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# HIGHER ASYMPTOTIC APPROXIMATIONS FOR NONLINEAR INTERNAL WAVES IN FLUIDS

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ABSTRACT. Nonlinear problems of wave-packet propagation along the interface between the two fluids of different densities with taking into account the surface tension are investigated. Two problems are considered, the one for two half-spaces, the another for the layer over a half-space. Asymptotic solutions are developed on the basis of the method of multiple scale expansions. Unlike previous investigations dealing with only three approximations in this paper four asymptotic approximations have been developed by using symbolic algebra. The evolution equations are obtained in the form of the nonlinear higher-order Schrödinger equations. The stability of solutions is investigated. As a result, the new region of stability for capillary waves and the new region of instability for gravity waves have been discovered in the case of the layer of finite thickness unlike the case of two fluid half-spaces.

## INTRODUCTION

Investigations of nonlinear and internal waves propagating in fluids lead to solving strongly nonlinear boundary value problems. Therefore the exact analytical solutions have been obtained only for a few particular problems. As a result asymptotic approaches have been extensively developed and they are of dominating tools to provide qualitative and quantative analysis. Nevertheless it is well-known that with increasing order of approximations the difficulties to obtain results catastrofically rise. That is why overwhelming majority of considerations are in the framework of classical evolution equations of KdV, Schrödinger type and others. At the same time, the analysis of contribution of higher harmonics to wave evolution is the problem of great interest due to the essential influence of higher harmonics on stability and existence of finite amplitude waves. One of the typical urgent example is a formation of freak waves in ocean.

This paper treats gravity waves on the interface between two infinite fluids which are assumed to be incompressible, inviscid and irrotational. The surface tension is taken into account unlike most previous investigations. The problem is analyzed on the basis of the method of multiple scale expansions. Basic analysis of the wave propagation on fluid interface between two semi-infinite fluids with taking into account the surface tension has been presented by Nayfeh[1]. The extension of this problem to the case of two-fluid system consisting of upper layer over lower semi-infinite fluid has been obtained by Avramenko and Selezov[2].

Investigations of interfacial waves in two-fluid systems have been in a focus of many researchers for a long time. Among of them it is necessary to select the problems taking into account the surface tension which essentially influences capillary-gravitational waves

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when the effects of high harmonics are of great importance. At the same time, recently many investigations have been devoted to formation of freak waves including both its evolution and stability [3], [4], [5]. It demands to consider the influence of high harmonics what essentially complicates the analysis.

In the article [6] the two-fluid system with both upper and lower fluids of finite depth was considered on the basis of original Euler equations in the assumptions that the layer thickness is small relatively to horizontal scale (wavelength), but the wave amplitude is considered to be constant. The solutions are found by expansions in small thickness parameter.

Nonlinear waves on the interface between two infinite fluids of different densities have been investigated in [7] on the basis of expansion of desired functions to Fourier series.

The extreme characteristics of waves were investigated including the case of small density of the upper fluid [8]. Propagation of interfacial solitary waves in two-fluid medium confined by upper and lower rigid planes without taking into account the surface tension has been studied. The solutions was constructed in the form of a power series expansions in terms of a parameter which depends on the reciprocal of Froude number.

Kakutani and Yamasaki [9] have investigated internal waves by the reductive perturbation method and obtained KdV equation instead of Schrödinger equation.

Recently, experimental investigations have been conducted by Huq for the three-layer fluid with a thin intermediate layer and in the presence of the uniform ambient current and the discharge source located within the intermediate layer. As a result, the generation of solitary waves in such a system has been discovered experimentally by Huq [10]. It has been shown that there are such parameters when the solitary waves are originated on the interface and then they propagate to the incident ambient flow. Initial analysis of the wave propagation in such a system [11] was based on the Stokes' type expansion leading to nonlinear Schrödinger equation.

The interfacial nonlinear waves have been investigated also in articles [12], [13], [14]. The analysis presented in this paper developes and extends the previous works [15], [16], [17].

The case of wave packets propagation along the interface of the fluid half-space and the fluid layer situated above was studied in [2], where the problem of the wave-packets stability is discussed. For the solution of the nonlinear boundary value problems the method of multiple scales expansions was used.

Here we consider the fourth-order problem of wave-packet propagation at the interface between two fluids. On this basis the new evolution equation in the form of the nonlinear third order Schrödinger equation has been obtained with a compact form of nonlinear part. The stability of the solutions is investigated in detail.

