problem (3)-(2).

We introduce Hilbert space $L_2(S_T; q, \lambda)$, q and λ some real numbers, function, which are determined in S_T for which norm

$$\|w; q, \lambda\| = \int\limits_0^T dt \int\limits_{\mathbb{R}^n} |w(t,x)|^2 \exp\{-2\lambda |x|^q\} dx$$

is finite.

THEOREM 3. *Let condition* H_1 *is valid. Then the problem* (3)–(2) *has:*

1. (a) *At* $q > 0$- *uniqueness solution* $u(t,x) \in L_2(S_T; q, \lambda)$ *at any positive* λ*;*
   (b) *At* $q \le 0$- *uniqueness solution* $u(t,x) \in L_2(S_T; q, 0)$.

Proof of the theorem 3 is based on the inequality (13). From (13) with the suppostions a) and b), and properties of the function $\xi(x,R)$ it follows that (at $R \to \infty$) integral maximum principle

$$\int\limits_{\mathbb{R}^n} \rho u^2(t_2,x) \exp\{-2\lambda |x|^2\} dx \le (3\mu + 1) \times \int\limits_{\mathbb{R}^n} \rho(x) u^2(t_1,x) \exp\{-2\lambda x^2\} dx, \tag{14}$$

$\eta \le t_1 < t_2 < s$, s satisfyes to the inequality (10) and at $q > 0$, $s < \frac{2C_1}{\lambda}$.

Sufficiently it is enough to prove, that from $u_0(x) \equiv 0$ and $u(t,x) \in L_2(S_T; q, \lambda)$ from $q > 0$, $\lambda > 0$, $u(t,x) \in L_2(S_T; q, 0)$ from $q < 0$ it follows that $u(t,x)$ is equal to zero almost everywhere in S_T. From inequality it follows that if $u_0(x) \equiv 0$, then $u(t,x) \equiv 0$, $\forall t$, $0 < t \le s$. If $s < T$ then with sames the argument as above in any layers $s \le t \le 2s$, $2s \le t \le 3s$ i.e.

In the case $q = 0$ theorem 3 can be changed to a stronger statement.

THEOREM 4. *The following assumptions wille be made:*
($\hat{H}$) The coefficients of equation (3) *are measurable in* $\bar{S}_T$ *functions,*

$$\exists \mu > 1: \ \mu^{-1}\xi^2 \le a_{ij}(t,x)\xi_i\xi_j \le \mu\xi^2, \ \forall \xi \in \mathbb{R}^n, \ \forall (t,x) \in \bar{S}_T.$$

There exists positive constants C_1 *and* $M \ge 1$ *such that*

$$\rho(t,x) \ge C_1 (1 + x^2)^{-1}, \ |a_i(t,x)|, |b_i(t,x)| \le M (1 + x^2)^{-1/2} \ln(3 + x^2),$$

$$c(t,x) \le M (1 + x^2)^{-1} \ln^2(3 + x^2).$$

Then any solutions $u(t,x)$ of the generalized Cauchy problem (3) – (2_0), for which

$$\int_0^T dt \int_{\mathbb{R}^n} |u(t,x)|^2 \exp\left\{-\lambda \ln^2\left(3+x^2\right)\right\} dx < +\infty,$$

with some positive constant λ, is equal to zero almost everywhere in $\bar{S}_T$.

Proof of theorem 4 is carried out according to the above-described scheme with $H((t,x)$ replaced by the function

$$-\frac{C_1 \ln^2\left(3+x^2\right)}{\chi\left[2(s-\eta)-(t-\eta)\right]}.$$

3. The uniqness classes classical Cauchy problem. We investigate classic solutions of the Cauchy problem

$$\rho(t,x)\partial_t u = P(t,x;\partial_x)u \equiv a_{ij}(t,x)\partial_{x_i}\partial_{x_j}u + b_i(t,x)\partial_{x_i}u + c(t,x)u, \quad (15)$$

$$(t,x) \in S_T = (0,T] \times R^n, u|_{t=0} = u_0(x), \qquad x \in R^n. \quad (16)$$

The problem (15), (16) with zero initial data will be denoted by (15), (16_0).