## 1. The system "layer – half-space"

The mathematical statement of the problem for wave-packet propagation along the interface between the upper layer and a half-space is presented as nonlinear boundary value problem with kinematic and dynamic conditions at the interface for desirable functions $\phi_j(x,z,t)$ and $\eta(x,t)$

$$\nabla^2\varphi_j = 0 \quad \text{in} \quad \Omega_j, \tag{1}$$

$$\eta_{,t} - \varphi_{j,z} = -\varphi_{j,x}\eta_{,x} \quad \text{at} \quad z=\eta(x,t), \tag{2}$$

$$\begin{gathered}\varphi_{1,t} - \rho\varphi_{2,t} + (1-\rho)\eta + 0.5(\nabla\varphi_1)^2 - 0.5\rho(\nabla\varphi_2)^2 - \\ -T\left(1+\eta_{,x}^2\right)^{-3/2}\eta_{,xx} = 0 \quad \text{at} \quad z=\eta(x,t),\end{gathered} \tag{3}$$

$$|\nabla\varphi_1| \to 0 \quad \text{as} \quad z\to-\infty, \tag{4}$$

$$\varphi_{2,z} = 0 \quad \text{at} \quad z=h, \tag{5}$$

where $\varphi_j$ $(j=1,2)$ are the velocity potentials; $\eta$ is the interface elevation; $\Omega_1 = \{(x,y,z),$ $-\infty<x<\infty,\ -\infty<y<\infty,\ z<0\}$ and $\Omega_2=\{(x,y,z),\ -\infty<x<\infty,\ -\infty<y<\infty,\ z>0\}$,

$\rho = \rho_2/\rho_1$. Dimensionless values are introdused using the characteristic length $L$, characteristic time $(L/g)^{1/2}$, density of the lower fluid $\rho_1$, where $g$ is the acceleration of the gravity. Introdusing the characteristic length $L$ leads to the dimensionless surface tension $T^* = T/(L^2 \rho g)$. The asterisk is dropped.

The approximate solution of the nonlinear problem (1)-(4) is determined using the method of multiple scale expansions for the fourth-order approximation

$$\eta(x,t) = \sum_{n=1}^{4} \varepsilon^n \eta_n(x_0, x_1, x_2, x_3, t_0, t_1, t_2, t_3) + O(\varepsilon^5), \tag{6}$$

$$\varphi_j(x,z,t) = \sum_{n=1}^{4} \varepsilon^n \varphi_{jn}(x_0, x_1, x_2, x_3, z, t_0, t_1, t_2, t_3) + O(\varepsilon^5), \quad (j = 1,2) \tag{7}$$

where $\varepsilon$ is small dimensionless parameter characterizing the steepness ratio of the wave, $x_n = \varepsilon^n x$, $t_n = \varepsilon^n t$.

The construction of the fourth-order approximation is connected with a huge combersome algebra. Here at first these difficulties were overcome using symbolic computations in software package.

Substituting (6) and (7) into (1)-(5) and equating coefficients of like powers of $\varepsilon$ yelds to four linear problems. The first, second and third-order problems were formulated, and also the solutions of the first- and second-order problems and solvability conditions of the second- and third-order ones were obtained in [2] for another dimensionless parameters, where the dimensionless surface tension was taken $T = 1$.

The results taking into account the dimensionless parameters above introduced are of the form:
the dispersion relationship

$$\omega^2 = (1 + \rho \mathrm{cth} kh)^{-1} k (1 - \rho + T k^2), \tag{8}$$

the solutions of the first-order problem

$$\eta_1 = A \exp i\theta - \bar{A} \exp(-i\theta), \tag{9}$$

$$\phi_{11} = -i\omega k^{-1} (A \exp i\theta + \bar{A} \exp(-i\theta)) \exp kz, \tag{10}$$

$$\phi_{21} = i\omega k^{-1} (A \exp i\theta + \bar{A} \exp(-i\theta)) \mathrm{ch} k(h - z) \mathrm{sh}^{-1} kh, \tag{11}$$

where $\overline{A}(x_1, x_2, x_3, t_1, t_2, t_3)$ is the complex conjugate of the complex envelope $A(x_1, x_2, x_3, t_1, t_2, t_3)$, $\theta = kx_0 - \omega t_0$, $k$ is the wave number and $\omega$ is the wave frequency of the center of the wave-packets.

For each of the three next approximations the solvability conditions have been obtained using a software package

$$A_{,t_1} + \omega' A_{,x_1} = 0, \tag{12}$$

$$A_{,t_2} + \omega' A_{,x_2} - 0.5 i\omega'' A_{,x_1 x_1} = -ik\omega^{-1} (1 + \rho \mathrm{cth} kh)^{-1} I A^2 \overline{A}, \tag{13}$$

$$\begin{aligned} &A_{,t_3} + \omega' A_{,x_3} - i\omega'' A_{,x_1 x_2} - \omega'''/6 \cdot A_{,x_1 x_1 x_1} = \\ &= k\omega^{-1} (1 + \rho \mathrm{cth} kh)^{-1} [J A \overline{A} A_{,x_1} - I (k\omega^{-1})' (A^2 \overline{A})_{,x_1}], \end{aligned} \tag{14}$$

where $\omega' = d\omega/dk$, $\omega'' = d^2\omega/dk^2$ $\omega''' = d^3\omega/dk^3$

$$I = k^2 \frac{a_3 T^3 k^6 + a_2 T^2 k^4 + a_1 T k^2 + a_0}{b_1 T k^2 + b_0}, \tag{15}$$

and

$$a_3 = 2\rho^2 \mathrm{sh}2kh + 2\rho^3 \mathrm{sh}^2 kh + 6\rho^3,$$

$$\begin{aligned} a_2 = &-17.5\rho^2(1-\rho)\mathrm{sh}2kh - 2\rho(5\rho^2 - 8\rho + 3)\mathrm{sh}^2 kh \mathrm{sh}2kh + \\ &+(\rho^3 + 3\rho^2 - 5\rho + 1)\mathrm{sh}^4 kh \mathrm{sh}2kh - 2\rho(\rho^3 - 5\rho^2 + 3\rho + 1)\mathrm{sh}^6 kh - \\ &-\rho(19\rho^3 - 39\rho^2 + 25\rho + 5)\mathrm{sh}^4 kh - \\ &-4\rho(11\rho^3 - 14\rho^2 + 5\rho - 2)\mathrm{sh}^2 kh - 33\rho^3(1-\rho) \end{aligned}$$

$$\begin{aligned} a_1 = &\,17\rho^2(1-\rho)\mathrm{sh}2kh + \\ &+\rho(0.5\rho^3 - 13\rho^2 + 24.5\rho - 12)\mathrm{sh}^2 kh \mathrm{sh}2kh + \\ &+(-6.5\rho^4 + 5\rho^3 + 10\rho^2 - 9\rho + 0.5)\mathrm{sh}^4 kh \mathrm{sh}2kh + \\ &+\rho(\rho^4 - 18\rho^3 + 20\rho^2 + 10\rho - 13)\mathrm{sh}^6 kh + \\ &+\rho(17\rho^4 - 74\rho^3 + 83\rho^2 - 12\rho - 14)\mathrm{sh}^4 kh + \\ &+2\rho(23\rho^4 - 58\rho^3 + 49\rho^2 - 32\rho + 4)\mathrm{sh}^2 kh + 36\rho^3(1-\rho)^2 \end{aligned}$$

$$\begin{aligned} a_0 = &-1.5\rho^2(1-\rho)^3\mathrm{sh}2kh - \\ &-2\rho(2\rho^4 - 9\rho^3 + 15\rho^2 - 11\rho + 3)\mathrm{sh}^2 kh \mathrm{sh}2kh + \\ &+(-8\rho^5 + 28\rho^4 - 40\rho^3 + 32\rho^2 - 16\rho + 4)\mathrm{sh}^4 kh \mathrm{sh}2kh - \\ &-4\rho(2\rho^5 - 8\rho^4 + 16\rho^3 - 20\rho^2 + 14\rho - 4)\mathrm{sh}^6 kh - \\ &-4\rho(5\rho^5 - 17\rho^4 + 29\rho^3 - 25\rho^2 + 11\rho - 2)\mathrm{sh}^4 kh + \\ &+\rho(-15\rho^5 + 57\rho^4 - 77\rho^3 + 39\rho^2 - 4)\mathrm{sh}^2 kh + 9\rho^3(1-\rho)^3 \end{aligned}$$

$$\begin{aligned} b_1 = &\,\mathrm{sh}^2 kh[16\rho^2(1-\rho)^2\mathrm{sh}2kh + 4(3\rho^3 - 3\rho^2 + \rho - 1)\mathrm{sh}^2 kh \mathrm{sh}2kh + \\ &+4\rho(5\rho^3 - 5\rho^2 + 7\rho - 7)\mathrm{sh}^2 kh - \\ &-8\rho(1-\rho)^3\mathrm{sh}^4 kh - 12\rho^3(1-\rho)] \end{aligned}$$