The following assumption will be made:

(H_2): The coefficients of equation (15) are continuons functions in $\overline{S}_T = [0,T] \times R^n$,

$$a_{ij}(t,x)\xi_i\xi_j > 0, (t,x) \in \overline{S}_T, \forall \xi = (\xi_1, ...\xi_n) \in R^n \setminus \{0\},$$

$$a_{ij}(t,x) \in L^\infty(\overline{S}_T), \rho(t,x) > 0 \text{ in } \overline{S}_T.$$

We describe the classes of equations of form (15) for which uniqueness theorems of the Cauchy problem are established.

We consider pairs of positive functions $A(r)$ and $B(r)$ which are defined on $[2,\infty)$ and suppose that $B(r)$ has the following properties:

α_1. $B(r) > 1, |B(r)| \geq MA(r)^2 r^{-2}, B(r) \uparrow \infty$ as $r \to \infty$,

α_2. $B(r)$ has two derivatives and satisfies the following estimates:

$$|B^{(l)}(r)| \leq MB(r)^{l/2} A(r)^{-l}, l = 1,2$$

We bring two concrete examples of classes of functions satisfying these conditions.

EXAMPLE 1.

$$A_{pq}(r) = Ar^{1-q/2}(\ln r)^{-p/2},$$

$$B_{pq}(r) = Br^q (\ln r)^p, q > 0, p \geq 0.$$

EXAMPLE 2.

$$A_p(r) = Ar(\ln r)^{p/2},$$

$$B_p(r) = B(\ln r)^{2-p}, p \in [0,1].$$

DEFINITION 1. Equation (15) belongs to the class (A,B) if its coefficients satisfy the conditions:

$$\rho(t,x) \geq MA(r)^{-2}, \qquad r^2 = x_1^2 + \ldots + x_n^2 + 2;$$

$$b_i(t,x)x_i \leq Mr(B(r))^{1/2} A(r)^{-1},$$

$$c(t,x) \leq MB(r)A(r)^{-2} \text{ in } \overline{S}_T.$$

The main result is the following

THEOREM 5. *Assume that equation* (15) *belongs to the class* (A, B) *and that conditions* H *hold. Then any solution* $u(t,x)$ *of the problem* (15), (16_0) *for which*

$$|u(t,x)| \le K \exp\{kB(r)\}$$

with some positive constants K *and* k, *is identically equal to zero in* $\overline{S}_T$.

The proof uses the maximum principle for solutions of eqaution (15) and the fact that the function

$$F(t,x) = \exp\{EB(r)(1+Qt)\}$$

for equations of the class (A, B) is a supersolution of equation (15) for any $E > 0$ and a suitable value of the positive constant $Q = Q(E)$, i.e. $\rho \partial_t F - P(t,x;\partial_t) F \ge 0$ in $\overline{S}_T$.

Now we give some corollaries from theorem 5 for concrete functions $A(r)$ and $B(r)$ from the examples 1 and 2.

COROLLARY 1. *Let conditions* H_2 *hold and assume that for some* $p, q, q > 0, p \ge 0$,

$$\rho(t,x) \ge M r^{q-2} (\ln r)^p,$$

$$b_i(t,x) x_i \le M r^q (\ln r)^p,$$

$$c(t,x) \le M r^{2q-2} (\ln r)^{2p}.$$

Then any solution $u(t,x)$ *of the problem (15),* (16_0)*, such that*

$$|u(t,x)| \le K \exp\{k r^q (\ln r)^p\}$$

with some positive constants K *and* k, *is identically equal to zero in* S_T.

COROLLARY 2. *Let condition* H_2 *hold and assume that for some* $p \in [0,1]$

$$\rho(t,x) \ge M r^{-2} (\ln r)^{-p},$$

$$b_i(t,x) x_i \le M (\ln r)^{1-p},$$

$$c(t,x) \le M r^{-2} (\ln r)^{2(1-p)}.$$

Then any solution $u(t,x)$ *of the Cauchy problem* (15), (16_0), *such that*

$$|u(t,x)| \le K \exp\left\{k (\ln r)^{(2-p)}\right\}$$

in $\overline{S}_T$ *with some positive constants* K *and* k, *is identically equal to zero in* $\overline{S}_T$.