$$\begin{aligned} b_0 = &\,\mathrm{sh}^2 kh[2\rho^2(1-\rho)^2\mathrm{sh}2kh + 2(4\rho^4 - 6\rho^3 + 4\rho^2 - 2\rho + "1)\mathrm{sh}^2 kh \mathrm{sh}2kh + \\ &+4\rho(\rho^4 - 2\rho^3 + 6\rho^2 - 8\rho + 4)\mathrm{sh}^2 kh + \\ &+4\rho(\rho^4 - 2\rho^3 + 4\rho^2 - 8\rho + 3)\mathrm{sh}^4 kh] \end{aligned}$$

In (15) the effect of the surface tension is presented by the terms containing $T$ in the first, second and third powers.

It can be noted that the coefficients $I$ and $J$ are connected with the expression

$$J = -i\partial I/\partial k. \tag{16}$$

Let's multiply equations (12) - (14) by $\varepsilon$, $\varepsilon^2$ and $\varepsilon^3$, respectively, and then add together all equations of this system. As a result, we obtain the evolution equation in the form of the high-order nonlinear Schrödinger equation

$$\begin{aligned} &A_{,t} + \omega' A_{,x} - i\omega''/2! \cdot A_{,xx} - \omega'''/3! \cdot A_{,xxx} = \\ &= -\varepsilon^2(1 + \rho \mathrm{cth} kh)^{-1}\{ik\omega^{-1}A\overline{A}[IA + I'A_{,x}] + (k\omega^{-1})'I(A^2\overline{A})_{,x}\}, \end{aligned} \tag{17}$$

where $I' = \partial I/\partial k$.

The solution of the equation (17) is of the form

$$A = a \exp\left(-\frac{i\varepsilon^2}{1+\rho\,\mathrm{cth}kh}\cdot\frac{k}{\omega}Ia^2t\right), \tag{18}$$

where $a = \mathrm{const}$.

Investigating the stability of (18) leads to the inequality

$$I\omega'' \le 0, \tag{19}$$

which is numerically analysed in the 3rd part.

## 2. The system "half-space – half-space"

The mathematical statement of the problem for the wave-packet propagation along the interface of two semi-infinite fluids is presented as follows

$$\nabla^2\varphi_j = 0 \quad \text{in} \quad \Omega_j, \tag{20}$$

$$\eta_{,t} - \varphi_{j,z} = -\varphi_{j,x}\eta_{,x} \quad \text{at} \quad z = \eta(x,t), \tag{21}$$

$$\begin{aligned} &\varphi_{1,t} - \rho\varphi_{2,t} + (1-\rho)\eta + 0.5(\nabla\varphi_1)^2 - 0.5\rho(\nabla\varphi_2)^2 - \\ &-T\left(1+\eta_{,x}^2\right)^{-3/2}\eta_{,xx} = 0 \quad \text{at} \quad z = \eta(x,t), \end{aligned} \tag{22}$$

$$|\nabla\varphi_j| \to 0 \quad \text{as} \quad z \to \mp\infty, \tag{23}$$

where $\Omega_1 = \{(x,y,z),\ -\infty < x < \infty,\ -\infty < y < \infty,\ z < 0\}$ and $\Omega_2 = \{(x,y,z),\ -\infty < x < \infty,\ -\infty < y < \infty,\ z > 0\}$. Dimensionless values are introdused like in the 1st part. The approximate solution of the nonlinear problem (20)-(23) was determined using the same method of multiple scale expansions (6), (7).

The dispersion equation and solvability conditions are of the form

$$\omega^2 = (1+\rho)^{-1}k(1-\rho+Tk^2) \tag{24}$$

$$A_{,t_1} + \omega' A_{,x_1} = 0, \tag{25}$$

$$A_{,t_2} + \omega' A_{,x_2} - 0.5i\omega'' A_{,x_1x_1} = -ik\omega^{-1}(1+\rho)^{-1}I_0A^2\overline{A}, \tag{26}$$

$$\begin{aligned} &A_{,t_3} + \omega' A_{,x_3} - i\omega'' A_{,x_1x_2} - \omega'''/6 \cdot A_{,x_1x_1x_1} = \\ &= k\omega^{-1}(1+\rho)^{-1}[J_0A\overline{A}A_{,x_1} - I_0(k\omega^{-1})'(A^2\overline{A})_{,x_1}], \end{aligned} \tag{27}$$