4. The triviality classes of weak positive solutions problem Cauchy. Now we consider positive solutions $u(t,x)$ for the weak Cauchy problem of the form:

$$\begin{aligned} \partial_t(\rho(t,x)u) &= P^*(t,x;\partial_x)u \\ &= \partial_{x_i}\partial_{x_j}(a_{ij}(t,x)u) - \partial_{x_i}(b_i(t,x)u) + c(t,x)u, (t,x) \in S_T, \quad (17) \\ u|_{t=0} &= u_0(x), x \in R^n. \quad (18) \end{aligned}$$

Here $u(t,x) = (u_1(t,x), \dots, u_N(t,x))$ is a complex-valued vector-function $\rho(t,x)$, $a_{ij}(t,x)$, $b_i(t,x)$, $c(t,x)$ are square matrix -functions of order N with complex-valued elements defined in $\overline{S}_T$.

By a weak Cauchy problem we mean, as usual, the problem of finding a weak solution of system (17) in S_T satisfying the initial condition (18) in the weak sense.

By positiveness of a solution $u(t,x)$ in S_T we mean that its range of values belongs to a given cone C of the space $\mathbb{C}^N$ for almost all $(t,x) \in S_T$.

To formulate conditions imposed on the coefficients of system (17) we again use the functions $A(r)$ and $B(r)$ supposing, in addition to α_1, α_2, that $B(r)$ has the following properties:
α_3. $|B'(r)|r \geq M$ for sufficeintly large r.
α_4. For some c, $r^{n+1}\exp\{-cB(r)\} \to 0$ as $r \to \infty$.

We remark that if $B(r) \geq M(\ln r)^{1+\varepsilon}$ for some positive ε, then condition α_4 is valid with any positive c.

DEFINITION 2. System (17) belongs to the class (A, B) if its coefficients satisfy the following conditions:
1). The elements of the matrices $\rho(t,x)$, $a_{ij}(t,x)$, $i,j = 1,...,n$, $b_i(t,x)$, $i = 1,...,n$, $c(t,x)$ are measurable locally bounded functions in S_T.
2). $\rho(t,x)$ is a non - singular matrix in $\overline{S}_T$. There exist positive constants M and m such that the norms of the matrices $\rho(t,x)$ and $\rho^{-1}(t,x)$ satisfy the following inequalities:

$$\|\rho(t,x)\| \leq M\exp\{mB(r)\},$$

$$\|\rho(t,x)^{-1}\| \leq MA(r)^2, m < c.$$

3). There exist a vector $f \in \mathbb{C}^N$ and a positive constant δ such that for any $\varphi \in \mathbb{C}^N$ and $(t,x) \in \overline{S}_T$

$$Re(\rho(t,x)f, \varphi) \geq \delta A(r)^{-2}\|\varphi\|,$$

(here $(\cdot,\cdot)$ denotes the scalar product in $\mathbb{C}^N$.)
4) $a_{ij}(t,s) \in L^\infty(S_T)$, $i,j = 1,...,n$.
5). $\|b_i(t,x)\| \leq MB(r)^{1/2}A(r)^{-1}$, $i = 1,...,n$; $\|c(t,x)\| \leq MB(r)A(r)^{-2}$ in S_T.

THEOREM 6. *Let system (17) belong to the class (A, B). Then any positive solution $u(t,x)$ of the weak Cauchy problem (17),(18_0), for which*

$$\int_0^T dt \int_{R^n} \|u(t,x)\|\exp\{-cB(r)\}dx < \infty$$

is equal to zero almost everywhere in S_T.

5. Comments. We would like to make two remarks.

Remark 1. In the previous theorems $\rho(t,x)$ is bounded from below by the radially symmetric power of $|x|$. Let us formulate one of the statements in which the function $\rho(t,x)$ bounded from below by the function which is not radially symmetric.

THEOREM 7. *Let conditions H_2 hold and for some $\{q_i\}, q_i > 0$, $1 \leq i \leq n$*

$$\rho(t,x) \geq M(1 + x_i^2)^{\frac{q_i-2}{2}} \tag{19}$$

$$b_i(t,x)x_i \leq M(1 + x_i^2)^{q_i-1}, \ c(t,x) \leq M(1 + x_i^2)^{q_i-1} \tag{20}$$

Then any solution $u(t,x)$ of the problem (15),(16_0) such that

$$|u(t,x)| < K\exp\{k(1 + x_i^2)^{\frac{q_i}{2}}\}$$

with some positive constants K and k, is identically equal to zero in S_T.