where

$$\begin{aligned} I_0 = k^2[&(1-6\rho+\rho^2)k^4T^2 + 0.5(1-31\rho+31\rho^2-\rho^3)k^2T + \\ &+4(1-2\rho+2\rho^2-2\rho^3+\rho^4)]/[2(1+\rho)^2(1-\rho-2Tk^2)], \end{aligned} \tag{28}$$

and the coefficients $I_0$ and $J_0$ are connected as

$$J_0 = -i\partial I_0/\partial k.$$

The evolution equation is the nonlinear Schrödinger equation

$$\begin{aligned} &A_{,t} + \omega' A_{,x} - i\omega''/2! \cdot A_{,xx} - \omega'''/3! \cdot A_{,xxx} = \\ &= -\varepsilon^2(1+\rho)^{-1}\{ik\omega^{-1}A\overline{A}[IA + I_0' A_{,x}] + (k\omega^{-1})' I_0 (A^2\overline{A})_{,x}\}\ , \end{aligned} \tag{29}$$

where $I_0' = \partial I_0/\partial k$.

Investigating the stability leads to the following stability condition

$$I_0\omega'' \le 0, \tag{30}$$

of the same form as for the system "layer – half-space".

## 3. Numerical calculations and conclusions

Numerical calculations have been carried out to investigate the stability of wave train envelope propagating along the interface of the two fluids: "half-space – half-space" and "layer – half-space". The results of calculations are presented at Figs.1,2 in the form of diagrams "density ratio $\rho = \rho_2/\rho_1$ – wave number $k^* = kL$".

Conclusions:

– surface gravity waves at the limiting case $k \to \infty$ are stable under the condition when the density of upper fluid (layer or half-space) is lesser than the density of lower semi-infinite fluid $\rho < 1$;

– in the case of the layer over semi-infinite fluid for surface gravity waves a new region of instability in the form of loop appears in the neighbourhood of the origin ($k = 0$, $\rho = 0$), while for capillary waves a narrow region of stability appears in the neighbourhood of the point ($k = 0$, $\rho = 0$);

– for the system "layer – half-space" for any fixed layer thickness $h$ the two unstable regions of capillary waves $(\sqrt{2} - 1)^2 < \rho < 1$ and $1 < \rho < (\sqrt{2} + 1)^2$ exist, while for the system "half-space – half-space" these regions combine to one $(\sqrt{2} - 1)^2 < \rho < (\sqrt{2} + 1)^2$;

– in general case the plane $\rho$, $k$ is separated onto the region of linear instability which in turn is separated onto such regions: three regions of nonlinear stability and five regions of nonlinear instability. The stability diagram for value of the surface tension coeffcient $T$ can be obtained by compressing corresponding diagram for $T = 1$ in $T^{1/2}$ time vertically;

– decreasing the layer thickness $h$ essentially narrows down the regions of nonlinear instability and extends the regions of nonlinear stability of capillary-gravitational waves;

– in the case of absence of the apper layer ($\rho = 0$) the surface tension leads to destabilization of waves for nondimensional wave numbers between $0.393 \cdot T^{-1/2}$ $0.707 \cdot T^{-1/2}$.

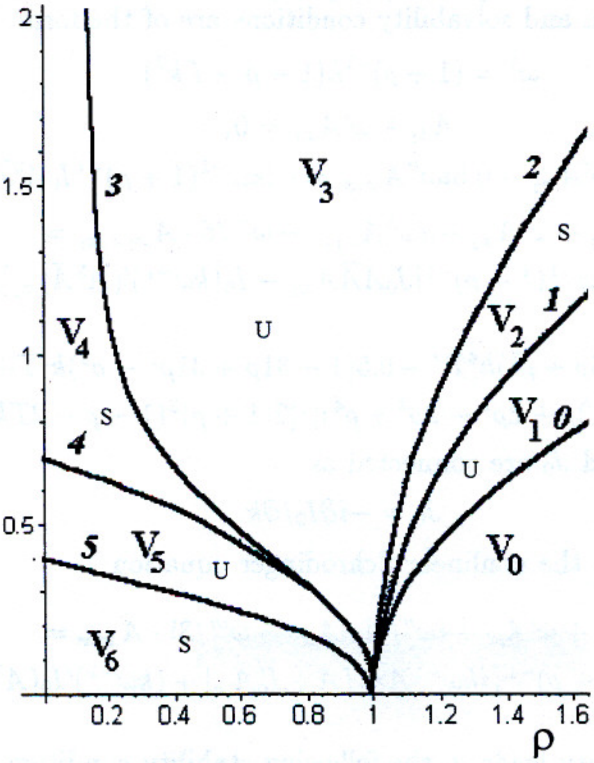


Fig.1 Stability diagram for system "half-space – half-space": nonlinear stable $S$, nonlinear unstable – $U$