Remark 2. The results obtained in Section 3 do not depend on the dimension. But the number of space variable n plays a role if $\rho(t,x)$ tends to zero as $|x| \to \infty$ sufficiently fast. Consider the example

EXAMPLE 3. Equation

$$|x|^{-2+q}\partial_t u = \Delta u \equiv \partial_{x_1}^2 u + \cdots + \partial_{x_n}^2 u, q < 0$$

has a self-similar solution

$$w(t,x) \equiv f(rt^{-1/q}) = \int_0^{rt^{-1/q}} exp\left\{-s^q q^{-2}\right\} s^{1-n} ds \ , r = |x|.$$

This solution is positive for any $t > 0$, $w(0,x) = 0$,

$$\lim_{|x|\to\infty} \frac{w(t,x)}{|x|} = t^{-1/q} \text{ for } n = 1,$$

$$\lim_{|x|\to\infty} \frac{w(t,x)}{\ln|x|} = 1 \text{ for } n = 2 \text{ and } t > 0,$$

$w(t,x)$ is a bounded function for any $n \geq 3$.

Let's consider now classical solutions of the equation

$$\rho(t,x)\partial_t u = \Delta u \ , \tag{21}$$

assuming that $\rho(t,x) \leq M\left(1+x^2\right)^{(q-2)/2}$ with some $q < 0$.

With the help of the function $w(t,x)$ it is easy to construct a nontrivial solution of the problem (21), (16_0), possessing the properties:

1) for $n = 1$ growing as $|x|$ with $|x| \to \infty$,
2) for $n = 2$ growing as $\ln|x|$ with $|x| \to \infty$,
3) bounded for any $n \geq 3$.

This example shows that if ρ tends to zero as $|x|^{q-2}, q < 0$ then in order to ensure the uniqueness of the Cauchy problem one has to prescribe some additional conditions for $|x|$ large. It was shown in [8],[9],[10] that for such ρ the Cauchy problem for equation (21) for $n \geq 3$ is well posed in the class of the function which tends to zero in some sense as $|x| \to \infty$. Below we present two results for one and two dimensional case.

Case n= 1. We study the Cauchy problem for the equation

$$\rho(t,x)\partial_t u = a(t,x)\partial_x^2 u + b(t,x)\partial_x u + c(t,x)u. \tag{22}$$

The following assertion is valid.

THEOREM 8. *Assume that* $|b(t,x)| \leq M\rho(t,x)(|x|+1)|$, $c(t,x) \leq M\rho(t,x)$ *in* $\overline{S}_T$. *Then any solution of the problem* (21), (18_0) *such that*

$$\lim_{|x|\to\infty}\left(\max_{t\in[0,T]} |u\left(t,x|/|x|\right)\right) = 0$$

is identically in $\overline{S}_T$ *equal to zero.*

Case n=2. Let us consider the equation

$$\rho(t,x)\partial_t u = \Delta u + b_i(t,x)\partial_{x_i} u + c(t,x)u, i = (1,2). \tag{23}$$

THEOREM 9. *If conditions* H, *holds and* $(b_i(t,x)x_i) \leq M(1+x^2)\ln(2+x^2)\rho(t,x)$, $(c(t,x)) \leq M\rho(t,x)$ *in* $\overline{S}_T$, *then any solution of the problem* (23), (18_0) *such that*

$$\lim_{|x|\to\infty}\left(\max_{t\in[0,T]} |u(t,x)|/\ln|x|\right) = 0$$

is identically in $\overline{S}_T$ *equal to zero.*

An extensive bibliography is devoted to the investigation of uniqueness classes of the Cauchy problem for parabolic equations and systems (see, for instance, [1-6], [8-12]). We note that parabolic equations degenerating at infinity appear in applications (see, for instance [8,9]). Propositions similar to theorem 6 are often called theorems of triviality . They contain conditions ensuring that some class of functions has no solution except for a trivial one.

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