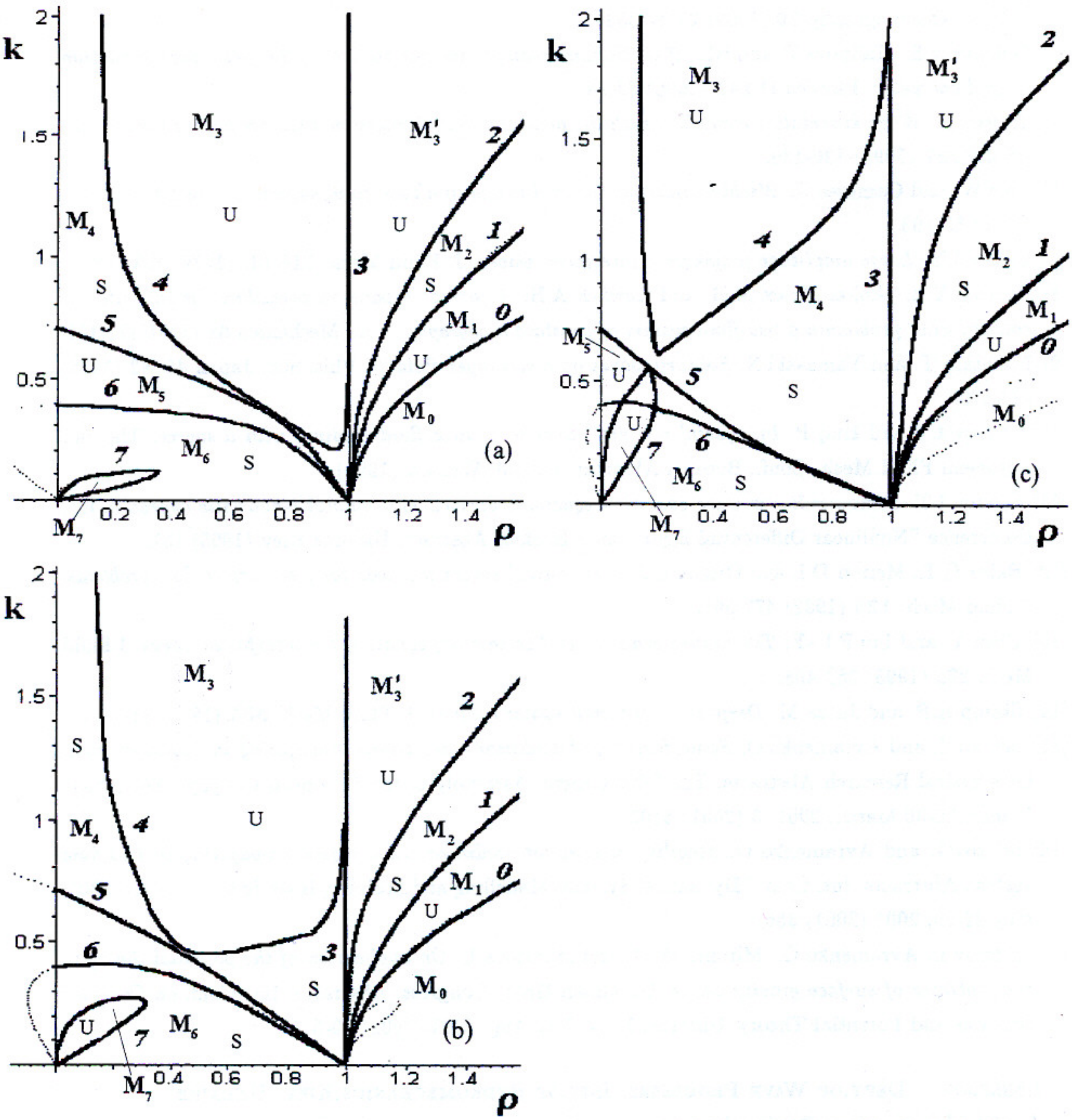


Fig.2 Stability diagram for system "layer – half-space" as $T^* = 1$
nonlinear stable $S$, nonlinear unstable – $U$
(a) $h^* = 5$ (b) $h^* = 2$ (c) $h^* = 1$

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